TOPOLOGY OF THE UNIVERSE

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ABSTRACT. Topology of the universe is the remains of quantum cosmology. The theoretical and observational aspects for the topology of the universe are discussed to show that the significances of topology of the universe in present observations can shed some light on the properties of the universe in the quantum cosmological era.

1. INTRODUCTION

"Is the Earth flat or spherical?" was a vexed question for ancient scholars. Today cosmologists face a similar vexation: what is the shape of the universe?

The difficulty of determining the shape of the Earth in the ancient days was due to that the observable area was much smaller than the whole surface of the Earth. Even in that time, several ancient scholars justfied the spherical shape of the Earth. This conclusion was obtained from the following two reasons:

a) The curvature is locally observable. It was found that the curvature radius of the Earth is finite.

b)The surface of the Earth is homogeneous, namely, the curvature is the same everywhere.

Similarly two points can also be found in modern cosmology:
a) spacetime curvature of the universe is locally observable;
b) cosmological principle, i.e. the space time curvature is the same everyzwhere in the universe. However, from the above-mentioned two points we would not be able to find definite conclusion on the shape of the universe, because the metric do not determine the topology of the spacetime as a whole.

For instance, the metric of a flat universe(k=0) is given by $ds^2=-c^2dt^2+R^2(t)(dx^2+dy^2+dz^2)$. The spacelike section of t=const. is an infinite 3-dimensional Euclidean geometry, $-\infty<x,y,z<\infty$. If we do the identification of points (x,y,z,t) with $(x+la_x, y+ma_y, z+na_z)$ for all integers l,m,n, the spacial section becomes then 3-dimensional torus T^3, in which the spacial volume is finite. Different topologies can be formed from a metric by different identifications. T^3 is one of 18 topologically different types of identifications in the case of flat

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universe

The following statement can ofen be found in text books: the spacial volume of open(k=-1) and flat universe are infinite, close universe(k=1) is finite. In fact, this statement implies that the topology of the universe is simply connected. But there is no reason, both observatioal and theoretical, to show that the topology of the universe must be simply connected. Therefore, a fundamental question in cosmology is: what is the topology of the universe?

The topology of the universe is not important in many cosmological topics, such as the origin of microwave background radiation, nuclear synthesis, formation of galaxies, etc.. In discussing these problems, we only need the cosmological principle as boundary conditions, while the topology is neglegible.

The development of quantum cosmology has made that the topology of the universe would not be neglectable. The aim of quantum cosmology is to study the universe in the Planck era, in which the main process was the formation of spacetime itself, i.e. spacetime as a whole becomes dynamical variable in quantum cosmology.

On the other hand, the theory of the topology of the universe is testable Since Einstein's equations of general relativity determine only the metric of spacetime, but not the topology, it means that the topology of spacetime is invariant in the period described by classical gravitation. Therefore, after the Planck time, i.e., the era of the formation of spacetime topology, the topology never changes; the topology observed in present universe is the same as that in Planck era, i.e., the era of cosmological age $\sim 10^{-43}$ sec.. In other words, cosmological topology is the remains of Planck era.

The significance of cosmological topology can be expressed by Table 1, in which we list all the important remains and eras of their formation. Cosmology studies the evolution of the universe from the

TABLE I. Cosmic Remains of Various Eras

Remains	Age of the universe
Topology of spacetime	10^{-44} seconds
Matter-antimatter asymmtry	10^{-34} seconds
Abundances of elements	3 minutes
Microwave background radiation	10^{5} years
Objects with large red-shift	$\sim 10^{10}$ years ago

remains left over various cosmic eras. Therefore, the topology problem may be the key of studying the universe at the earliest era, i.e. the era of creation of the universe. From Table 1 one can also find a systematic property: the earlier the remains, the stronger(or harder) the associated interaction; the chronological order of the remains is particle-nuclear-radiation-galaxy, which corresponds to the order of interaction intensity, strong-electroweak-gravitation. Spacetime is the hardest. In most problems of physics, spacetime is considered as a fixed plateform on which all physical processes take place. This is the same as to assume that spacetime is the hardest among all physical entities. We should therefore search for the remains of Genesis in the properties of space-time. This is just the topology of the universe.

2. POSSIBLE TOPOLOGIES OF THE UNIVERSE

Immediately after Einstein gave the cosmological model which has S^3 as the configuration of the space, Klein noticed that it is equally possible for the space to have P^3, the 3-dimensional projective space as the configuration. But the fact is that models with S^3 topology have received far more considerations than those with P^3 topology. The reason for this comes more from human's prejudice than from physical considerations. This preference eliminates from our interests many models that are as possible to be a cosmological model as the ones we prefered.

The number of the candidates is very large, as matter of fact, mathematicians have given an almost complete classification of the topologies for 3-dimensional spaces(Scott 1983), before physical considerations, they are all possible candidates for the 3-dimensional space-like hypersurfaces of spacetime, so we in fact have an infinite number of candidates.

In cosmology, model chooing must rely on physical considerations and observations. There are many physical considerations, such as the orientability and the differentiability of the spacetime, etc., we will give here physical considerations related to the operations of identifications to give various topologies. In static spacetime, the identifications can be made at any time, because the geometry as well as the topology does not change with time. But for a model with expanding as our universe is, in order that the identification at one time be the same as that at another time, the universe must be expanding conformally or the identification is made in some special way. Three classes of models of this kind is of special interest they are the ones with constant curvature. The one with positive curvature k=1(class I) is locally diffeomorphic to S^3, the one with null curvature k=0(class II) is locally difeomorphic to E^3 and the one with negative curvature k=-1 (class III) is locally difeomorphic to H^3. They correspond to the density parameters of $\Omega > 1, =1, < 1$ respectively. the metrices for these three classes are given by

$$dl^2 = dr^2 + f^2(r)(d\theta^2 + \sin^2\theta \, d\varphi^2) \tag{1}$$

where

$$f^2(r) = \begin{cases} \sin^2 r & \text{(class I)} \\ r^2 & \text{(class II)} \\ \mathrm{sh}^2 r & \text{(class III)} \end{cases}$$

As is the mathematical result, all spaces of the three classes can

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be classified(not completely) topologically according to the discrete subgroups of the isometry groups of S^3, E^3, H^3 respectively; they are the quotient spaces of the kinds S^3/Γ , E^3/Γ , H^3/Γ respectively with Γ the discrete subgroups acting without fixed points. It is equally to say that they can be obtained from properly identifying points in S^3 E^3 and H^3 .

Class I. This class is obtained from properly identifying points in S^3 , so one can write compactly the class as $\Sigma_1 \approx S^3/\Gamma_1$ where Γ_1 is the discrete subgroup of the isometry group SO(4) acting without fixed points. Noticing that SO(4) is diffeomorphic to $SO(3)\times SO(3)/Z_2$. So if we have the discrete subgroups for SO(3) , we can find Σ_1 by binary identifications in both SO(3) group manifolds. It is well known that there are 7 kinds of such discrete subgroups for SO(3) , they are, Z_2 , the 2-cyclic group; Z_n (n>2), the n-cyclic groups; D_m (m>2), the dihedral groups; T, the regular tetrahedron groups; O, the regular octahedron group; I, the regular icosahedron group. So there are infinite number topologies for Σ_1. To see the identifications more clearly, one can represent SO(3) manifold diagrammatically as follows: choosing two balls S_1 , S_2 of radius π with antipodal points on each surface identified. Rotation in the direction (θ, φ) by $\alpha<2\pi$ is given by a point with radical coordinate α and azimuthal coordinates (θ, φ) in S_1 , while rotations in the direction of (θ, φ) by $2\pi \leq \alpha < 4\pi$ is given by a same point in S_2 . Rotation with $\alpha=4\pi$ will return to the original point. If one identifies tow points on S_1 and S_2 with α-coordinates differ by 2π , one thus obtains P^3 which is diffeomorphic to SO(3) manifold. So the two balls thus constructed is diffeomorphic to S^3 , the identifications in S^3 can be achieved(topologically) by binary identification of points in the two balls.

Because of the topology complexity, it is very difficult to find for all cases the coordinates of the identified points in terms of (r,θ,φ) in equ.(1). But for the simplest case with P^3 configuration, which is the only one in this class that has the same symmetry as S^3 , the identification can be found. If one considers S^3 as the 3-sphere in 4-dimensional space with equation $x^2+y^2+z^2+w^2=1$, then P^3 is the identification $(x,y,z,w)\equiv(-x,-y,-z,-w)$ on S^3 , or, in terms of intrinsic coordinates, the identification of $(r,\theta,\varphi)\equiv(\pi-r,\pi-\theta,\pi+\varphi \pmod{2\pi})$ on S^3 .

Because of the compactness of the space in this class, multiple images can even occur in S^3 case. As matter of fact, because the geodesic for light is the great cycle on S^3 , it would be possible to find a conter image in the opposite direction to the objects. But it has been proved that for an expanding universe, is is impossible for such a conter-image to occur in S^3 case, because in this case the horizon size will be smaller than the length of the semicycles. But it is possible for P^3 case, if the deceleration parameter q_0 at present time were greater than 1, there would be one or more images for one object(Narlikar and Seshadri 1985).

Class II. This class is obtained by properly identifying points in E^3 . The spaces in this class can be written as $\Sigma_2 \approx E^3/\Gamma_2$ where Γ_2 is the discrete subgroups of the isometry group for E^3 , that is $R^3\times SO(3)$ acting without fixed points. So if E^3 is considered to be an infinite crystal constituted of lattice cells, then class II can be obtained as identifications of points on the opposite faces of each cell(sometimes with a screw). The classification of this class is complete, there are 18 kinds of Γ_2, so we can have 18 different kinds of topologies. The first group consists of six compact. orientable members. They are the ones obtained by proper identifications of points on the opposite faces of rectengular lattice that is (i) identification of opposite surfaces that gives T^3, the 3 torus. (ii) identification of opposite faces. with one pair screwed by an angle π, (iii) identification opposite faces with one pair scewed by an angle $\pi/2$, (iv) identification opposite faces with all pairs screwed by an angle π and those obtained from the identifications of points on the lattice made by translating a hexagonal lattice plane a certain distance perpendicular to the plane, that is (v) identification of the two hexagonal faces with one screwed by an angle $2\pi/3$ with respect to the other (vi) identification the two hexagonal faces with one screwed $\pi/3$ with respect to the other The second group consists of four compact, nonorientale members they can be obtained in the same way as in cases (i)-(iv) but with some reflections. The third group consists of four orientable but noncompact members, it is obtained from identifications(with screws) in less than three pairs of lattice faces. They are (i) type $\mathcal{E}=E^3$, with no identification at all, (ii) type $\mathcal{S}_i^{\theta}$, with Γ_2 generated by a translation and a screw motion of an angle θ, (iii) type $\mathcal{T}_1$, with Γ_2 generated by two independent translations and (iv) type $\mathcal{K}$, with Γ_2 generated by a translation and a screw motion of an angle π in the direction perpendicular to the translation. The four non-orientable, noncompact manifolds can be obtained in the same way. It should be pointed out that the only one which preserves the maximal symmetry $R^3 \times SO(3)$ is E^3 itself. others are homogeneous but anisotropy because all identifications can not commute with the rotational group SO(3).

The coordinates of the identified points in E^3 for some cases can be found without great difficulty. For example, in order to get T^3-topology, one can identify point (x,y,z) in E^3 with points $(x+nL_1, y+mL_2, z+lL_3)$ with (n,m,l) the lattice numbers and (L_1, L_2, L_3) the lengthes of the rectangular lattice.

One reason for considering models in this class is that the inflation in the very early universe gave a flat spacetime, and, as will be discussed in the next section, compactness of the 3-space is prefered by quantum cosmology. If such is the case, we will inevitably have one of the six manifolds in the first group as our candidates. But they are anisotropic, so one must be very careful about the effects of such an anisotropy, when one considers an inhomogeneous model.

Whether or not one or more images will be observed for an object depends on whether or not the present horizon is larger than the smallest length of the lattice. But theoretically, one can choose so small a length scale for the universe as to have one or more images for one object, we do not face the problem as in class I.

Class III. This class can be obtained by properly identifying points in H^3. But the classification has not been completed in mathematics. There are three ways to proceed. The first is given by Löbell, Löbell(1931) found that H^3 can be fitted just once by the fundamental region formed by 14-sided figure with two sides are regular rectangular hexagons and the rest twelve rectangular pentagons. So we can find non-trivial topologies by properly identifying points in these fundamental regions. The second is given by noticing that the metric of 3-space with constant negative curvature can locally be written as(Ellis 1967)

$$d\sigma^2 = dr^2 + ch^2 r d\tau^2 \tag{2}$$

where $d\tau^2$ is the metric on the 2-surface H^2 with constant negative curvature. So the total 3-space can be written as $M^3 \approx R\times H^2$. Now the identification on H^2 can be obtained, for example, by attaching hundles on the surface, we thus have a group of 3-space with constant negative curvature in the form of direct product. The final way is given by the "translation" or "translation plus screw motion" on H^3 . As matter of fact, H^3 can be globally imbedded in Minkowski space F^4 ; it is invariant under a simply transitive group of Bianchi type V and a 1-parameter family of simply transitive group of Bianchi type VII_h. So one can find the discrete subgroups of these isometry groups and obtain the quotient space by these discrete subgroups. There are infinite number of 3-space in this class.

Astronomical observations made so far gives an open universe($\Omega \leq 1$), this plus the compactness of the three space required by quantum cosmology (as is mentioned before) lead inevitably to a space with nontrivial topology, this is one reason to study models in this class. The determinations of the coordinates for the identified points are extremely difficult. As a simple example, Fagundes(1985a) considered an unrealistic model with topology $H^2\times E$, and found the images in the 2-dimensional hyperbolic geometry H^2. As for the identification in H^3 , so far as we know, researches are remaining to be done.

We have just given a relatively detail list of 3-spaces which are made from properly identifying points in the maximally symmetrical spaces S^3 ,E^3 ,H^3 . The list is far from complete. There are many other alternatives which can not be excluded from our candidates(for a more complete list, see Fagundes(1985b)). The list given above only provides an illustration of how the constructions of models with different topologies proceeds and of how many alternatives we can have for the models of the universe.

3. CONSTRAINTS ON TOPOLOGY BY QUANTUM COSMOLOGY

We have seen in the last section how large a number of variants can be for the candidates of the models of the universe. But we believe that our universe exists with a certain specific topology. Now one question is why our universe has such a topology not the other. To answer this question, we therefore first turn to the observational results. That is the candidates chosen must be consistent with the observations we have for the universe today. The observational significances of topology of the universe, or reversely speaking, the constraints on the possible topologies of the universe by observations will be our main concern for our latter sections. In this section we will consider the problem theoretically, that is the constraints on topology by quantum cosmology.

If we speak quantum cosmology in the sense of quantum events on a background spacetime, the constraints come from the instability of some background spacetimes. For example, some authors have considered the instability of de Sitter space, they found that quantum corrections of the energy-momentum tensor causes an instable de Sitter space; only anti-de Sitter space is stable under the quantum correction. Although the constraint on the topology(especially the multiply connectedness) of the space is very loose, it indeed sheds light on the problem. Remember that in the nontrivial topology, there is a nontrivial constraint to the energy-momentum tensor of the fields from the gravitational casemir effect. The instability is thus related to the global properties of the spacetime.

If one considers quantum cosmology in the sense of quantum creation of the universe, interesting constraints can be found.

First we discuss the problem in a broad sense, that is, we admit that the state of the universe is given by a wavefunction(cosmic wave-function) which gives the probability amplitude for the universe to distribute with respect to some characteristic parameters. Topology being a very important property of the universe, so quantum cosmology, intuitively speaking, will give a choice of the topology. The dynamics of the wavefunction is given by Wheeler-DeWitt equation, the quantum cosmological analog of the Schrodinger equation. So one can find the possibility for the universe to transite from the quantum euclidean era (motions with imaginary time) to the classical lorentzian era. The amplitude for such transition is given by the WKB wavefunction $\exp(-|I_{cl}|)$ where I_{cl} is the euclidean action on the classical trajectories. Then if one wants to find the possibility of a universe with a certain topology, one must know how the topological effects can enter the action I_{cl}. Since I_{cl} is the integral of lagrangian density over the spacetime manifold, global properties of the spacetime that is implied in the action is the volume of the universe. For pure gravitational field, $I_{cl} \propto V^{2/3}$ where V is the volume of the universe(t=const.). One immediately find that the wavefunction is zero for $V \to \infty$, that means that the universe with infinite large volume can not occur from the quantum tuneling. One result of this is that, universe with null curvature(k=0) and negative curvature(k=-1) can occur with nonzero possibility only if it exists with nontrivial topology so that the space is compact. As mentioned before, models with k=0,-1 are preferred by both observations and inflationary theory, therefore nontrivial topology seems to be inevitable in this sense.

In the consideration of the transition, one must have in the field configuration a spacelike hypersurface which separates the field configuration into euclidean and lorentzian sections. This gives strigent constraints on the form of classical solutions of the model, this in turn gives some constraints on the topology. For example, if one considers the isotropic expanding universe E^3, if the effective cosmological constant Λ is positive, the expanding factor will be proportional to exp(Ht) with $H^2 = \Lambda/3$. Although $t \to -\infty$ is a singularity, there is no point in the scale factor that can separate the euclidean region and lorentzian region, the universe could then create from the classical

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singularity at $t \to -\infty$. If one considers such a model with T^3-topology, the space will be geodesically complete, so there is no singularity at $t \to -\infty$. In these cases, there is no sense for the quantum creation. But if one takes into account the gravitational casemir effect of the matter fields due to the nontrivial topology, the situation will change. For example, in the T^3-topology, the contribution of the casemir effect to the energy-momentum tensor will lead to a classical trajectory with the form $a(t) \propto (ch2Ht)^{1/2}$(Zeldovich,et al. 1984), there indeed exists a minimum a_{min} for a, in this case the region $0 < a < a_{min}$ corresponds to the euclidean era and the region $a > a_{min}$ corresponds to the lorentzian region. There is no problem for the interpretation of the quantum creation of the universe. It concludes that if one insists on the quantum creation of the universe, there are some constraints on the topology.

Now we turn to a specific quantum cosmology theory which is invented by Hawking and his colleagues. The corner stone in this theory lies in the ground state proposal of Hartle and Hawking(1983) which states that the ground state wavefunction for the universe (Σ, h_{ij}, ϕ) is given by the path integral over all compact manifolds M(compact euclidean space after continuation) which has Σ as its boundary with metric h_{ij} and matter field ϕ on it. One can see that the compactness of M poses very strigent constraints on the forms of the classical solution for the models chosen, these constraints are the same as that posed by the existences of the barrier separating the euclidean and lorentzian regions, so the early discussion still holes in this case. There are other two questions in this proposal:

(i) for a given Σ , does M always exist with Σ as its boundary?

(ii) For a field on Σ : $\phi : \Sigma \to \Phi$, can the field be extended to all of M such that there exists an induced mapping $\bar{\Phi} : M \to \Phi$?

The answer to the first question is positive if one considers a 4-dimensional spacetime, since for any 3-dimensional closed manifold Σ, there exists a 4-dimensional manifold M with its boundary diffeomorphic to Σ. But if one considers high-dimensional models, one can find manifolds which are not boundaries of any manifold. These cases should be excluded by the qantum cosmology.

The answer to the second question is not certain. For example, the conjectures made by Mkrtchyan(1986)(proved for some cases by Li(1986)) shows that if the topology of Σ is S^3 and the matter field assumes values on the group G, then the mapping $\bar{\phi} : M \to G$ which satisfies the conditions implied in Hartle-Hawking's proposal exists if and only if the solitonic charge of the mapping $S^3 \to G$ is zero. But this solitonic charge corresponds to the baryonic number in the model. So being zero is a contradiction. One way out of this may be the considering of spacetime manifolds with complicated topologies, but whether or not they can give nonzero baryonic number remains to be proved. The discussion here only serves to illustrate how quantum cosmology can give constraints on the topology of the universe.

We have seen that quantum cosmology indeed gives some interesting constraints(although very loosely) on the topology of the universe. The purpose of writing this section is not so ambitious as to give an affirmative answer to the question, but rather to show that, as to the choice of topology of the universe, we can no longer rely on our costums or prejudice, there are theories(the quantum cosmology) by our hands.

4. OBSERVATIONS OF THE COSMOLOGICAL TOPOLOGY

As mentioned in the introduction, the spacetime topology is not a dynamical variable in classical gravity. Topology is an invariant after the Planck era. Therefore, the topology of the present universe is the same as that formed by the event of the birth of the universe. The prediction on the topology given by quantum cosmology may then be testable by observations.

Up to now quantum cosmology would not be able to predict the detail of the topological type, but only on whether or not the universe is multiply connected. So we limit the discussion on the observable properties which can be used to distinguish simple and multiple connecticity of the cosmological space. In a simply connected space, the geodesic between any two points is unique, namely, each object can be observed only in one direction. While in a multiply connected space, the geodesics between two points are sometimes multiple, namely, each object can be observed simutaneously in several different directions. Let uz consider a two-dimensional torus which can be constructed from a plane by identifying point (x,y) with points (x+la,y+mb) with all integers l and m. An observer in such a torus will find that the observed picture is the same as a plane, but the distribution of matter is periodic with "wavelength" a in the x-direction and b in the y-direction, and the picture is like a plane lattice.

Is the above-mentioned periodicity in the distribution of matter observable? It depends on the length scale of the horizon and of the size of the universe. In a compactified space, the size of the universe is given by the distance L between the most widely separated points. In T^3 universe, $L \sim R(t)a_x, R(t)a_y, R(t)a_z$. Therefore, global properties which are due to the effect of topology should have a length scale larger than L. On the other hand, the scales of all observable properties should be smaller than the horizon L_H. So the necessary condition for the topological effects to be observed is

$$L_H > L \tag{3}$$

Since $L_H \sim cH_0^{-1}$, we have

$$L < cH_0^{-1} \tag{4}$$

This condition can also be obtained in the field theoretic regime. Let $\mathcal{E}_t$ be subspace of synchonous space Σ_t. Let Φ_1 and Φ_2 be two fields defined on Σ_t and identified on $\mathcal{E}_t$:

$$\Phi_1(\vec{x}) = \Phi_2(\vec{x}), \qquad \text{for } \vec{x} \in \mathcal{E}_t \tag{5}$$

But they can differ with each other on $\Sigma_t - \mathcal{E}_t$. If the observer can determine the properties of Φ_1 and Φ_2 only through interactions on $\mathcal{E}_t$.

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Then there is no way to distinguish Φ_1 and Φ_2. The question is whether Φ_1 and Φ_2 can be defined on spaces with different topologies. If it is the case, the observer can not rely on the Φ_1, Φ_2 interactions on $\mathcal{E}_t$ to determine the topology. It has been shown that, in general, Φ_1, Φ_2 can be defined with different topologies(Unwin 1982), hence for one to determine the topology through interactions of Φ_1, Φ_2, the interacting region $\mathcal{E}_t$ must be equal or larger than Σ_t. $\mathcal{E}_t$ is nothing but the horizon, so the condition is the same as eq .(3).

In the case of eq .(4), the light rays cross the whole volume of the universe more than one time, an object can be seen in more than one directions.

An important result in observational cosmology is that eq .(4) can not be ruled out by observations. The lower limit of L can be obtained from the fact of no image twins in some surveys of galaxies,such as Shane-Wirtanen sample(Shane and Wirtanen 1967) which gives $L \gtrsim 200h_0^{-1}$Mpc, where h_0 is the Hubble constant in unit of 100 km/s.Mpc. This lower limit is smaller than the present horizon L_H, which is about $3000h_0^{-1}$Mpc. It concludes that present observations do not exclude the possibility of the multiply connecticity of the cosmological topology.

5. POSSIBLE EVIDENCES FOR MULTIPLY CONNECTED TOPOLOGY

The lower limit of the cosmic size given in last section is even smaller than the distance of objects with high redshifts, such as quasars. Therefore, from the distribution of quasars one can already find some evidences which seems to show the multiple connecticity of the cosmic space.

5.1. Periodicity in the Distribution of Quasar Redshifts

Resently, it has been demonstrated from the statistical analysis of the observational data that the distribution of the emission line red shifts of quasars have a periodic feature with respect to the argument $x \equiv F(z,q_0)$ defined by(Fang 1982)

$$F(z,q_0)=\int_0^z \frac{dz}{(1+z)(1+2q_0 z)^{1/2}} \qquad (6)$$

where q_0 denotes the deceleration parameter. In the distribution of the red shifts there exists a set of peaks at z_n given by

$$F(z_n,q_0)=An+B \qquad (7)$$

where n is zero or positive integer and A and B are constants.

This above periodicity might be interpreted as the existence of a large scale periodic perturbation in the density distribution of cosmic matter. According to this interpretation, A is related to the "wave-length" of such periodic perturbation as(Fang 1982)

$$A=H_0\lambda_0/c \qquad (8)$$

the subscript 0 denoting the present value. From the statistical analysis, λ_0 has been estimated as

$$200h_0^{-1}\,\mathrm{Mpc} \lesssim \lambda_0 \lesssim 600h_0^{-1}\,\mathrm{Mpc} \qquad (9)$$

The constant B in eq .(7) is related to our position relative to the perturbation waves. However, it is very difficult to explain why B is not random depending on direction. If we were located in a preferred position near center of the large-scale spherical wave perturbation, the above relation would be explained but such interpretation would be not acceptable from the point of view of the cosmological principle.

If we insist on the simply connected universe, it will be very difficult to overcome this difficulty. But such a type of result can be obtained natually if we assume the multiply connected universe.

Let us consider the flat universe with topology T^3 for a simple example(Fang and Sato 1983). The redshifts of the multiple images of the source located at x_S in the x-direction is given from

$$x_S + na_x = \int_{t_n}^{t_0} \frac{c\,dt}{R(t)} \qquad \text{and} \qquad z_n + 1 = \frac{R(t_0)}{R(t_n)}$$

as

$$z_n + 1 = \left[1 - \frac{R(t_0)x_S}{2(c/H_0)} - \frac{R(t_0)a_x}{2(c/H\ \ )}n\right]^{-2} \qquad (10)$$

From eq .(7) for q =1/2, $A=R(t_0)a_x/(c/H_0)$ and $B=R(t_0)x_S/(c/H_0)$. Considering the $R(t_0)a_x/2\sim 400\mathrm{Mpc}$ and $R(t_0)x_S < a_x/2$, the quasars with $z < 3$ are the original image(n=0) or the ghosts of n=1 or n=2.

In the case of the whole-sky correlation, however, the situation is more complicated. For example, in the case of $a_x=a_y=a_z=a$, the units of space periodicities are Ra, $\sqrt{2}$Ra, $\sqrt{3}$Ra,..., depending on the observed directions. Then, the superposition of the two periodicities of Ra and $\sqrt{2}$Ra produces an approximate periodicity of 0.5Ra. In the actual situation, the observed periodicity may be the superposition of a few fundamental space periodicities.

In spite of the above ambiguity, we can predict as a general tendency that the "wavelength" of periodicity for the quasars in the given direction should be larger than the "wavelength" estimated from the whole-sky data. This result might be used to interpret the following observational evidence: the wavelength of the whole-sky data is about a half of the wavelength estimated from the quasars listed by Savage and Bolton(1979), which are the quasars in the two given directions of the south Galactic Polar region of $02^h00^m,-50^\circ00'$ and $22^h04^m,-18^\circ55'$.

5.2. Periodicity in the Distribution of Absorption Line Redshifts of Quasars

If the absorption lines in the quasar spectrum are due to absorptions of intervening objects, the distribution of absorption line redshifts should also show periodicity in a multiply connected universe.

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It has been found(Chu, Fang and Liu 1984) that there are several peaks in the number distribution of absorption lines with respect to β given by

$$\beta = \frac{(1+z_{em})^2 - (1+z_{ab})^2}{(1+z_{em})^2 + (1+z_{ab})^2} \quad (11)$$

where z_{em} and z_{ab} are the redshifts of emission and absorption, respectively. The peaks in β distribution were explained by so called "line-locking" mechanism. But to us, the meaning of such peaks is to provide more possibility of finding evidence of multiple connecticity. Indeed, the distribution of the peaks seems to be periodic. From the locations of the peaks, one can find the size of the universe L by the same way as that in the emission line case. An interesting result is that the size of the universe determined by β-distribution is about the same as that determined by emission line distribution.

5.3. Close Pairs of Quasars

It has been shown(Burbidge, Narlikar and Hewitt 1985) that the possibility of finding so many close pairs of quasars with different redshifts by chance projection is as small as $\lesssim 10^{-4}$. It seems too low to support the belief of cosmological origin of the redshifts. However, in a multiply connected universe, original images and its ghost images can locate in about the same direction. Considering this mechanism, one can show, close pairs of quasars do not contradict with cosmological hypothesis, it may be an evidence for multiply connected topology of the cosmological space.

In a simply connected universe, the probability of finding pairs with angular separation $< \theta$ by chance projection is given by

$$< s >_c = 2.4 \times 10^{-7} \Gamma(<m) N f^2 \theta^2 \quad (12)$$

where $< s >_c$ is the expected number of close pairs with angular separation less than θ, $\Gamma(<m)$ is the sky density of quasars brighter than magnitude m, expressed in units of $(\text{arcdeg})^{-2}$, N is the total number of quasars listed in catalogues which are brighter than m, f indicates the fraction of such quasars whose fields have been searched for close companions out to θ.

In a multiply connected universe, the probability $< s >_c$ should be enhanced by a factor

$$\mathcal{T} = (1+\delta)^{l_m} \quad (13)$$

where δ is the amplitude of inhomogeneity in quasar distribution, l_m is the the maximum order of ghost images. For weak clustering of quasars, $1+\delta \sim 1.3$-1.5. For the λ_0 given in eq .(9), $l_m \sim 3$-5. Therefore, the enhancement factor $\mathcal{T}$ is about four, which relax strongly the contradiction of the observed number of close quasar pairs with the expect number of eq .(12).

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5.4. Association between Quasars and Galaxies

Since 1971, after the first finding of bright galaxies associated with 3CQSO, many statitical analyses have been done to check a significance level of association between quasars and galaxies. Some analyses show a positive correlation(Chu and Zhu 1983; Seldner and Peebles 1979). It is very difficult to explain why the objects with such different redshifts are correlated in their positions. A possible way out of this problem also lies in the assumption of multiple connecticity. In the multiply connected universe, we can sometimes see the original image and its ghost images in the same direction. If we see some clump where the formation of quasar and galaxy were active, the association in appearance between the small redshifts galaxies of the original image and the large redshift quasars of the ghost images will result naturally. Since the quasar is thought to be a short-lived phenomena lasting only a few million years, we will see only one image of the quasar if $R_H \sim$ 400Mpc and $z < 3$. According to this interpretation, the associated regions may provide us a good sample for studying the evolutionary relationship between galaxy and quasar: the remnents of the quasars should belong to the same clump of the associated galaxies.

5.5. Length Scale of Clustering

In a simply connected topology, the length scale of clustering is admissible,in principle, up to t_0 , the size of the horizon $cH_0^{-1} \sim 3000h_0^{-1}$Mpc. However, in a compactified universe, the upper limit of length scale of clustering is given by the size of the universe. Namely, if the result of eq .(9) is correct, we can then predict that no clustering has scale larger than $\sim 400h_0^{-1}$Mpc.

The largest clustering found today is superclusters which have the length scale of about $100h_0^{-1}$Mpc. The scale of voids is also less than $100h_0^{-1}$Mpc. It has been found from the distribution of quasars that the clustering of quasars is much weaker than galaxies. All these seem to show that the prediction on the evidence of a cut-off in the clustering scale can be accepted by present observations. More conclusive result could be obtained in the near future from the Space Telescope.

5.6. Isotropy in a Multiply Connected Universe

The identification of finding multiply connected topology from Robertson-Walker metric will, in general, destroy the isotropy of the universe as a whole. For instance, a T^3 universe of $a_x=a_y=a_z$ is anisotropic, but with a symmetry of a cubic lattice. Since, even in this case, the expansion of the universe can be isotropic, the small scale isotropy of microwave background radiation is still ensured, as in the simply connected case.

Multipole component of the background radiation is sensitive to the global anisotropy. Therefore, we should discuss whether or not the present observations on quadrupole component of background radiation is inconsistent with the multiply connected model. It has been shown (Fang and Mo 1986) that, at least in the case of $L_H > L$, the anisotropically multiply connected universe can not be ruled out by the observation of quadrupole component.

The large scale fluctuation in the background radiation temperature $T(\theta,\varphi)$ can be described by multipole expansion, i.e., spherical harmonic expansion:

$$\frac{T(\theta,\varphi)}{T_b} = 1+ \sum_{l,m} a_l^m Y_l^m(\theta,\varphi) \tag{14}$$

In a simply connected universe, the multipole component a_l is given by(Peebles 1982)

$$\langle a_l^2 \rangle = \frac{3\pi(2l+1)}{2l(l+1)}\delta^2 \tag{15}$$

where δ is the density fluctuation at small k. From eq .(15) one finds $a_l \sim \delta$.

In a T^3 universe of $a_x=a_y=a_z=a$, we have(Fang and Mo 1986)

$$\langle a_l^2 \rangle \cong \frac{4\pi\ (2l+1)}{(4\pi c/H_o \lambda_o\ )^6}\delta^2 \tag{16}$$

Therefore, in the case of $\lambda_0 < cH_0^{-1}$ (i.e., $L_H > L$) one have $a_l < \delta$, namely, in multiply connected universe, the multipole component of the background temperature will even be smaller than that in simply connected case.

6. CONCLUSIONS

All the evidences mentioned in section 5 are only very tentative. Anyhow the further statistical analyses of quasar redshift will be important to check the "positive" evidence of the compactified universe or to improve the lower limit on its size. All these results will be important for quantum cosmology regargingless whether or not there are positive or negative evidences for multiple connecticity.

The significance of this research is, more importantly, in the aspect of methodology. It teaches us that the birth of the universe can also be studied by the following methods:

(i) in observational cosmology, to determine the spacetime topology of the universe as a whole by means of the distributions of objects, such as quasars and galaxies;

(ii) in theoretical cosmology, to find the model of the birth of the universe, which can explain why the spacetime topology is just as it is.

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DISCUSSION

NARLIKAR: In 1984 Seshadri and I discussed elliptical topology of closed Friedmann models. We found that a QSO could have two images in diametrically opposite directions. However, the effect could operate only for $q_o>1$ and for redshifts under certain limits. Do you have any such theoretical constraints in your models?

FANG: For T^3 topology q_o must be equal to 0.5, namely only that universe admits torus topology. In this case, there is no constraint on redshift for the formation of images with opposite directions. However different images are related in general, to different times of the sources.

LOH: In a torus universe, does volume increase approximately as z^3 even if $cz/H_o > L$, where L is the "period" of the universe?

FANG: Yes, the geometrical properties of a torus universe are the same as those of the simply connected $K = 0$ universe.

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