REPRESENTATIONS OF FINITE ABELIAN GROUPS $C^n_{m,p}$

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1. Introduction. Let $C^n_m = C_m \times C_m \times \ldots \times C_m$ be the finite abelian group of order m^n generated by n elements $w_1, \ldots, w_n$ of order m. Let $\mathbf{C}$ be the field of complex numbers and P a projective representation of G with factor set α over $\mathbf{C}$ (see Morris [**2**]). Further let

$$\mu(w_i) = \prod_{j=1}^{m} \alpha(w_i^j, w_i) \quad (1 \leqslant i \leqslant n)$$

and

$$\alpha'(w_i, w_j) = \alpha(w_i, w_j)\alpha^{-1}(w_j, w_i) \quad (1 \leqslant i, j \leqslant n).$$

Then, it can be easily shown that the factor set α can be chosen in such a way that $\mu(w_i) = 1$, for $i = 1, \ldots, n$ and $\alpha'(w_i, w_j)$ $(1 \leqslant i \leqslant j \leqslant n)$ is an mth root of unity. In the case when m is even and $\alpha'(w_i, w_j) = -1$ $(1 \leqslant i \neq j \leqslant n)$, Morris [**2**] determined the complete set of inequivalent irreducible projective representations of C^n_m with factor set α. His results served as a structure theorem in the study of the projective representations of the generalized symmetric group (see Read [**3**]). This paper deals with the linear and projective representations of the subgroups of C^n_m given by

$$C^n_{m,p} = \left\{ w_1^{a_1} \ldots w_n^{a_n} : \sum_{i=1}^{n} a_i \equiv 0 \pmod{p} \right\}$$

with respect to the restriction of the factor set α to $C^n_{m,p}$, where α satisfies $\alpha'(w_i, w_j) = -1$ $(1 \leqslant i \neq j \leqslant n)$ and $p \mid m$. The only non-trivial factor set which we get by restricting factor sets of $G(m, p, n)$ to $C^n_{m,p}$ is α (see Read [**5**]). This provides a base for the study of projective representations of the finite imprimitive unitary reflection groups (see [**6**] and [**7**]). The case of irreducible linear representations of $C^n_{m,p}$ is simple and has been considered by Read [**4**]. His results are given in Lemma 5.1.

The group $C^n_{m,p}$ may be generated by the following set of generators:

$$\{w_i w_j^{-1}, w_i^p : i, j = 1, \ldots, n\}.$$

In what follows, the restriction of α to $C^n_{m,p}$ is denoted by α itself and $\mathbf{C}^*$ denotes the multiplicative group of non-zero complex numbers. Unless otherwise specified, all the projective representations considered in this paper are taken with respect to the factor set α.

2. We first give a result on representations in subgroups of index 2 which is used in the construction of representations of $C^n_{m,p}$.

Definition 2.1. Let $H \subset G$ be finite groups such that $|G:H| = 2$ and let T be an irreducible projective representation (henceforth written as i.p.r.) of G. We say that T

splits in H if and only if $T = P \dot{+} \bar{P}$ on H, where P and $\bar{P}$ are G-conjugate i.p.r.'s of H and $\dot{+}$ denotes the direct sum of representations.

LEMMA 2.2. *The following are equivalent.*
(i) *An i.p.r. T of G splits in H.*
(ii) $T(x) \simeq (-1)^{\theta(x)} T(x)$ *for all* $x \in G$, *where* $\simeq$ *stands for "is equivalent to" and*

$$\theta(x) = \begin{cases} 0 & \text{if } x \in H, \\ 1 & \text{otherwise.} \end{cases}$$

(iii) *If* χ_T *is the character of* T, $\chi_T(x) = 0$ *for all* $x \in G \backslash H$.

Proof. See Read [**3**, page 122].

3. The irreducible representations of C_m^n.

LEMMA 3.1. *The irreducible linear representations (i.l.r.'s) of* C_m^n *are given by* $\{F_1 \otimes \ldots \otimes F_n\}$ *where* F_i *is an i.l.r. of* $C_m^{(i)}$, *the* i*-th copy of the cyclic group* C_m *of order* m.

Proof. Well-known.

Since all the i.l.r.'s of $C_m^{(i)}$ are of degree 1, $F = F_1 \otimes \ldots \otimes F_n$ is an i.l.r. of degree 1 and may be identified with its character. If F_i is defined by $F_i(w_i) = \xi^{a_i}$ where ξ is a primitive mth root of unity, w_i is a generator of $C_m^{(i)}$ and $a_i \in \{1, \ldots, m\}$ then F may be identified with the linear representation $\theta_{(a_1, \ldots, a_n)}$ of C_m^n defined by

$$\theta_{(a_1, \ldots, a_n)}(w_1^{b_1} \ldots w_n^{b_n}) = \xi^{a_1 b_1 + \ldots + a_n b_n}$$

for all $a_i, b_i \in \{1, \ldots, m\}$, $i = 1, \ldots, n$.

Let

$$\sigma = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}, \quad \rho = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \tau = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where $i = \sqrt{(-1)}$. Then

$$\sigma^2 = \rho^2 = \tau^2 = \varepsilon, \quad \sigma\rho\tau = i\varepsilon \quad \text{and} \quad \sigma\rho = -\rho\sigma, \quad \rho\tau = -\tau\rho, \quad \tau\sigma = -\sigma\tau.$$

For any positive integer k, a set of $2k+1$ matrices $N_1, \ldots, N_{2k+1}$ of degree 2^k is defined by

$$N_{2i} = \tau \otimes \ldots \otimes \tau \otimes \sigma \otimes \varepsilon \otimes \ldots \otimes \varepsilon$$
$$N_{2i-1} = \tau \otimes \ldots \otimes \tau \otimes \rho \otimes \varepsilon \otimes \ldots \otimes \varepsilon$$

for $i = 1, \ldots, k$ and

$$N_{2k+1} = \tau \otimes \ldots \otimes \tau \otimes \tau \otimes \tau \otimes \ldots \otimes \tau$$

where $\otimes$ denotes the tensor product of matrices.

It is easily verified that:
(i) $N_j^2 = I$, $j = 1, \ldots, 2k+1$,
(ii) $N_j N_h = -N_h N_j$, $j \neq h$,
(iii) $N_1 \ldots N_{2k+1} = i^k I$,

(iv) no other product of distinct matrices $N_{j_1}\dots N_{j_t}=\zeta I$, for any $\zeta\in\mathbf{C}^*$ apart from a reordering of (iii),

(v) $N_{j_1}\dots N_{j_t}$ has nonzero trace if and only if $N_{j_1}\dots N_{j_t}=\zeta I$ for some $\zeta\in\mathbf{C}^*$, (where I is the identity matrix of degree 2^k).

See also Read [**3**, Lemma 1.11] and references given there.

LEMMA 3.2. *Let $k=[\frac{1}{2}n]$ and $\{N_1,\dots,N_{2k+1}\}$ be the set of matrices defined above. Define a projective representation $T_{(a_1,\dots,a_n)}$ of C^n_m $(a_i\in\{1,\dots,m\}, i=1,\dots,n)$ by*

$$T_{(a_1,\dots,a_n)}(w_i)=\xi^{a_i}N_i,\quad i=1,\dots,n.$$

Then $T_{(a_1,\dots,a_n)}$ is an i.p.r. of C^n_m.

(a) *When n is even, a full set of inequivalent i.p.r.'s of C^n_m is given by*

$$\{T_{(a_1,\dots,a_n)}: a_i\in\{1,\dots,\tfrac{1}{2}m\}, i=1,\dots,n\}.$$

(b) *When n is odd, a full set of inequivalent i.p.r.'s of C^n_m is given by*

$$\{T_{(a_1,\dots,a_n)}: \textit{either all } a_i\in\{1,\dots,\tfrac{1}{2}m\} \textit{ or all } a_i\in\{\tfrac{1}{2}m+1,\dots,m\}\}.$$

Proof. See Morris [**2**].

DEFINITION 3.3. Let T be the i.p.r. of C^n_m defined by $T(w_i)=N_i$, $i=1,\dots,n$, i.e. $T=T_{(a_1,\dots,a_n)}$ where $a_i=m$, $i=1,\dots,n$. We shall call T the basic projective representation of C^n_m.

LEMMA 3.4. *Let χ denote the projective character of the basic projective representation T as above. Then*

(i) *if n is even,*

$$\chi(w_1^{2b_1}\dots w_n^{2b_n})=2^{n/2},$$

where $b_i\in\{1,\dots,\frac{1}{2}m\}$, $i=1,\dots,n$ and χ has value 0 on all other elements;

(ii) *if n is odd,*

$$\chi(w_1^{2b_1}\dots w_n^{2b_n})=2^{(n-1)/2},$$
$$\chi(w_1^{2b_1+1}\dots w_n^{2b_n+1})=(2i)^{(n-1)/2},$$

where $b_i\in\{1,\dots,m\}$, $i=1,\dots,n$ and χ has value 0 on all other elements.

Proof. We note that

$$T(w_1^{2b_1}\dots w_n^{2b_n})=I$$

and

$$T(w_1^{2b_1+1}\dots w_n^{2b_n+1})=N_1,\dots,N_n=i^{(n-1)/2}I \text{ if } n \text{ is odd}.$$

and $T(w)\neq cI$, $c\in\mathbf{C}^*$ in any other case. The result now follows from the properties of the matrices $N_1,\dots,N_n$.

COROLLARY 3.5. *In the above notation*

(i) $T_{(a_1,\dots,a_n)}=\theta_{(a_1,\dots,a_n)}T$,

(ii) *if $\chi_{(a_1,\dots,a_n)}$ denotes the projective character of $T_{(a_1,\dots,a_n)}$ then $\chi_{(a_1,\dots,a_n)}=\theta_{(a_1,\dots,a_n)}\chi$.*

Proof. This is immediate from the definitions of the representations involved.

4. The restrictions of the i.l.r.'s of C_m^n to $C_{m,p}^n$ are clearly irreducible and $\theta_{(a_1,\ldots,a_n)}\downarrow C_{m,p}^n$ will also be denoted by $\theta_{(a_1,\ldots,a_n)}$. But the i.p.r.'s of C_m^n when restricted to $C_{m,p}^n$ are not always irreducible. This problem is analysed in the following result.

THEOREM 4.1. (i) *If either n or p is odd, then $T_{(a_1,\ldots,a_n)}\downarrow C_{m,p}^n$ is an irreducible representation of $C_{m,p}^n$.*

(ii) *If n and p are both even then $T_{(a_1,\ldots,a_n)}\downarrow C_{m,p}^n$ is the sum of two inequivalent C_m^n-conjugate irreducible representations of $C_{m,p}^n$.*

Proof. We note that

$$C_m^{n-1} \cong C_{m,m}^n \subseteq C_{m,p}^n \subseteq C_m^n$$

and if n is odd then the i.p.r.'s of C_m^n and C_m^{n-1} are of the same degree, viz $2^{(n-1)/2}$. Therefore the restrictions of i.p.r.'s of C_m^n to $C_{m,p}^n$ must be irreducible.

If n is even then the i.p.r.'s of C_m^n and C_m^{n-1} are respectively of degrees $2^{n/2}$ and $2^{(n-1)/2}$. Thus the i.p.r.'s of C_m^n when restricted to $C_{m,p}^n$ can decompose into at most two components and 2 must divide p (see [**1**, p. 82]). This proves (i).

If n and p are both even then $C_{m,p}^n \subseteq C_{m,2}^n \subset C_m^n$ and to obtain (ii) it is sufficient to prove that every i.p.r. of C_m^n splits in $C_{m,2}^n$. For this we need only consider the basic projective representation T of C_m^n and show that T splits in $C_{m,2}^n$. We note that $|C_m^n : C_{m,2}^n| = 2$ and $\chi(w) = 0$ for all $w \in C_m^n \backslash C_{m,2}^n$ where χ is a projective character of T. The result now follows by Lemma 2.2.

DEFINITION 4.2. Let n be even and $C_{m,m}^n \cong C_m^{n-1}$ be the subgroup of C_m^n generated by $\{v_i = w_i w_n^{-1} : i = 1, \ldots, n-1\}$. We may choose a transversal of C_m^{n-1} in $C_{m,p}^n$ given by

$$t(C_{m,p}^n : C_m^{n-1}) = \{w_n^{rp} : r = 1, \ldots, q\},$$

where $q = m/p$. If $T^{(1)}$ is the basic projective representation of $C_{m,m}^n \cong C_m^{n-1}$ then define

$$T^{(-1)}(v_1^{b_1} \ldots v_{n-1}^{b_{n-1}}) = (-1)^{b_1+\cdots+b_{n-1}} T^{(1)}(v_1^{b_1} \ldots v_{n-1}^{b_{n-1}})$$

and

$$T_p^{(1)}(vw^{rp}) = T^{(1)}(v), \qquad T_p^{(-1)}(vw^{rp}) = T^{(-1)}(v),$$

for all $v \in C_{m,m}^n$ and $r = 1, \ldots, q$. $T_p^{(1)}$ and $T_p^{(-1)}$ are i.p.r.'s of $C_{m,p}^n$ called C_m^{n-1}-associate representations.

THEOREM 4.3. *Let n and p be even. Then*

$$T\downarrow C_{m,p}^n = T_p^{(1)} + T_p^{(-1)}$$

and hence

$$\begin{aligned} T_{(a_1,\ldots,a_n)}\downarrow C_{m,p}^n &= (\theta_{(a_1,\ldots,a_n)} T)\downarrow C_{m,p}^n \\ &= \theta_{(a_1,\ldots,a_n)}(T\downarrow C_{m,p}^n) \\ &= \theta_{(a_1,\ldots,a_n)} T_p^{(1)} + \theta_{(a_1,\ldots,a_n)} T_p^{(-1)}. \end{aligned}$$

Proof. If $\chi^{(1)}$ and $\chi^{(-1)}$ denote the projective characters of $T^{(1)}$ and $T^{(-1)}$ respectively, then

$$\chi^{(1)}(v_1^{2b_1}\dots v_{n-1}^{2b_{n-1}})=2^{(n-1)/2}$$
$$\chi^{(1)}(v_1^{2b_1+1}\dots v_{n-1}^{2b_{n-1}+1})=(2i)^{(n-1)/2}$$

and $\chi^{(1)}$ has value zero on all other elements. Similarly

$$\chi^{(-1)}(v_1^{2b_1}\dots v_{n-1}^{2b_{n-1}})=2^{(n-1)/2}$$
$$\chi^{(-1)}(v_1^{2b_1+1}\dots v_{n-1}^{2b_{n-1}+1})=-(2i)^{(n-1)/2}$$

and $\chi^{(-1)}$ has value zero on all other elements. The result now follows easily.

5. For the complete solution of the problem posed in this paper we now turn our attention to find the sets of i.l.r.'s and i.p.r.'s of C^n_m which coincide when restriced to $C^n_{m,p}$.

LEMMA 5.1 (Read [**4**]). *Let $\theta_{(a_1,\dots,a_n)}$ and $\theta_{(a'_1,\dots,a'_n)}$ be two i.l.r.'s of C^n_m. Then $\theta_{(a_1,\dots,a_n)}\downarrow C^n_{m,p}=\theta_{(a'_1,\dots,a'_n)}\downarrow C^n_{m,p}$ if and only if*

$$\text{(i)}\qquad a_i-a'_i\equiv a_j-a'_j\ (\text{mod } m),$$

$$\text{(ii)}\qquad p(a_i-a'_i)\equiv 0\ (\text{mod } m),$$

for all $i,\ j=1,\dots,n$.

Proof. $C^n_{m,p}$ is generated by $\{w_iw_j^{-1},\ w_i^p: i,j=1,\dots,n\}$. Thus $\theta_{(a_1,\dots,a_n)}$ and $\theta_{(a'_1,\dots,a'_n)}$ are identical on $C^n_{m,p}$ if and only if

$$\theta_{(a_1,\dots,a_n)}(w_iw_j^{-1})=\theta_{(a'_1,\dots,a'_n)}(w_iw_j^{-1})\quad\text{and}\quad \theta_{(a_1,\dots,a_n)}(w_i^p)=\theta_{(a'_1,\dots,a'_n)}(w_i^p),$$

i.e. $\xi^{a_i-a_j}=\xi^{a'_i-a'_j}$ and $\xi^{pa_i}=\xi^{pa'_i}$ for all $i,\ j=1,\dots,n$. These imply $a_i-a'_i\equiv a_j-a'_j\ (\text{mod } m)$ and $p(a_i-a'_i)\equiv 0\ (\text{mod } m)$.

THEOREM 5.2. *Let n or p be even. Let $T_{(a_1,\dots,a_n)}=\theta_{(a_1,\dots,a_n)}T$, $T_{(a'_1,\dots,a'_n)}=\theta_{(a'_1,\dots,a'_n)}T$ be two i.p.r.'s of C^n_m. $T_{(a_1,\dots,a_n)}\downarrow C^n_{m,p}$ is equivalent to $T_{(a'_1,\dots,a'_n)}\downarrow C^n_{m,p}$ if and only if*

$$\text{(i)}\qquad a_i-a'_i\equiv a_j-a'_j\ (\text{mod } \tfrac{1}{2}m),$$
$$\text{(ii)}\qquad p(a_i-a'_i)\equiv 0\ (\text{mod } m_1)$$

for all $i,\ j=1,\dots,n$, where

$$m_1=\begin{cases}\tfrac{1}{2}m & \text{if } p \text{ is odd,}\\ m & \text{if } p \text{ is even.}\end{cases}$$

Proof. By Lemma 3.4 and Corollary 3.5, $T_{(a_1,\dots,a_n)}\downarrow C^n_{m,p}\simeq T_{(a'_1,\dots,a'_n)}\downarrow C^n_{m,p}$ if and only if

$$\text{(i)}\qquad \theta_{(a_1,\dots,a_n)}(w_1^{2b_1}\dots w_n^{2b_n})=\theta_{(a'_1,\dots,a'_n)}(w_1^{2b_1}\dots w_n^{2b_n})$$

for all $b_i \in \{1, \ldots, \frac{1}{2}m\}$, $i = 1, \ldots, n$, with $\sum_{i=1}^{p} 2b_i \equiv 0 \pmod p$, and

(ii) (only if n is odd)

$$\theta_{(a_1,\ldots,a_n)}(w_1^{2b_1+1} \ldots w_n^{2b_n+1}) = \theta_{(a'_1,\ldots,a'_n)}(w_1^{2b_1+1} \ldots w_n^{2b_n+1})$$

for all $b_i \in \{1, \ldots, \frac{1}{2}m\}$, $i = 1, \ldots, n$, with $\sum_{i=1}^{n} (2b_i + 1) = 0 \pmod p$.

If n is even, let $b_i = 1$, $b_j = \frac{1}{2}m - 1$ for arbitrarily fixed indices i and j and put $b_k = \frac{1}{2}m$ for $k \neq i, j$. Thus a necessary condition for $T_{(a_1,\ldots,a_n)}\downarrow C^n_{m,p} \cong T_{(a'_1,\ldots,a'_n)}\downarrow C^n_{m,p}$ is

$$a_i - a'_i \equiv a_j - a'_j \pmod{\tfrac{1}{2}m}.$$

Similarly, substituting

$$b_i = \begin{cases} \frac{1}{2}p & \text{if } p \text{ is even} \\ p & \text{otherwise,} \end{cases}$$

we get another necessary condition:

$$\begin{aligned} p(a_i - a'_i) &\equiv 0 \pmod m) && \text{if } p \text{ is even,} \\ p(a_i - a'_i) &\equiv 0 \pmod{\tfrac{1}{2}m} && \text{otherwise.} \end{aligned}$$

Conversely, if $T_{(a_1,\ldots,a_n)}$ and $T_{(a'_1,\ldots,a'_n)}$ satisfy the above conditions, then

$$\begin{aligned} &\theta_{(a_1,\ldots,a_n)}(w_1^{2b_1} \ldots w_n^{2b_n}) \cdot \theta^{-1}_{(a'_1,\ldots,a'_n)}(w_1^{2b_1} \ldots w_n^{2b_n}) \\ &= \xi^{2a_1b_1+\ldots+2a_nb_n}\xi^{-(2a'_1b_1+\ldots+2a'_nb_n)} \\ &= \xi^{\Sigma 2(a_i-a'_i)b_i} \\ &= \xi^{(a_i-a'_i)\Sigma 2b_i} \quad \text{(by (i))} \\ &= \xi^{p(a_i-a'_i)} \text{ or } \xi^{2p(a_i-a'_i)} \quad \text{(according as } p \text{ is even or odd)} \\ &= 1 \quad \text{(by (ii)).} \end{aligned}$$

Similarly, if n is odd, we get the two necessary conditions as above.

In addition we need to consider elements of the form $w_1^{2b_1+1} \ldots w_n^{2b_n+1}$, $b_i \in \{1, \ldots, \frac{1}{2}m\}$, $i = 1, \ldots, n$ with $\sum_{i=1}^{n} (2b_i + 1) \equiv 0 \pmod p$. If p is even then the elements of this form do not belong to $C^n_{m,p}$ and therefore, in this case, the above two conditions are sufficient as well.

THEOREM 5.3. *If n and p are both odd then $T_{(a_1,\ldots,a_n)} \simeq T_{(a'_1,\ldots,a'_n)}$ as representations of $C^n_{m,p}$ if and only if*

(i) $a_i - a'_i \equiv a_j - a'_j \pmod{\frac{1}{2}m}$,

(ii) $p(a_i - a'_i) \equiv 0 \pmod{\frac{1}{2}m}$,

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(iii) $(p-n)(a_n-a'_n)+A \equiv 0 \pmod m$,

where $A=\sum_{i=1}^n a_i-\sum_{i=1}^n a'_i$.

Proof. Conditions (i) and (ii) are obtained by considering elements of type $w_1^{2b_1}\dots w_n^{2b_n}$ as in the case n even. In this case, we further need to consider elements of the type $w_1^{2b_1+1}\dots w_n^{2b_n+1}$ where $b_i\in\{1,\dots,\frac{1}{2}m\}$, $i=1,\dots,m$ and $\sum_{i=1}^n(2b_i+1)\equiv 0$ $\pmod p$. Let $b_i=0$, $i=1,\dots,n-1$ and $2b_n+1=p-n+1$. Then $\sum(2b_i+1)\equiv 0 \pmod p$ and therefore $w=w_1\dots w_{n-1}w_n^{p-n+1}\in C^n_{m,p}$. Equating the values of the projective characters of $T_{(a_1,\dots,a_n)}$ and $T_{(a'_1,\dots,a'_n)}$ on w, we obtain the third necessary condition as required.

Conversely, conditions (i) and (ii) are sufficient to prove that

$$\theta_{(a_1,\dots,a_n)}(w_1^{2b_1}\dots w_n^{2b_n})=\theta_{(a'_1,\dots,a'_n)}(w_1^{2b_1}\dots w_n^{2b_n})$$

for all $w_1^{2b_1}\dots w_n^{2b_n}\in C^n_{m,p}$.

If $w_1^{2b_1+1}\dots w_n^{2b_n+1}\in C^n_{m,p}$ then $\sum_{i=1}^n(2b_i+1)\equiv 0 \pmod p$. Since $\sum_{i=1}^n(2b_i+1)$ and p are both odd, there exists an odd integer $2k+1$ such that $\sum(2b_i+1)=(2k+1)p$. We show that $\sum_{i=1}^n(2b_i+1)(a_i-a'_i)\equiv 0 \pmod m$. This will complete the proof of the theorem.

$$\begin{aligned}\sum_{i=1}^n(2b_i+1)(a_i-a'_i)&=\sum_{i=1}^n 2b_i(a_i-a'_i)+\sum_{i=1}^n(a_i-a'_i)\\&\equiv(a_n-a'_n)\sum_{i=1}^n 2b_i+A \pmod m \quad \text{(by (i))}\\&\equiv(a_n-a'_n)((2k+1)p-n)+A \pmod m\\&\equiv 2kp(a_n-a'_n)+(a_n-a'_n)(p-n)+A \pmod m\\&\equiv(a_n-a'_n)(p-n)+A \pmod m \quad \text{(by (ii))}\\&\equiv 0 \pmod m \quad \text{(by (iii))}.\end{aligned}$$

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