

# MATHEMATICAL PROCEEDINGS

*(formerly Proceedings)*

*of the  
Cambridge Philosophical Society*

VOLUME 177



 **CAMBRIDGE**  
UNIVERSITY PRESS

Published by the Press Syndicate of the University of Cambridge  
The Pitt Building, Trumpington Street, Cambridge CB2 1RP, United Kingdom

CAMBRIDGE UNIVERSITY PRESS

University Printing House, Shaftesbury Road, Cambridge CB2 8BS, United Kingdom  
32 Avenue of the Americas, New York, NY 10013–2473, USA  
477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
C/Orense, 4, planta 13, 28020 Madrid, Spain  
Lower Ground Floor, Nautica Building, The Water Club, Beach Road,  
Granger Bay, Cape Town 8005, South Africa

© Cambridge Philosophical Society 2024

Printed and bound by CPI Group (UK) Ltd, Croydon, CR0 4YY

INDEX FOR VOLUME 177

	PAGE
<b>Adams, C., Romrell, Z., Bonat, A., Chande, M., Chen, J., Jiang, M., Santiago, D., Shapiro, B. &amp; Woodruff, D.</b> Generalised knotoids . . . . .	67
<b>Andrew, N. &amp; Martino, A.</b> Centralisers of linear growth automorphisms of free groups . . . . .	219
<b>Bérczes, A., Bugeaud, Y., Győry, K., Mello, J., Ostafe, A. &amp; Sha, M.</b> Multiplicative dependence of rational values modulo approximate finitely generated groups . . . . .	149
<b>Blomme, T.</b> Tropical curves in abelian surfaces I: enumeration of curves passing through points . . . . .	109
<b>Boden, H. U., Elmacioglu, C., Guha, A., Karimi, H., Rushworth, W., Tang, Y.-C. &amp; Wang Peng Jun, B.</b> On knots that divide ribbon knotted surfaces . . . . .	439
<b>Boroński, J. P.</b> Linked orbits of homeomorphisms of the plane and Gambaudo–Kolev Theorem . . . . .	103
<b>Carvalho, M., Rodrigues, F. B. &amp; Varandas, P.</b> Topological and metric emergence of continuous maps . . . . .	525
<b>Conlon, D. &amp; Lee, J.</b> Domination inequalities and dominating graphs . . . . .	167
<b>Diao, Y.</b> The ropelength conjecture of alternating knots . . . . .	367
<b>Garcia-Calciñes, J. M. &amp; Vandembroucq, L.</b> A study of the Ganea conjecture for topological complexity by using weak topological complexity . . . . .	241
<b>Gerin, L.</b> The Ulam–Hammersley problem for multiset permutations . . . . .	23
<b>Gorodetsky, O.</b> Smooth permutations and polynomials revisited . . . . .	455
<b>Hu, D., Kaneko, I., Martin, S. &amp; Schildkraut, C.</b> On a Mertens-type conjecture for number fields . . . . .	481
<b>Käenmäki, A. &amp; Nissinen, P.</b> Non-invertible planar self-affine sets . . . . .	49
<b>Kasprowski, D., Powell, M. &amp; Ruppik, B.</b> Homotopy classification of 4-manifolds with finite abelian 2-generator fundamental groups . . . . .	263
<b>Ko, H. &amp; Mazorchuk, V.</b> Graded extensions of Verma modules . . . . .	285
<b>Lee, Y.</b> Discrepancy bounds for the distribution of $L$ -functions near the critical line . . . . .	313
<b>Leng, J., Sah, A. &amp; Sawhney, M.</b> Improved bounds for five-term arithmetic progressions . . . . .	371
<b>Lindemann, D. &amp; Swann, A.</b> Special homogeneous surfaces . . . . .	333
<b>Ma, S.</b> Differential forms on universal $K3$ surfaces . . . . .	1
<b>Paterson, R.</b> The Failure of Galois Descent for $p$ -Selmer Groups of Elliptic Curves . . . . .	185
<b>Roche–Newton, O.</b> A better than $3/2$ exponent for iterated sums and products over $\mathbb{R}$ . . . . .	11
<b>Yan, J.</b> Computing the Cassels–Tate pairing for genus two jacobians with rational two-torsion points . . . . .	415
<b>Zakharov, D.</b> Most integers are not a sum of two palindromes . . . . .	363

# INSTRUCTIONS TO AUTHORS

## 1. Preparation of Manuscripts

A paper should be submitted electronically to [mpeditor@hermes.cam.ac.uk](mailto:mpeditor@hermes.cam.ac.uk) in pdf form only. Authors are encouraged to prepare their manuscripts in LaTeX 2<sub>ε</sub> using the PSP class file. The class file, together with a guide, `PSP2egui.tex`, and sample pages, `PSP2esam.tex`, can be downloaded from <ftp://ftp.cambridge.org/pub/texarchive/journals/latex/pspcls> in either packed or unpacked form. These files will be updated periodically; please ensure that you have the latest version.

A cover page should give the title, the author's name and institution, with the address to which mail should be sent.

The title, while brief, must be informative (e.g. *A new proof of the prime-number theorem*, whereas, *Some applications of a theorem of G. H. Hardy* would be useless).

Authors are asked to provide an abstract as a basis for search on the Web. This may be an explicit abstract at the start of the paper. Otherwise the first paragraph or two should form a summary of the main theme of the paper, providing an abstract intelligible to mathematicians. Please note that the abstract should be able to be read independently of the main text. References should therefore not be included in the abstract.

Authors are encouraged to check that where references are given, they are used in the text. Experience has shown that unused references have a habit of surviving into the final version of the manuscript.

For a typescript to be accepted for publication, it must accord with the standard requirements of publishers, and be presented in a form in which the author's intentions regarding symbols etc. are clear to a printer (who is not a mathematician). Please also check the Cambridge University Press website for information about the style in which the paper should be submitted.

## 2. Notation

Notation should be chosen carefully so that mathematical operations are expressed with all possible neatness, to lighten the task of the compositor and to reduce the chance of error. For instance  $n$  sub  $k$  is common usage, but avoid if possible using  $c$  sub  $n$  sub  $k$ . Fractions are generally best expressed by a solidus. Complicated exponentials like:

$$\exp\{z^2 \sin \theta / (1 + y^2)\}$$

should be shown in this and no other way.

It helps if displayed equations or statements which will be quoted later are numbered in order on the right of their line. They can then be referred to by, for example 'from (7)'.  
The author must enable the printer (if necessary by pencilled notes in the margin) to distinguish between similar symbols such as  $o, O, o, O, 0; x, X, \times; \phi, \Phi, \emptyset; 1, 1; e, k, \kappa, k$ .

Footnotes should be avoided.

Please use typewriter font for all addresses and email addresses.

Omit \* from the end of proofs.

In listing assertions, conclusions, etc. do not use a vertical column of dots and do not follow (a) or (i) by a capital letter (eg. (i) the absolute value . . .)

In making references precise use [3, theorem 5.1]

## 3. Diagrams

Diagrams should be in black ink or from a high-quality laser printer and should not be larger than 30 cm by 45 cm. Lettering to be inserted by the printer should be shown clearly on copies of the figures rather than on the original drawings. Please note that a charge may be made if hand-drawn diagrams need to be re-drawn for publication.

Figure 1 here

A typed list of captions may be provided at the end of the manuscript in the following format:

Figure 1. *A basis for ...*

Note that there is no point at the end of the heading. All headings should be centred.

## 4. Tables

Tables should be numbered (above the table) and set out on separate sheets. Indicate the position of each in the text as for figures:

Table 3 here

Heading for tables should be shown in the following way:

Table 1. *A basis for ...*

Note that there is no point at the end of the heading. All headings should be centred over columns.

## 5. References

References should be collected at the end of the paper numbered in alphabetical order of the authors' names. Where references are given, they should be used in the text. Titles of journals should be abbreviated as in *Mathematical Reviews*. The following examples show the preferred style for references to a paper in a journal, a paper in a proceedings volume, a book and an unpublished dissertation:

- [1] J. F. ADAMS. On the non-existence of elements of Hopf invariant one. *Ann of Math.* (2) **72** (1960), 20-104.
- [2] M. P. FOURAM and D. S. SCOTT. Sheaves and logic. In *Applications of Sheaves* Lecture Notes in Math. vol. 753 (Springer-Verlag, 1979), pp. 302-401.
- [3] P. T. JOHNSTONE. *Stone Spaces*. Cambridge Studies in Advanced Math. no. 3 (Cambridge University Press, 1982).
- [4] F. W. LAWVERE. Functional semantics of algebraic theories. Ph.D. thesis. Columbia University (1963).

## 6. Submission of papers accepted for publication

When a paper has been accepted for publication the relevant TeX files of the final version, accompanied by a pdf file, should be sent to the Editor by e-mail.

This journal issue has been printed on FSC™-certified paper and cover board. FSC is an independent, non-governmental, not-for-profit organization established to promote the responsible management of the world's forests. Please see [www.fsc.org](http://www.fsc.org) for information.

# MATHEMATICAL PROCEEDINGS

*of the  
Cambridge Philosophical Society*

VOLUME 177 PART 3, pages 371–551, November 2024

## CONTENTS

JAMES LENG, ASHWIN SAH & MEHTAAB SAWHNEY Improved bounds for five-term arithmetic progressions . . . . .	371
JIALI YAN Computing the Cassels–Tate pairing for genus two jacobians with rational two-torsion points . . . . .	415
HANS U. BODEN, CEYHUN ELMACIOGLU, ANSHUL GUHA, HOMAYUN KARIMI, WILLIAM RUSHWORTH, YUN-CHI TANG & BRYAN WANG PENG JUN On knots that divide ribbon knotted surfaces . . . . .	439
OFIR GORODETSKY Smooth permutations and polynomials revisited . . . . .	455
DANIEL HU, IKUYA KANEKO, SPENCER MARTIN & CARL SCHILDKRAUT On a Mertens-type conjecture for number fields . . . . .	481
MARIA CARVALHO, FAGNER B. RODRIGUES & PAULO VARANDAS Topological and metric emergence of continuous maps . . . . .	525

©The Cambridge Philosophical Society 2024

### Cambridge Core

For further information about this journal  
please go to the journal website at:

[cambridge.org/psp](https://www.cambridge.org/psp)



**CAMBRIDGE**  
UNIVERSITY PRESS