

STATISTICAL TEST OF THE DISTRIBUTION OF PERIHELION POINTS
AND ITS IMPLICATION FOR COMETARY ORIGIN

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ABSTRACT. The distribution of perihelion points of long-period comets is known to cluster towards the solar apex, and some authors ascribe it to north-south asymmetry in the distribution of observers. Validity or otherwise of this alleged selection effect is tested by randomly picking up the same number of perihelia in the southern ($\delta < 0$) as those in the northern ($\delta > 0$) hemisphere. It is shown that the observed clustering cannot be ascribed to the asymmetry of observers. Further, 67 comets which are *new* in Oort's sense are tested similarly. The character of their distribution is similar to that of all the known comets. It appears difficult to interpret the clustering in terms of a recent stellar disturbance of the Oort cloud.

1. Introduction

Whether the comets originated in the primitive solar nebula and hence represent the most primitive form of the solar system material or they originated in the interstellar medium is one of the most important problems in cometary research. The most direct approach to this problem would be to measure the isotope abundance ratio of one of the comets, which may be achieved by the GIOTTO mission. However, careful analysis of the observational data as well as their proper interpretation by taking into account the dynamical evolution of the cometary orbits can provide an answer. I have recently (Yabushita 1983) reviewed the processes of dynamical evolution, and it need not be repeated here. Relevant problems are;

1. Is the distribution of original $1/a$ values as calculated by Marsden, Sekanina & Everhart (1978) consistent with the assumption of steady state Oort cloud ?
2. How does the planetary perturbation change the distribution of $1/a$ values ?

I have shown (Yabushita 1983) that the present cometary population cannot be regarded as being in a steady state. There is too much excess of new comets in Oort's sense. Everhart (1979) earlier pointed out that three out of four *new* comets are not observed, if the cloud is to be in a steady state. On the other hand, it has been shown that if

comets are injected into the observable region 6 ~ 9 million years ago, the planetary perturbation can bring the $1/a$ distribution to what it is observed now (Yabushita 1979a). Napier & Clube (1979) argue that there was a comet capture event 10 million years ago, as the solar system passed through the Gould belt.

If the cometary capture from interstellar medium is a correct theory, then there should be peculiarities arising from the solar system motion relative to the nearby stars.

2. Statistical test.

It has been known (see for instance, Hasegawa 1976) that the perihelion points are not distributed randomly, but clustered toward the solar apex. Let $(\bar{l}, \bar{m}, \bar{n})$ be the direction cosines given by

$$\begin{aligned}\bar{l} &= \frac{1}{N} \sum \cos L_i \cos B_i, & \bar{m} &= \frac{1}{N} \sum \sin L_i \cos B_i \\ \bar{n} &= \frac{1}{N} \sum \sin B_i\end{aligned}$$

where (L_i, B_i) are the ecliptic longitude and latitude of the perihelion point of the i -th comet. We write (λ, β) to denote the longitude and latitude specified by $(\bar{l}, \bar{m}, \bar{n})$.

Tyror (1957) obtained $\lambda = 261^\circ$, $\beta = 71^\circ$, while Yabushita (1979b) obtained $\lambda = 259.7^\circ$, $\beta = 66^\circ$ from Marsden's (1972) catalogue.

On the other hand, the ecliptic coordinates of the solar apex can be calculated from the data given by Allen (1976);

$$L = 271.5^\circ, \quad B = 53.4^\circ.$$

Thus, it is apparent that the direction of the solar apex is away from the direction of the clustering of the perihelia by less than 20 degrees, and the closeness of the two directions have been noted by all of the authors referred to above. On the assumption that the Oort cloud is primordial, one would expect a uniform distribution of perihelia over the sky. The probability of the two directions being separated by less than 20 degrees is $(1 - \cos 20^\circ)/2 \sim 0.03$ on the null hypothesis of uniform distribution.

Some authors (cf. Kresak 1975) argue that since there are more observers in the northern hemisphere and since comets are bright while close to perihelia and are more likely to be discovered, the closeness of the two directions merely reflects the asymmetry of the distribution of observers over the globe; that β is positive might reflect the asymmetry of the distribution of the observers.

In order to investigate if the asymmetry gives any bias concerning the calculated values of λ and β , the present author (Yabushita 1979b) proposed the following test.

A larger number of comets have perihelia in the north of the ecliptic ($B > 0$) than in the south. For instance, in Marsden's (1972) catalogue, there are 307 comets with $B > 0$, while there are 196 comets with perihelia in the south of the ecliptic. In order to eliminate the alleged bias which might arise from the larger number of comets with $B > 0$, take equal numbers of comets with $B \geq 0$ and those with $B < 0$, and calculate the average direction cosines $(\bar{l}, \bar{m}, \bar{n})$. The direction so

obtained should be free from the effect which might arise from the inequality in the numbers of comets with $B > 0$ and those with $B < 0$. In case of 503 comets contained in Marsden's catalogue (1972), it would be appropriate to take 196 comets among those with $B > 0$, since there are 196 comets such that $B < 0$. However, since there are many ways of picking 196 comets among the 307 comets with $B > 0$, the direction cosines $(\bar{l}, \bar{m}, \bar{n})$ are not uniquely calculated. The 196 comets may be randomly chosen among the 307 comets by a Monte Carlo method. One will then have a distribution of $(\bar{l}, \bar{m}, \bar{n})$, or a distribution of (λ, β) . (see Figs. 5,6,7 of Yabushita 1979b)

If the direction of the solar apex is not far from a region where the calculated points (λ, β) are densely distributed, the alleged effect due to the asymmetry of the distribution of the observers cannot be accepted, and the closeness of the two directions (solar apex and the direction (λ, β)) will be real. In the earlier paper, I have calculated the directions by

$$\begin{aligned} \bar{l} &= \frac{1}{2} (-0.1232 + \sum \ell_i / 196), & \bar{m} &= \frac{1}{2} (-0.0331 + \sum m_i / 196) \\ \bar{n} &= \frac{1}{2} (-0.4366 + \sum n_i / 196) \end{aligned} \tag{2.1}$$

where (ℓ_i, m_i, n_i) are the direction cosines of 196 randomly chosen comets among 307 comets with $B > 0$, and $(-0.1232, -0.0331, -0.4366)$ are the average direction cosines of 196 comets with $B < 0$.

Now, the ecliptic is inclined to the equator so that the north-south asymmetry over the globe is not the same as the asymmetry with respect to the ecliptic. So, it will be more appropriate to divide the comets according as $\delta > 0$ or $\delta < 0$, where δ is the declination of a cometary perihelion. The following table gives the average direction cosines of 503 comets in the Marsden catalogue.

Table 1. Numerical values of $(\bar{l}, \bar{m}, \bar{n})$ calculated from Marsden's (1972) catalogue of cometary orbits.

N	\bar{l}	\bar{m}	\bar{n}	classification
293	0.04534	0.1568	0.4872	$\delta > 0$
210	-0.08896	-0.3643	-0.3481	$\delta < 0$
503	-0.01073	-0.06076	0.1385	$\delta \geq 0$
174	0.03151	0.1359	0.5026	$q < 1 \text{ AU } \delta > 0$
134	-0.05956	-0.3374	-0.4012	$q < 1 \text{ AU } \delta < 0$
119	0.06556	0.1875	0.4647	$q > 1 \text{ AU } \delta > 0$
76	-0.1408	-0.4119	-0.2546	$q > 1 \text{ AU } \delta < 0$

Figs. 1 ~ 3 show the distribution of (λ, β) which are calculated from equations similar to (2.1). Since a set of (λ, β) is obtained from one random sampling, we get a distribution of (λ, β) and it is possible to judge if the solar motion is related to the clustering direction of perihelion points. Figs. 1 ~ 3 show the distribution of (λ, β) so obtained.

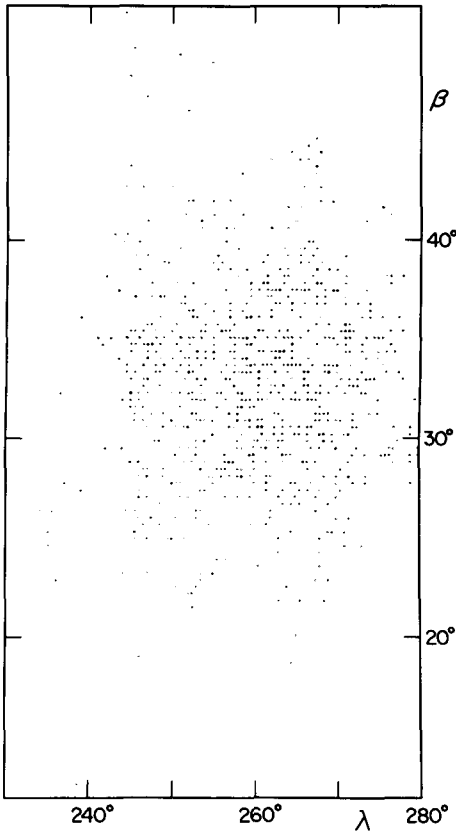


Figure 1

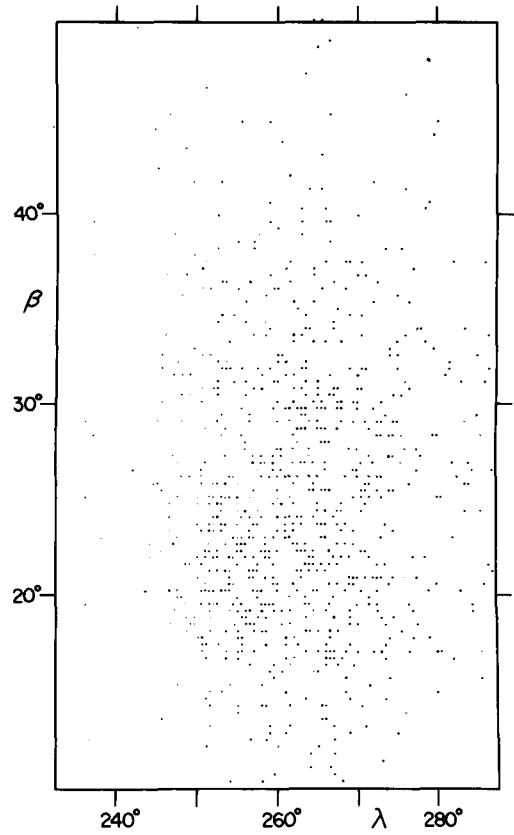


Figure 2

Fig.1. Distribution of directions calculated by adopting the same number of comets with perihelion points in the north ($\delta > 0$) and those in the south ($\delta < 0$). The comets are not classified according to the perihelion distance, q . 1000 random directions have been generated.

Fig.2. Distribution of average directions calculated by adopting the same number of comets with perihelion points in the north as those in the south. Those comets such that $q < 1$ a.u. are taken into account.

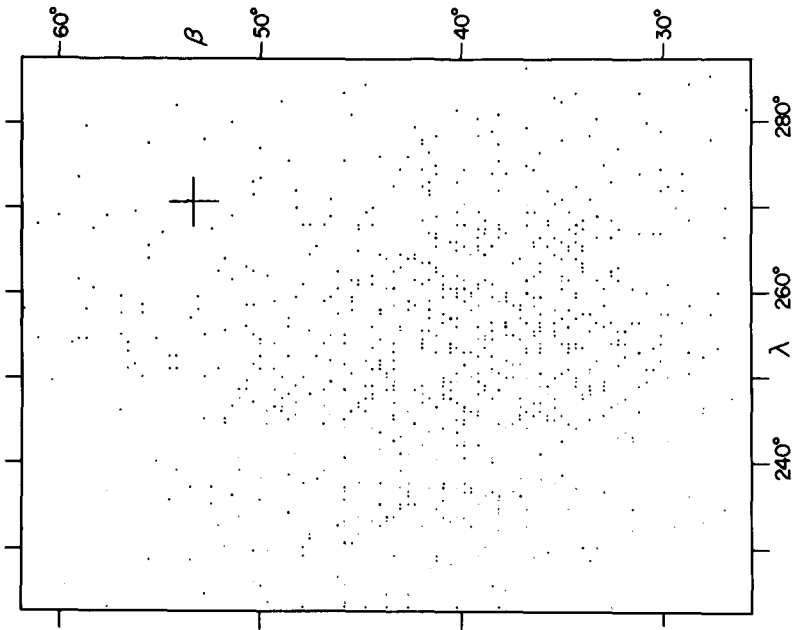


Figure 3

The same as Fig.2, except that comets with $q > 1$ a.u. are taken into account. The cross denotes the solar apex.

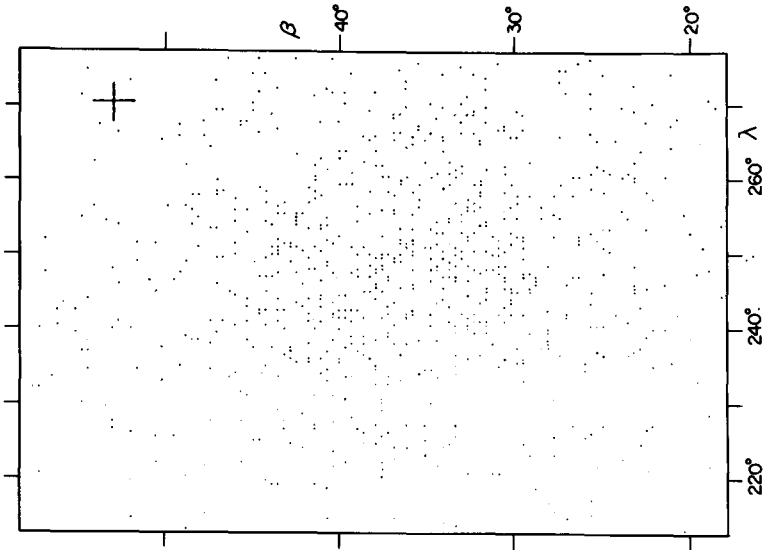


Figure 4

Distribution of average directions by taking the same number of comets with perihelia in the north as those in the south. New comets in Oort's sense are taken into account.

One may note that the direction of the solar apex, ($L = 271^{\circ}5$, $B = 53^{\circ}4$), is not far from the direction where (λ, β) points are densely distributed. In Fig.1, which include comets with q (perihelion distance) ≥ 1 AU as well as, $q < 1$, the distance between the two is some 18 degrees or so. In Fig.2, which corresponds to comets such that $q < 1$ AU, the distance between the two is 25 degrees. On the other hand for $q > 1$ AU, the two directions are separated by 15 degrees or so, as one can see from Fig.3. Almost the same was obtained earlier, where comets were divided into two groups according to the latitude (B).

Next we consider *new* comets in Oort's sense. There are 67 comets such that original $1/a$ is less than $5 \times 10^{-5} \text{AU}^{-1}$ ($a \geq 2 \times 10^4 \text{AU}$) including those whose original $1/a$ values are negative. A test similar to the previous ones has been done and the result is shown in Fig.4. Here, one does not see any marked difference from Figs. 1 ~ 3. We may conclude that the distribution of the *new* comets is almost similar to the distribution of the comets as a whole. Thus, it seems difficult to interpret them as star stracks through the Oort cloud, as suggested by Biermann *et al.* (1983).

One also notes that among the *new* comets, the number of those with q (perihelion distance) greater than 1 AU is greater than those with $q < 1$ AU. This reflects that they are intrinsically brighter than the others.

Table 2. Average direction cosines of 67 comets such that original $1/a$ as calculated by Marsden, Sekanina & Everhart is less than $5 \times 10^{-5} \text{AU}^{-1}$.

N	\bar{l}	\bar{m}	\bar{n}	classification
36	-0.01064	0.07554	0.5387	$\delta > 0$
31	-0.1151	-0.3606	-0.3026	$\delta < 0$
67	-0.05898	-0.1262	0.1495	$\delta \lesssim 0$
47				$q > 1 \text{ AU}$
20				$q < 1 \text{ AU}$

3. Summary and discussions.

It is known that perihelion points of long-period comets cluster toward the solar apex. Some authors criticize that this could be due to the north-south asymmetry in the distribution of observers over the globe. A statistical test has been made in order to see the validity or otherwise of the criticism. It has been shown that when equal numbers of comets with perihelia in the north and in the south of the equator are adopted, they still cluster toward the solar apex. This remains true even if comets are divided into two groups, namely $q > 1$ AU and $q < 1$ AU. Further, the same is true with *new* comets in Oort's sense.

That the preferred direction of cometary perihelia is close to the solar apex is difficult to be reconciled with the intra-solar system origin of comets, Biermann *et al.* (1983) regard this trend of the new comets as the result of recent stellar encounters, but the present study

shows that the statistical character of the distribution of perihelia of the new comets is not different from that of the rest. Thus, a capture of interstellar comets (by the action of Jupiter, or by direct interaction with the molecular clouds which contain comets (Clube & Napier (1984)) appears to be in better accord with the present study.

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DISCUSSION

Kresák: Did you also consider the longitude-dependent biases? For example, the Holetschek effect, when combined with the discovery opportunities varying as a function of solar longitude (length of the night, seasonal variations of cloudiness at the contributing observatories), should make comet discoveries more probable for particular perihelion longitudes.

Yabushita: I agree with you in that such an effect may exist. This problem is now under investigation by I. Hasegawa and myself using a most recent database.

Lüst: 1) I would like to underline the comment just made by Dr. Weissman. It should be interesting to compare the small arc around the apex with the overall distribution on the complete sky. There may be other clusters of comparable density.

2) The results depend on the material. Dr. Yabushita worked with a comet sample different from that of our investigation in which we found a somewhat different result. It further more depends on the exact position adopted for the apex. If we shift that position for some degrees, it is also located in a dense region of perihelia. This is, however, not meaningful because there are about 5 such dense "groups" distributed over the whole sky.