SELF-SIMILAR COSMOLOGICAL MODELS WITH A COSMICAL CONSTANT

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The presence of the cosmical constant introduces a fundamental scale and prevents there being any simple self-symmetry. Henriksen, Emslie and Wesson (HEW)[1], who studied spherically-symmetric models with a positive cosmical constant, have, however, demonstrated the possible existence of a self-similarity of the second kind and identified the similarity variable. They obtained interesting analytic solutions which are homogeneous in density, but not in pressure. We have extended this work and investigated the general behaviour of these cosmological models which possess a self-similarity of the second kind and in which the requirement of homogeneity is relaxed.

The similarity symmetry allows Einstein's field equations to be reduced to four ordinary differential equations in the similarity variable. It is found that this system of equations admits two general integrals, represented by an inhomogeneity parameter and a parameter C, which gives the constant proper ratio of unit coordinate distances in the radial and transverse directions. All models are spatially closed, possessing a finite proper volume for a fixed value of the time coordinate. Their temporal behaviour depends on the value of the parameter C. When C is less than unity, the models expand monotonically and tend asymptotically to the de Sitter metric. When $C > 1$, the scale factor is bounded and tends to a finite non-zero value, corresponding to an Einstein static solution. We find that some of these models encounter shell-crossing singularities when the scale factor reaches a maximum value, while in others the scale factor increases monotonically - this distinction is investigated elsewhere[2]. No equation of state is imposed in deriving the self-similar models, and it is found that all inhomogeneous solutions fail to meet the dominant energy criterion at early times. They may, however, be patched on to the singular solution derived by HEW, and the continuity conditions across the patch select a unique inhomogeneous solution for a given value of C.

[1] R.N. Henriksen, A.G. Emslie and P.S. Wesson, Phys. Rev. D,27, 1219 (1983)

[2] D. Alexander, R.M. Green and A.G. Emslie (1987) to be submitted to Astron. Astrophys.

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