ON THE ROLE OF DOUBLE LAYERS IN ASTROPHYSICAL PLASMAS

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Limitations of current knowledge of plasma double layers create difficulties in extrapolating double-layer concepts for application to astrophysical models. Some problems of this sort are described, and some central issues in structure and dynamics of double layers are identified, which must be addressed in astrophysical contexts. These include the determination of kinetic boundary conditions, and the relations of time and length scales of local dynamics and structure to those of the global circuit in which the double layer is contained.

There is widespread interest in double layers (DL) as a possible acceleration mechanism in various energetic phenomena in space and astrophysical plasmas. They have been invoked in such diverse contexts as terrestrial auroral discharges, magnetospheric substorms, solar flares, Jovian radio emission, and extragalactic radio sources. In a thought-provoking series of discussions, Alfvén (1979, 1981, 1982) has considered DL to be a central paradigm in plasma astrophysics.

Our current knowledge of DL physics, however, is insufficient for us to judge with much confidence what roles DL may play in astrophysics. This knowledge is derived from a growing but still limited number (and perhaps more important, a limited class) of experimental, theoretical, and numerical investigations. The application, however tentative, of this knowledge to the scales and conditions of astrophysical phenomena requires conceptual extrapolations which must be quite judicious and which are, in our present state of ignorance, most probably unwarranted. Many basic questions must be addressed before DL can become the fruitful astrophysical paradigm envisioned by Alfvén.

A first indication of the nature of these questions can be inferred from the current literature. In this paper I shall discuss a few central points concerning both the structure and dynamics of double layers. Consideration of such questions may be essential in assessing

M. R. Kundu and G. D. Holman (eds.), Unstable Current Systems and Plasma Instabilities in Astrophysics, 113–123.

the applicability of models invoking DL to various astrophysical phenomena, from the standpoints of both basic physics and applications.

Before the work of Sato and Okuda (1980), the common concept of the DL was that of a "strong", laminar, BGK-type potential structure (Figure 1). Steady-state fluid analyses yield well-known necessary boundary conditions for the existence of such structures. In the asymptotic forms usually cited (Block, 1978), these are the "Bohm criteria"

$$U_{e1}^2 > (\gamma_e T_{e1} + T_{i1})/m_e \quad ; \quad U_{i2}^2 > (\gamma_i T_{i2} + T_{e2})/m_i \tag{1}$$

and the Langmuir condition

$$F_{e1}/|F_{i2}| = (m_i/m_e)^{1/2} , \tag{2}$$

where $\gamma_{e,i}$ are the electron and ion adiabatic indices (i.e., $p_{e,i} \sim n_{e,i}^{\gamma_{e,i}}$) and subscripts 1,2 refer to the cathode ($\phi=0$) and anode ($\phi=\phi_{DL}$) sides of Figure 1; U_j and F_j are, respectively, the velocity moment and flux of free particles of species j entering the DL. The Bohm criteria relate to monotonicity of the potential $\phi(x)$; the Langmuir condition relates to overall charge neutrality integrated across the DL.

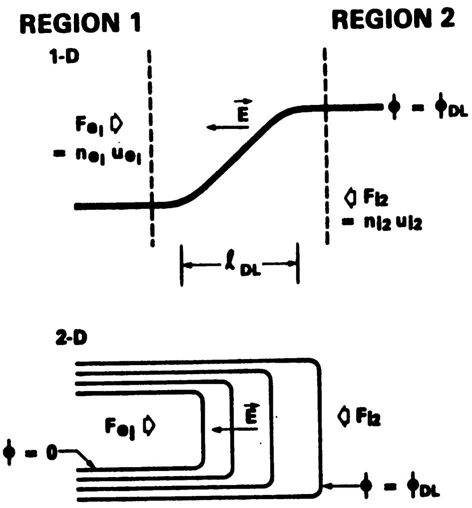

Fig. 1. Schematic of the classical BGK double layer concept, in 1-D (potential profile) and 2-D (potential contours). Plasma in regions 1 and 2 is uniform with respect to the direction of current flow.

In fluid analyses, U_{e1} and U_{i2} are the bulk velocities of electron and ion fluids with temperatures T_{e1} and T_{i2}. As was shown by Levine and Crawford (1980), however, application of adiabatic fluid theory is self-consistent only when the Bohm criteria are in fact satisfied. Moreover, because the DL structure is bounded, its general description requires a kinetic analysis in which the boundary conditions are applied to the free particles on the appropriate half-spaces in velocity; in the geometry of Figure 1, these are $v>0$ at $\phi=0$ and $v<0$ at $\phi = \phi_{DL}$. Thus, for example, in judging whether the boundary conditions for the existence of a BGK DL are met in some situation, it would be erroneous to argue that a relative bulk drift of velocity $(U_{e1} - U_{i2})$ must exist in the plasma. For example, Hubbard and Joyce (1979) found quasi-steady DL in numerical simulations in which the free particles were injected from the half-spaces of nondrifting Maxwellians.

Therefore, although the existence of the BGK double layer must depend on some conditions analogous to (1) and (2), these conditions must be formulated kinetically. Knorr and Goertz (1974) constructed a steady-state waterbag model of DL; to my knowledge, a more general kinetic formulation has not been done. Nor can the Bohm criteria simply be re-interpreted to apply to kinetic moments on a half-space in velocity. Besides the limitations pointed out by Levine and Crawford on the validity of fluid analyses of DL, the indices $\gamma_{e,i}$ in the fluid equation of state really have no analogs in the kinetic formulation: simulations (Smith 1983) reveal that both electrons and ions transport substantial heat flux through the double layer.

In addition to expressing boundary conditions for the existence of BGK DL, the as-yet unknown kinetic Bohm/Langmuir conditions must be either formulated *a priori* with reference to plasma conditions external to the DL boundaries, or considered *a posteriori* in this context. The consideration here is the *stability* of the DL. For example, Hubbard and Joyce (1979) observed *disruption* of the DL due to trapping of the electron influx in low-frequency waves produced by the accelerated ions. The extent of such trapping will depend on the distributions of the inflowing particles.

Considerations of the Bohm criteria (1) have motivated some useful work in which the question of accessibility of the DL state was addressed, with the Bohm criteria viewed as an *initial* condition in a current-carrying plasma with no initial dc electric field. (As we shall see below, however, in the evolutionary problem the final boundary conditions are not identical to the initial conditions!) Smith and Goertz (1978) noted that the Bohm criteria (1) were compatible with the threshold condition

$$|U_e - U_i| \gtrsim 1.3\ V_e \qquad (3)$$

for the Buneman instability, where V_e is the electron thermal velocity. They suggested that DL evolve nonlinearly in an inhomogenous, Buneman-unstable plasma. When (3) is initially met, this turns out to be the case (Smith 1982a,b; 1983); I shall describe the evolutionary dynamics below. Sato and Okuda (1980) suggested that for $|U_i - U_e| < V_e$, DL-like structures may be driven by particles accelerated in DC electric fields supported by anomalous resistivity provided that the system be "sufficiently long".

In simulations using short systems, Sato and Okuda observed only anomalous resistivity supported by ion-acoustic turbulence. Upon lengthening the system, they found sharp potential spikes embedded in the turbulence (Figure 2); they named these spikes "ion-acoustic double layers" (IA DL). In contrast to the strong, laminar, BGK DL, the IA DL are weak ($e\phi_{DL} \sim T_e$), turbulent, and unstable to emission of ion sound. In very long systems, (Sato and Okuda 1981), they recur on some characteristic length scale which I shall call ℓ_{IA} (in the simulations ℓ_{IA} is of order 1000 λ_e, but this value may be an artifact of the numerical parameters, such as m_i/m_e). On this characteristic scale, IA DL seem to be statistically stationary in the sense that as individual structures decay, others spontaneously appear.

The existence of the scale length ℓ_{IA} is crucial and must be linked to the ion dynamics. Unlike the essentially monotonic BGK DL, the structure of the IA DL includes a sharp negative spike at the leading edge. This negative spike, indicating a local "ion hole" in the ion phase space, dynamically leads to the subsequent potential rise by reflecting current-carrying electrons (Hasegawa and Sato 1982; Schamel 1982; Chanteur et al. 1983). The scale length ℓ_{IA} may be simply a correlation length for the spontaneous formation of an ion hole by constructive interference of random-phased ion acoustic waves (W. Lotko, private communication).

Our current knowledge of DL, then, concerns two strikingly different limiting cases: the classical BGK paradigm and the more

Fig. 2. Recurrent weak double layers embedded in ion-acoustic turbulence. After Sato and Okuda (1981).

recently discovered ion acoustic double layers. Recent spacecraft observations of electric fields in the broad inverted-V auroral region have been interpreted as IA DL (Temerin *et al.* 1982); BGK DL may exist in the narrower discrete auroral arcs, but observations on this question are not definitive. Nothing that we currently know rules out the possibility that both types of DL exist in space and astrophysical plasmas. There is much, however, that we do not yet know concerning DL structure under different conditions and the dynamic accessibility of the final DL state. Related to these questions is the possibility that the BGK and IA DL are limiting cases of a sequence of intermediate states linked by transitions in structure and underlying dynamics.

Such questions can only be completely addressed by studying the DL as part of a complete electric circuit, a point that has been stressed by Alfvén and others and on which, I believe, there is growing concurrence. The reasons are several; I shall not attempt to compose a formal list, but shall give some examples relating to time and length scales and to the intrinsic nature of DL evolution in a circuit.

Let us first consider some dynamical issues that arise when a DL evolves in a circuit. For visualization, consider the simple model circuit of Figure 3a, which may (or may not!) be of interest for the

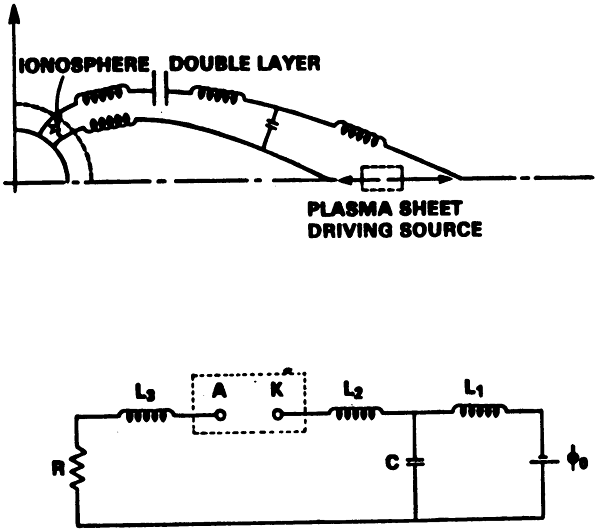

Fig. 3. (a) Schematic of a simplified model circuit for the terrestrial aurora. The circuit parameters are distributed. (b) Lumped network model of the circuit in (a).

terrestrial aurora and, perhaps, topologically similar astrophysical situations. For simplicity, I shall assume here that this circuit can be represented by the lumped-element network of Figure 3b. In Figure 3a the DL is an active nonlinear element, which in Figure 3b is represented by the anode-cathode pair A-K with variable voltage $\phi_{DL}(t)$. Analyzing this circuit, we find three time constants $\tau_m(R, C, L_1, L_2, L_3)$, where m = 1, 2, 3. The current density $J_0(t)$ through the DL is given by

$$J_0(t) = \sum_{m=1}^{3} J_m e^{t/\tau_m} + \int^t dt' \, G \, [\phi_{DL}(t'), \frac{t'}{\tau_1}, \frac{t'}{\tau_2}, \frac{t'}{\tau_3}], \qquad (4)$$

where the J_m are constants and G is a functional which, were it exhibited, would look more complicated than it really is.

We see from (4) that the global currents in the circuit depend on the time history of the DL itself. But $J_0(t)$ provides the boundary condition on the DL evolution, and so there is a mutual feedback effect between the DL and the circuit. Moreover, this feedback involves a nonlocal influence of the DL on the kinetic distributions of the plasma that enters the DL, in order that they provide contributions to $J_0(t)$ that are self-consistent with the contributions of the particles that have traversed the DL. Thus, over the course of the DL evolution the distributions of the incoming particles, which correspond to the boundary conditions in a stationary BGK description, can change considerably from their initial values. To simulate DL evolution in space and astrophysical circuits, boundary conditions which model this nonlocal feedback must be employed. This has not yet been done.

As a second example, the ion transit time

$$\tau_{tr} \simeq \ell_{DL}/U_i \qquad (5)$$

is an important time scale for the DL evolution. In (5), ℓ_{DL} is the length of the DL region and U_i the characteristic speed of a free ion entering the DL (ℓ_{DL} and U_i may be time-dependent, in general). The dynamics of the evolution may depend heavily on the value of τ_{tr} relative to the circuit time constants τ_m.

Smith (1982a) simulated DL evolution in a simple LR circuit with a constant voltage source; this circuit has one time constant, τ_{ind} = L/R. In runs with $\tau_{tr} \simeq \tau_{ind}$ and with the initial and injected distributions chosen to satisfy to Bohm criteria (1), Smith observed the following dynamics: (i) In the initial phase, a linear Buneman-like instability develops, with the unstable wave envelope growing spatially in the direction of electron drift; (ii) As electrons become trapped, the potential profile rectifies and its dominant length scale changes from the short wavelength of the unstable waves to a longer scale; (iii) Next, ions become trapped, developing holes in the ion phase space. In this stage, the DL potential is already "strong"; i.e., $e\phi_{DL} >> T_e$. There are rapid overshoots and undershoots of the potential $\phi_{DL}(t)$, involving complex phase - space dynamics; (iv) The ions became detrapped, damping the potential oscillations and leading to the transition to a strong, laminar BGK state. In the steady state, the current density has diminished by a factor of 15 from its initial value, and ϕ_{DL} is slightly less than the driving potential ϕ_0. The time history of $\phi_{DL}(t)$ in one such case is shown in Figure 4a.

In contrast to Smith, Belova et al. (1980) did not employ a circuit model, but attempted to simulate a constant current source by injecting constant electron and ion distributions, which were also Buneman-unstable. A constant current source corresponds to an LR circuit with $L \rightarrow \infty$, and thus $\tau_{ind} \rightarrow \infty$. Belova et al. observed recurrent explosive development of the potential, the explosions occuring within a few ion plasma periods and recurring on approximately the τ_{tr} time scale (Figure 4b). The reason for these contrasting results is not yet fully understood, but seems to be related to the differing values of τ_{tr}/τ_{ind}. In the case $\tau_{ind} \simeq \tau_{tr}$ the DL-circuit feedback stabilizes the dynamics, while in the case $\tau_{ind} \rightarrow \infty$ the DL continues to grow until the influxes can no longer sustain it (Yu. Sigov, private communication).

Finally, let us consider the influence of length scales on DL structure. A well-known scaling result for strong DL, first found by Goertz and Joyce (1975), is a relation between the potential ϕ_{DL}, scale length ℓ_{DL}, and upstream density n_{e1} (c.f. Figure 1):

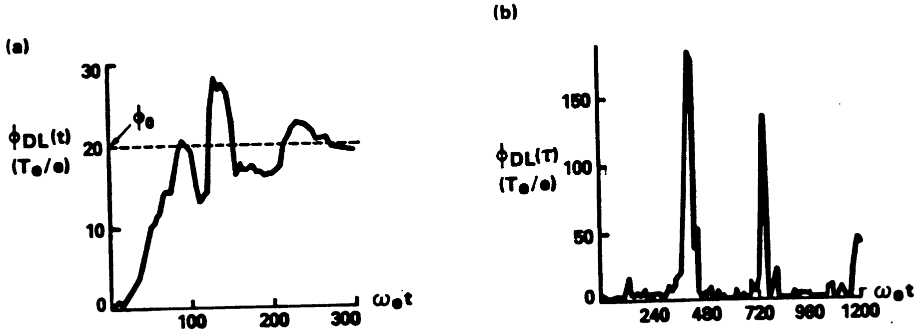

Fig. 4. Time histories of the double layer potential in two numerical simulations. (a) LR circuit with $\tau_{ind} \simeq \tau_{tr}$. After Smith (1982a). (b) Constant injected current, $\tau_{ind} \rightarrow \infty$. After Belova et al. (1980).

$$\phi_{DL} = \alpha \; n_{e1} \; \ell_{DL}^{2} \; , \tag{6}$$

where α is an empirical constant. Recently, however, Chan and Hershkowitz (1982) showed that there is some length ℓ_* such that, for a given ϕ_{DL} applied across a system of length ℓ_{sys}, a BGK structure of the type described by Eq. (6) develops when $\ell_* > \ell_{sys} > \ell_{DL}(\phi_{DL})$. When the ordering is changed to $\ell_{sys} > \ell_* > \ell_{DL}\ (\phi_{DL})$, however, the laminar structure splits into a "multiple-DL" structure (Figure 5) in which both electron plasma and ion-acoustic oscillations develop. This phenomenon is not well understood. In particular, we don't know what determines ℓ_*.

We note, through, that the same basic physical phenomena, involving current interruption by particle trapping, seem to be important in both the strong Buneman DL and the weak ion-acoustic DL. The very different phenomenology between the two types indicates important differences in the underlying dynamics, however. The principal reason seems to be that in the Buneman cases of Smith (1982 a, b; 1983), the inertia of the drifting ions causes them to be trapped later than the electrons; the nonlinear phase of the evolution is controlled first by the electrons, while the intermediate and final stages are dominated by the ions. In the ion-acoustic regime, the ion dynamics dominate throughout. On physical grounds, therefore, one expects that in any circuit, the laminar length scale $\ell_{DL}\ (\phi_{DL})$ is determined by kinematics, ℓ_* is determined by a transition from kinematics to dominance by the ion dynamics, and ℓ_{IA} is determined solely by ion dynamics. The available system length ℓ_{sys}, together with the time constants τ_m, are governed by the circuit topology.

I have chosen to discuss these issues of DL structure and dynamics not only because I believe that they are fundamental ones, but because they may relate to observable consequences for astrophysical models of double layers. For example, the time-dependence of radiation signatures may be related to the dynamical considerations involving the time scales for local and global evolution of circuits containing DL. The

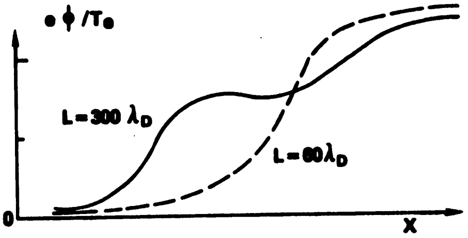

Fig. 5. Transition from single to "multiple" double layers as system length increases (Courtesy of C. Chan and N. Hershkowitz).

spectral characteristics of radiation may be related to the distributions of accelerated particles, which will be nearly monoenergetic for some distance after having transversed a strong BGK DL, but much more thermalized if they have transversed a turbulent region containing ion-acoustic DL.

I shall close this discussion on a frankly speculative note. The transition from BGK to "multiple" DL, with the attendant appearance of IA oscillations, leads one to wonder whether a gross extrapolation from laboratory scales to space or astrophysical scales might lead to the aforementioned sequence of structures, in which the BGK and IA DL are limiting cases. Such a sequence is indicated schematically in Figure 6. This would be appealing, but some caveats should be borne in mind. First, the experimental results on "multiple-DL" are few in number and are limited to showing two such structures, so we don't know whether a "many-DL" structure such as shown in Figure 6c could exist. Second, although it seems plausible to hypothesize that successive fractalizations of the sequence in Figure 6 would involve increasing levels of LF turbulence and might lead to a turbulent region of multiple IA DL as in Figure 6d, it is not yet established that recurring IA DL give a cumulative potential drop.

Such caveats, in fact, are illustrative of the introductory theme; at present, discussion of double layers in space and astrophysical contexts must rely on uncomfortably large extrapolations of our current lore of theory, experiment, and simulation. Such extrapolations require judicious appraisal of the effects of experimental factors (grids, walls, volume ionization, etc.) and boundary conditions on the interpretation of these different types of results. A deeper

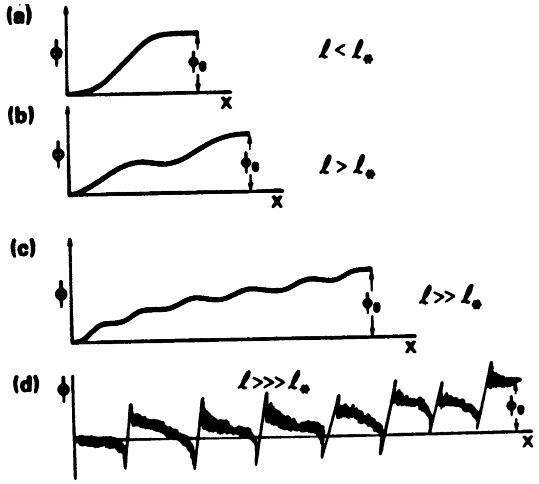

Fig. 6. Hypothetical sequence of double layer structures ordered by system length, with BGK and ion-acoustic double layers as limiting members.

realization of the difficulties in translating our current knowledge of DL into astrophysical contexts, however, can help us direct future work toward considerations of importance for space and astrophysical applications. The experiments of Stenzel, Gekelman, and Wild (1983) already point in this direction. Numerical simulations can put more emphasis on elucidating structure and dynamics under boundary conditions modeling global circuits. Finally, but crucially, all these approaches must ultimately consider observable tests for DL in astrophysical models. Then we can begin to work on Alfvén's paradigm.

I am grateful to C. Chan, N. Hershkowitz, W. Lotko, K. Papadopoulos, D. Spicer, and L. Vlahos for useful and stimulating discussions. In addition, I thank C. Chan and N. Hershkowitz for permission to use the previously unpublished Figure 5. This work was supported by and performed at the Naval Research Laboratory, under Contract No. N00014-83-C-2034.

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DISCUSSION

Kundu: Can you give some example of astrophysical situations where the concept of double layer has been applied in detail?

R. Smith: Double layers have been proposed to be important in applications such as auroral arcs, solar flares, the Io-Jupiter circuit, etc., and this morning we saw a beautiful experimental observations of double layers in a reconnecting current sheet. But I think its fair to say that although some authors have tried to approach the evolution analytically, one really can't go very far beyond treating models of possible triggering mechanisms, i.e., one can't take the analysis into the nonlinear stage. As for considering questions of the sort I've tried to raise here, no papers have addressed them.

van Hoven: What sets the width of the spikes you showed for the case in which the external-circuit time constant is very long?

R. Smith: It is between a few ion plasma periods and the ion crossing time. That is because the ion inertia governs the phase-space dynamics both in the later stages of evolution and in the breakup.

Sturrock: What is the nature of the "turbulence" which shows up in systems with multiple double layers?

R. Smith: In the so-called "stairstep" double layers, there is both high frequency (electron plasma frequency) and low frequency turbulence, which I believe the authors identify as ion-acoustic.

Wentzel: If one reads the J. Geophys. Res., one gets the impression that double layers are well established. Why do you disagree?

R. Smith: Generally speaking, one finds two types of articles concerning double layers in the current literature. First, there are various simulation papers with titles referring to double layers in the aurora, for example. They contain some useful results, but one must be very careful because their boundary conditions are not appropriate to treat auroral dynamics, and they are also not related to any circuit concepts. I've published these comments elsewhere. Second, there are reported observations of double layers, from S3-3. These concern the so-called "ion-acoustic" double layers to which I referred at the end. These are very different from the "classical" D.C. concept about which I've been speaking, and seem to be linked with anomalous resistivity.

Bratenahl: Can you quickly tell me what is wrong with the Bohm and Langmuir conditions, i.e., why they are not applicable?

R. Smith: These conditions emerge from fluid-theory analyses, assuming adiabatic acceleration (the Bohm criterion, e.g., is written in terms of the adiabatic indices) and in the limit of infinite potential. But the double layer problem is bounded, and therefore, on the one hand, there is significant heat flux through the double layer, and on the other hand one must consider kinetic descriptions on half spaces in velocity at the boundaries. In general, though there is some condition such as the Bohm condition, it is not straight forward to find, and thinking in terms of fluid drifts is quite misleading.