1 Introduction

Quantum mechanics is one of the fundamental subjects in physics. It is a key component for any undergraduate and graduate curricula, and it is required for students to obtain a good degree of understanding in order to conduct meaningful research. From a student's immediate perspective, however, this goal may be difficult to achieve without taking fundamental concepts and applying them in practice problems.

As part of our teaching experiences as professors, we have had students constantly asking for more practice problems. The consensus is that having a variety of available exercises illustrating key concepts *more than one time in different scenarios* can give students that extra knowledge. This skill is strongly desired, especially in quantum mechanics as being one of the pillars in physics. On the other hand, from a practical point of view of teaching the course and having to give multiple exams during the semester, it is useful if a diverse set of problems reflecting modern developments in quantum mechanics is available.

There are many books on quantum mechanics available in the literature. In addition to textbooks adapted by universities, professors develop custom-made notes emphasizing a particular topic or set of topics, and when it comes to practice problems, the majority of books cover rather standard topics, such as 1D Schrödinger equations, perturbation theory, addition of angular momenta, the simple harmonic oscillator and hydrogen atom Schrödinger equations, and scattering. However, new research areas are emerging that rely on many concepts that are less represented in this list (which is not meant to be exhaustive, of course).

Here are some examples. Recent discoveries of many new materials with topologically nontrivial nature have elevated the importance of symmetry and geometry in quantum mechanics. There is a great need to better understand Berry phases and related properties in free space and periodic environments. Manipulating functions of operators and various transformations in this context also becomes important. Our experience in the classroom, as students and professors now, shows that geometrical phases are given rather peripheral attention and are not much studied beyond the standard Aharonov–Bohm effect. But since topology with its underlined symmetry and group theory is gaining prominence in various areas, we feel that a larger body of introductory exercises is needed. After all, future researchers can benefit tremendously if these concepts are taught in quantum mechanics classes with the opportunity for extra practice.

Quantum information is forming into a separate scientific field, which uses Dirac notation with theories relying heavily on the density operator. Describing a given

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system with a density matrix is in fact more general than the wave function formalism, for which many problems already exist throughout the literature. However, the statistical nature of quantum particles, processes, and measurements is much better captured using the density operator. This formalism is also necessary for the probabilistic nature of entropy, pure and entangled states, and measurements for quantum information science. Although the density operator with some of its basic properties is discussed in standard textbooks, there is a shortage of practice problems at a more advanced level.

Perhaps the most challenging problems in quantum mechanics involve identical particles. It is not easy to understand how to apply symmetrization/antisymmetrization procedures to composite systems that contain two, three, four, seventeen, or an infinite number of fermions or bosons. Most books focus on the quantum mechanical states of two identical fermions and bosons making a connection with addition of angular momenta. For this purpose, the Clebsch–Gordan coefficient table is conveniently introduced. Our experience shows, however, that students cannot easily translate this knowledge when they are dealing with three or four identical particles and their angular momenta have to be added. The problem of handling a large but finite number of fermions becomes especially important in atoms for which the electron shell structures are needed. Even though this is well known in chemistry, here we tried to reinforce that the electron shells in atoms from the periodic table are a consequence of the nature of identical particles. When dealing with quantum mechanical states that represent essentially an infinite number of identical fermions or bosons (such as in macroscopic materials), second quantization becomes necessary, which is seldom discussed in standard courses. Here we try to give problems showing how second quantization is performed in relatively simple Hamiltonians by giving the reader opportunity for extra practice and to make broader connections in the context of the statistical nature of quantum mechanics. The problems of second quantization are also quite necessary for students to see how current research can become textbook material. Much of the research in the past several years has been graphene-driven, thus it would be beneficial to see how its Hamiltonian quantization occurs. Starting with "simpler" problems and delving into statistical quantum mechanical concepts, the problems we work out in detail give an exemplary connection between textbook quantum mechanics and quantum field theory.

Understanding relativistic effects in quantum mechanics has become especially important in light of many recently discovered new materials, in which the Dirac equation plays a prominent role. The most important example perhaps is graphene, whose properties can be described and understood in terms of relativistic quantum mechanics, where the speed of light is substituted by an effective velocity of massless particles. Scattering of relativistic particles as well as topologically nontrivial edge states also require knowledge of the Dirac equation and its variations. In addition to these new developments, relativistic problems are always left at the end of quantum mechanics courses and students are shortchanged. The problems in the last chapter of the book give different variations of the Dirac equation, which gives an opportunity for practice in the context of contemporary research situations.

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These emerging fields of quantum research have created new challenges for teaching and practice. Universities are currently creating programs and curricula to answer these demands and train the future workforce with diverse background, so students are better prepared for cutting-edge research and industry of tomorrow. The collection of problems in this book emphasizes less-represented topics in the mainstream literature. At the beginning of each chapter, a brief introduction of basic concepts is provided. The reader will notice that this is not a textbook format; rather, it is a brief summary of some important formulas and relations with concise explanations (a more in-depth discussion can be found in textbooks). The problems in each chapter are of varying difficulty and we tried to arrange them by starting with simpler examples. Some problems are created from scratch, while others are revised from the research literature in order to make them suitable for practice and instruction.

We certainly hope that the reader will find this book useful. Technical details and connections that a student or a beginning researcher were expected to make on their own are now shown explicitly, which is quite beneficial in the interest of saving time and building skills. We also hope readers will find the book enjoyable. Even though we have tried very hard to make all notation uniform throughout, it is possible that improvements are necessary. Some typos are also likely to occur (not many, we hope) simply because of the nature of the content. We would appreciate it if readers communicate with us in case they find such unintentional errors. It would also be great if readers send us variations of the current problems or new ones based on their research to include in a new edition of this book in the future.