THE LIMITING SENSITIVITY AND VISIBILITY LOSS IN A SMALL APERTURE AMPLITUDE INTERFEROMETER

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ABSTRACT

We analyze the behaviour of an amplitude interferometer with aperture diameters of the order of r_o. A simple optical correlator can be used to measure the source coherence function, and the limiting magnitude is found to depend principally on the optical bandwidth. The basic instrument appears to have a limiting magnitude $m = +8$, but this can be improved by more elaborate correlators. The effects of atmospheric turbulence on such an interferometer can be substantially reduced by an active optical system which compensates for the wavefront tilts in the incident light, and it appears that the limiting magnitude of the interferometer, when atmospheric effects are taken into account, is around $m = 7 - 8$.

1. INTRODUCTION

A small aperture stellar interferometer is one which employs entrance pupils with diameters of the order of r_o (~10 cm). The attraction of such instruments is twofold: they should be substantially cheaper to construct than large aperture speckle type instruments which may have pupils of one meter or more, and they appear to be relatively insensitive to the effects of atmospheric turbulence, so that accurate measurements of the coherence function should be feasible. On the other hand, the light gathering power of these interferometers is clearly limited, and in practice a compromise aperture size must be found which will be large enough to yield a satisfactory signal to noise ratio but which will not be so large as to cause excessive loss of fringe visibility due to turbulence. As I will show below, even for an instrument with $d = r_o$ the effects of "seeing" are not negligible, and some means of active compensation of the wavefront is essential. With small apertures, however, the active optics need not be elaborate, and only a small amount of post-detection correction of the data should be necessary.

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2. DESCRIPTION OF THE BASIC INSTRUMENT

In common with any high resolution amplitude interferometer, the small aperture instrument will consist of two telescopes or coelostats separated by the baseline distance D, transfer optics, and the variable path length compensators which are required to maintain the temporal coherence of the two wavefronts. One can envisage a variety of methods for actually bringing about the interference of the two signals and for measuring the source coherence (or fringe visibility), but perhaps the simplest approach is that embodied in the interferometer shown in Fig. 1. This is essentially the same as the Monteporzio stellar interferometer (see the following paper), and is similar to the Maryland interferometer.[1] In the Figure only the central portion of the interferometer is shown for simplicity.

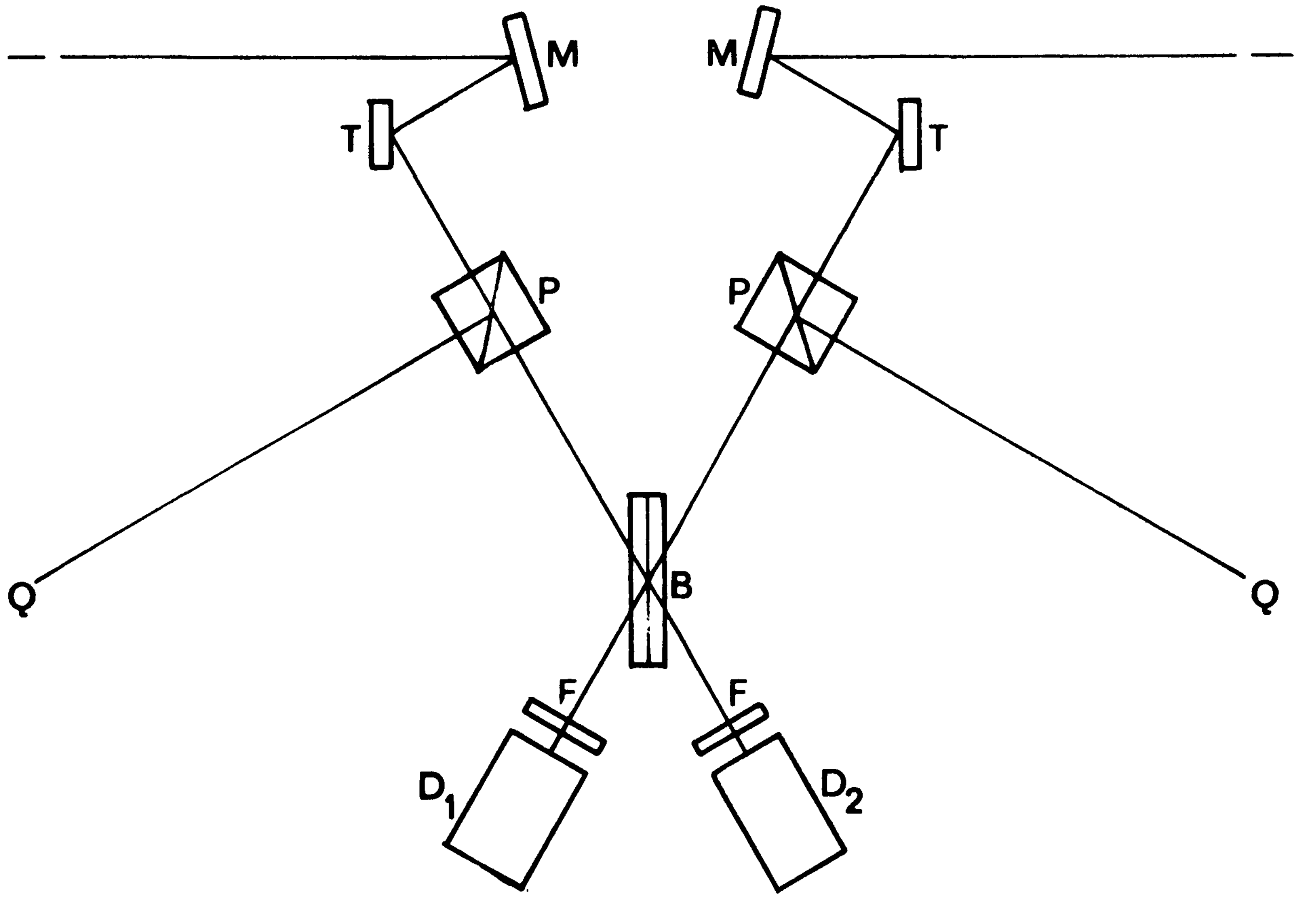

Figure 1. Optical layout of the interferometer. M, T are mirrors, the polarizing beamsplitters are indicated by P, the main beamsplitter by B, and the optical filters by F. The two photodetectors are D_1 and D_2, and the quadrant detectors which are used to guide the tilting mirrors T are located at Q.

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The mirrors M and T direct the two beams, which we assume have been reduced to a convenient diameter, to two polarizing beamsplitters (P). These beamsplitters select a single polarization state for interference; the other component is directed to the quadrant detectors Q which form part of the active optical compensating system. The wavefronts interfere at the main beamsplitter B. The beams are arranged to interfere at nominally zero angle, so that the complimentary exit pupils of the interferometer will, in the absence of aberrations, be uniformly illuminated. The filters (F) define the optical bandwidth of the system, and the simple photodetectors D_1 and D_2 are used to measure the total flux in each beam.

Consider first the behavior of the interferometer when one can neglect the aberrations introduced by atmospheric turbulence. In this situation it is easy to see that the signals from the two detectors will be proportional to

$$I(1 + |\Gamma|\cos\Phi) \tag{1a}$$

and

$$I(1 - |\Gamma|\cos\Phi) \quad , \tag{1b}$$

where I is the incident irradiance, $|\Gamma|$ is the modulus of the source coherence function, and Φ is a randomly varying phase which is the result of fluctuating optical paths through the atmosphere, errors in the path compensation, etc. We assume that this phase is nearly constant or "frozen" over a time τ, estimated to be of the order of 1 - 10 msec.[2] The photocounts registered by the detectors during each τ second sample time are processed by a correlator which basically measures the square of the difference between the two signals. The data are integrated for a total observing time T, after which the output of the correlator is analyzed to yield the normalized correlation $\bar{c}_N^2$, assuming that the counting statistics are known. The apparent correlation $\bar{c}_N^2$ will be equal to $2|\Gamma|^2\langle\cos^2\Phi\rangle$, where the brackets indicate an average over the T second observing period, and if we assume that the phase is a uniform random variable the normalized correlation will be equal to $|\Gamma|^2$, the modulus squared of the source coherence function.

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The only limit to our ability to measure the correlation under these rather idealized conditions will be photon noise, which can be estimated from the properties of the Poisson distribution. If N_o is the mean counting rate from both detectors then in the situation when $(N_o\ \tau) < 1$ it can be shown that the signal to noise ratio of the correlator is given by:[3]

$$S/N(|\Gamma|^2) = (N_o T)(\tau/8T)^{\frac{1}{2}}|\Gamma|^2 \quad . \qquad (2)$$

It should be noticed that the signal to noise ratio is proportional to N_o and not $N_o^{\frac{1}{2}}$ when the light level is very low. This is due to the fact that $\bar{c}_N^2$ is determined by the second order statistics of the counting distribution.

Experience shows that the minimum useful signal to noise ratio is about 3:1 in an hour. For a 1 msec sample time, this gives a minimum counting rate of $N_o = 4.5\ s^{-1}$. For a source which is partially resolved significantly higher rates must be used. Taking typical values for the quantum efficiency, etc., apertures of 10 cm diameter, and an optical bandwidth of 2.5×10^{11} Hz, one finds that the limiting magnitude of the stellar interferometer is about +8.5.

The bandwidth of 2.5×10^{11} Hz corresponds to a few Angstroms. This value was chosen since it allows the presence of path errors up to about 100 μm without significant loss of temporal coherence. In a large interferometer this tolerance on path length seems realistic. The sensitivity could be greatly improved, of course, by increasing the bandwidth, and there seem to be two ways of doing this. The first is to replace the optical filters with prisms or gratings, and to focus the dispersed spectra onto detector arrays. This gives a large number M of independent and relatively narrow band optical channels; the data from the M channels are incoherently added in the data processing to give an effective optical bandwidth $M^{\frac{1}{2}}$ times that of a single channel. The second approach would employ some means of tracking the white light fringe in order to reduce the path errors. This would allow a much wider coherent bandwidth. In practice this technique would probably require detector arrays as well[4] (this type of interferometer is closely related to Shao and Staelin's "astrometric" interferometer[5]).

3. THE EFFECTS OF ATMOSPHERIC TURBULENCE

When atmospheric turbulence is taken into account, the accurate measurement of the coherence function becomes more difficult. Turbulence causes both phase and amplitude aberrations which distort the interfering wavefronts, and as a result an irregular and rapidly varying fringe pattern will be seen in the exit pupils in place of the uniformly illuminated fields which we considered above. The effect of this on our simple optical correlator is a reduction in the apparent fringe visibility. This means that longer integration times will be necessary, since the signal is reduced. Further, and more seriously, unless one can either estimate accurately the loss in fringe contrast or somehow reduce it to an acceptable level, it will be impossible to make quantitative measurements of the source coherence.

One approach to the problem has been given by Greenaway and Dainty,[6] who suggest that the simple two channel optical correlator be replaced by two dimensional detector arrays, thus dividing the apertures into a large number of subapertures. Corresponding subapertures are correlated, so that one has effectively many independent small interferometers. If the projected subaperture size is much smaller than r_o it can be shown that the measured correlation is independent of the atmospheric turbulence. It seems, however, that the signal to noise ratio of such an interferometer will be reduced by a factor of $K^{-\frac{1}{2}}$ with respect to the two channel instrument, where K is the number of subapertures.[7] To recover this loss of sensitivity one would require quite large primary apertures.

An alternative approach, proposed by Twiss and used in the Monteporzio interferometer, is to use a simple active optical system to remove the major part of the aberrations before the wavefronts actually interfere. Some residual aberration will remain to cause a loss in correlation, but this can be estimated by auxiliary observations. Since the correction term will be relatively small, these observations of the "seeing" need not be done with high accuracy.

A detailed analysis of the interferometer shows that the correlation which is observed is given by

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$$\overline{c}_N = \eta|\Gamma| \quad , \tag{3}$$

where η is a loss factor which is a function of the aberrations in the optical system. It can be expressed as an integral involving the fourth order moments of the phase and amplitude fluctuations during the T second observing period. As usual, one can estimate its average value by assuming that the turbulence follows a Komolgorov model, that it is stationary and that the fluctuations at the two apertures are uncorrelated. Then one finds that the loss is given by the relatively simple expression

$$\eta^2 = (\pi d^2/4)^{-1} \int d^2\underset{\sim}{u}\ T(\underset{\sim}{u})\ B^2(\underset{\sim}{u})/B^2(0) \quad , \tag{4}$$

where $T(\underset{\sim}{u})$ is the optical transfer function of a single aperture and $B(\underset{\sim}{u})$ is the second moment of the wave aberration; i.e. it is equal to $<U(\underset{\sim}{r}+\underset{\sim}{u})U^*(\underset{\sim}{r})>$, where $U(\underset{\sim}{x})$ is the complex wave amplitude at a point $\underset{\sim}{x}$ in the plane of the apertures. $B(\underset{\sim}{u})$ has been given by Fried[8] for Komolgorov turbulence, and the resultant loss η is shown as curve (a) in Figure 2, where it is graphed as a function of (d/r_o).

The drastic loss of fringe visibility is clearly evident from the Figure, and for $d = r_o$ there is a 50% reduction. For apertures of order r_o, it is well-known that most of the aberration takes the form of wavefront tilt, which is readily removed by having a tiltable mirror in the optical system. We therefore use the quadrant detectors situated at Q in Fig. 1 to sense the motion of the image centroid and actuate piezo-electric drivers which cause the mirrors T to tilt. The loss can be recalculated for this situation simply by using the moment $B(\underset{\sim}{u})$ for the fluctuations with tilt removed. This is just the "short exposure" moment which has also been found by Fried, and the loss for an interferometer with tilt correction is given as curve (b) in Fig. 2.

It will be seen that for $d = r_o$ the loss in fringe visibility is now only about 15%, but this may still be unacceptably high for precise work. It should be possible, however, to obtain a fairly good estimate of the loss from ancillary observations. In particular, it should be noted that

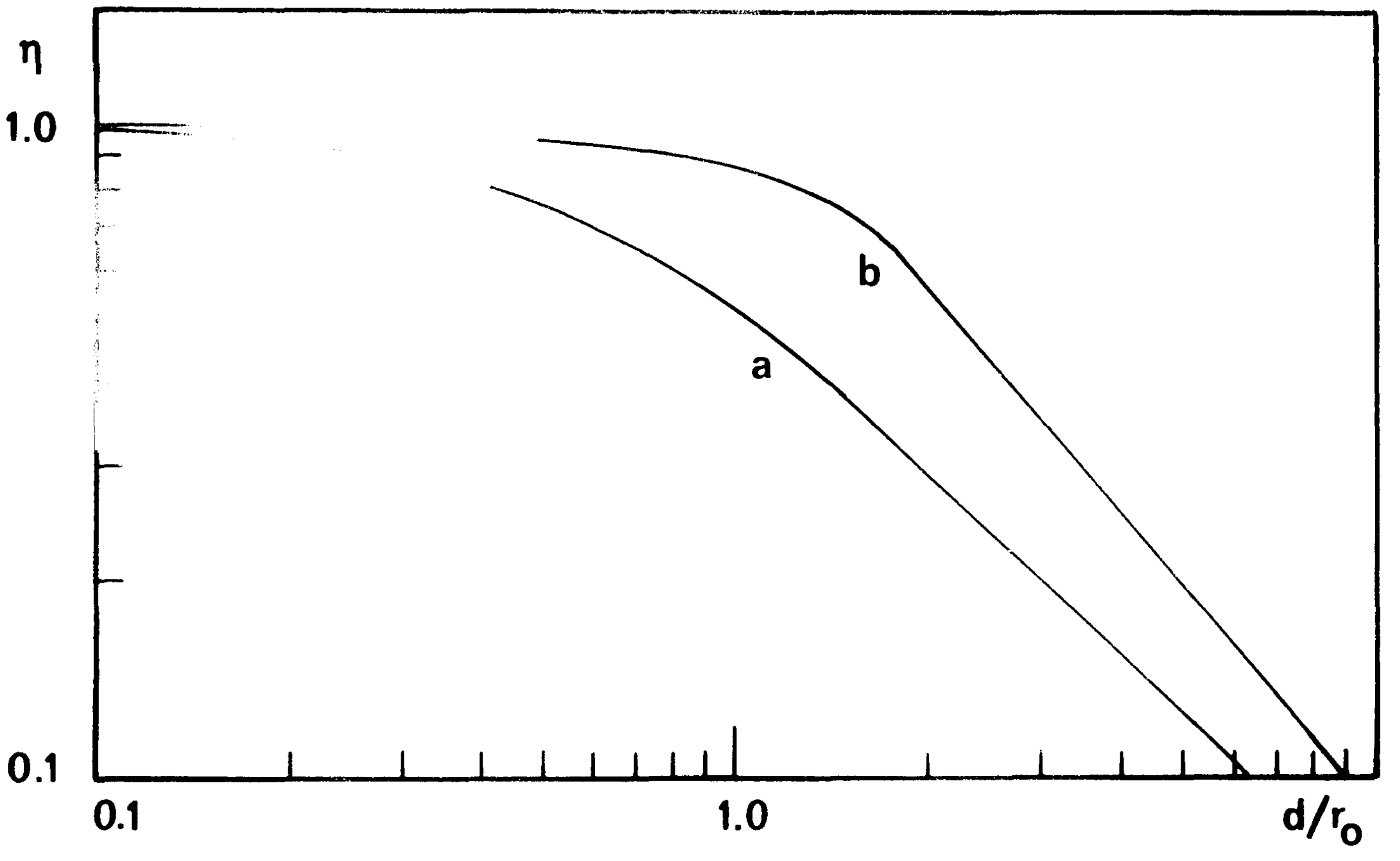

FIGURE 2

the Strehl ratio, i, which is a common figure of merit for image quality, is also expressible in terms of $T(\underset{\sim}{u})$ and $B(\underset{\sim}{u})$:

$$i = (\pi d^2/4) \int d^2\underset{\sim}{u}T(\underset{\sim}{u})B(\underset{\sim}{u})/B(0) \quad . \tag{5}$$

Although i and η^2 have different dependences upon $B(\underset{\sim}{u})$, because of the exponential character of the moment the only difference between i and η is a change of scale. Thus it should be possible to estimate η from observations of the Strehl ratio or another equivalent measure of image sharpness, and such observations might well be done using the light reaching the quadrant detectors.

The use of active tilt correction imposes a further constraint on the sensitivity of the interferometer, for shot noise in the servo system will cause the tilting mirrors to dither randomly This introduces a loss in correlation which will be proportional to the shot noise amplitude and the

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noise bandwidth of the servo system. By limiting the bandwidth the amount of system noise can be controlled but then another problem can arise. The frequency spectrum of the wavefront tilts caused by atmospheric turbulence may be broader than the servo bandwidth, and as a result the mirrors will be unable to follow the high frequency tilt fluctuations. In practice there will be an optimum servo bandwidth which will produce the maximum correlation. This bandwidth will depend of course on the details of the servo system and such factors as the wind speed during the observation, but one can get a rough idea of the magnitude of the visibility loss due to the servo noise from simple models. Tango and Twiss[3] have done this using a simple servo response function, $d = r_o$, a wind speed of 5 m/s, and an optical bandwidth for the servo of 100 nm. The results are shown in Table I. The percentage loss of visibility and the servo cutoff frequency f_o are given as functions of the magnitude of the star under observation.

TABLE I

Visibility loss due to uncompensated tilt fluctuations and shot noise

m	$(\eta - 1) \times 100$	f_o (Hz)
+5	1.8	100
6	3.2	70
7	5.7	50
8	9.8	30
9	16.7	20

As was the case with the residual wavefront curvature, the losses due to the failure of the servo to completely correct for the tilt can be compensated for in the subsequent data analysis, for the loss, which is proportional to the rms shot noise and to the residual rms tilt fluctuations, can be deduced directly from the error signals from the quadrant detectors.

4. SUMMARY

In summary, the limiting sensitivity depends on both the signal to noise ratio of the optical correlator that is used to measure $|\Gamma|$ and the noise characteristics of the tilt correcting servo system. It is also dependent upon how large a correlation loss we are willing to accept in the raw data. A large loss means increased integration times and also means that the final result is correspondingly more sensitive to how well we have estimate the loss from our auxiliary "image quality" monitors. It seems realistic to limit the loss from each of these causes to about 10%, and thus the limiting magnitude of the interferometer for quantitative work is about $m = +8$.

ACKNOWLEDGMENT

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DISCUSSION

D.G. Currie: Why did you choose an interferometer bandwidth of a few Angstroms?

W.J. Tango: This is based on the assumptions that (a) no fringe tracking is used and (b) internal path errors will be of the order of 100 μm. This figure for the path error is probably a realistic estimate for a practical interferometer. A fringe tracker would allow a much wider bandwidth and we will be investigating this further.

K.M. Liewer: Do your estimates of limiting magnitude include instrumental losses?

W.J. Tango: Yes. I omitted to say that the data in Table I is based upon an instrumental transmission of 0.25 and a detector quantum efficiency of 0.25.

J.C. Dainty: Our measurements at a good site (Mauna Kea, Hawaii) indicate that r_o can vary considerably through a single night, both on short (minute) and long (hour) timescales. In practice one might have to use a smaller aperture to get seeing independent results.