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Editorial

The mathematical theory of Optimal Transport (OT) is now one of the most visible and also most active fields in nonlinear analysis, with two Fields medal winners Cedric Villani and Alessio Figalli among its protagonists. The origins of OT date back to the works of Gaspard Monge in the 18th century and those of Leonid Kantorovich in the '1940s. However, what is considered as the foundations of the modern theory have mostly been established in several ground-breaking works at the end of the 20th century: among the most prominent contributors, there is Yann Brenier, who characterized optimal transport in terms of connections with PDEs and hydrodynamics in the 1980's; there is Robert McCann, who introduced the powerful concept of displacement convexity in the 1990's; there is Felix Otto, who demonstrated the implications of OT to nonlinear evolution equations around the millennium.

In the past twenty years, a huge number of young researchers entered into the field, and did not only greatly improve and expand the theory, but also revealed more and more connections of OT to other research areas in and outside of mathematics. These applications include nonlinear functional analysis, differential geometry, statistics, big data, fluid mechanics, economics, meteorology, design problems, image processing and many more. By now, interdisciplinarity has become one of the most characteristic features of the current developments in and around OT. The theory and its application attract researchers with very different backgrounds.

Although many of the aforementioned areas of application for OT have been identified already ten to twenty years ago, it had been very difficult to draw any practical use from these connections until recently. The bottleneck has been the lack of efficient numerical methods to compute distances and optimal maps. For instance, the linear programming methods that follow directly from the ideas of Kantorovich cannot be applied to large-scale problems. The breakthrough took place just a few years ago, when several novel methods were developed, or existing ones were improved to become precise or scalable. In that context, entropic regularization has become a versatile tool since it allows for amazingly quick computation of approximate optimal plans in very high dimensions. Alternative methods aim at solving the PDEs related to the optimality conditions and mostly feed on the rich machinery from computational fluid mechanics. Simultaneously, a new framework appeared: the semi-discrete computations based on computational geometry tools, which allow for precise, and provably converging, algorithms which work in low dimension but with almost arbitrary resolution levels.

The present volume collects some recent results presented by specialists of OT in areas of interest for applications. The subject being currently too wide to prepare a single volume spanning all the applications, the focus is mainly on those closer to the analytical taste of the editors, but all the papers have been selected in view of their interest for applied mathematicians.

The papers [1] and [2] are devoted to numerical methods for OT-related problems, using computational fluid dynamics and entropic regularization, respectively. The use of OT as a tool to prove existence and study properties of solutions to evolution equations is demonstrated in papers [3] and [4]. The modeling of multi-agent systems on a large scale, so that the use of evolving

measures is required, is a nowadays very common application of OT, as we can see in papers [5] and [6]. Finally, variants of the Monge-Kantorovich problem can be found in papers [7,8,9,10] and [11]: multi-marginal problems with applications in quantum chemistry, unequal dimension cases with applications in economics, and various problems motivated by stochastic interpretations. Actually, it is difficult to classify each paper into a single area of applications of OT, and most of these studies are at the interface between different emerging theories.

In presenting this special issue of EJAM, we wish to give a taste of some current directions that the research in this field is pursuing.

The guest editors
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