U, Θ being current coordinates on the tangent. After time δt, let the angle θ become $\theta + \delta\theta$. The inverses of the radii vectores to the curve and to the tangent are now

$$u + u_1 \delta\theta + u_2 (\delta\theta)^2/2$$

and
$$u(1 - (\delta\theta)^2/2) + u_1 \delta\theta$$

to the second order of small quantities. Thus, the distance between the curve and the tangent in the direction $\theta + \delta\theta$ is, to the same approximation,

$$(u + u_2)(\delta\theta)^2/2u^2.$$

This represents the distance moved in the time δt under an acceleration ρ towards the centre of force, the initial velocity in this direction being zero.

Hence $$(u + u_2)(\delta\theta)^2/2u^2 = \rho(\delta t)^2/2,$$

i.e. $$\rho = (u + u_2)(d\theta/dt)^2/u^2.$$

But $$d\theta/dt = hu^2.$$

Thus we get $$\rho = h^2 u^2 (u + u_2).$$

Routh ("Dynamics of a Particle," p. 199) mentions the fact that $(u + u_2)$ indicates the convexity or concavity of a curve. It seems to me that the method of proof here given is as short as the one generally given, and has the advantages of being really intelligible to any student, and of indicating clearly the underlying dynamical principles.

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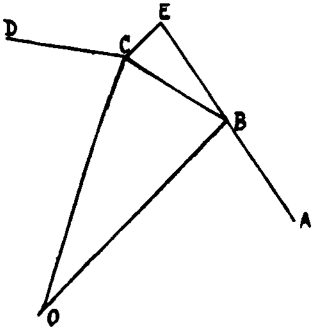
Elementary Proof of the Formula $\frac{V^2}{R}$.

Let O be the centre of a regular polygon $ABCD$, round the perimeter of which a point P moves with uniform speed V.



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Let AB be produced to meet in E a straight line through C drawn parallel to OB.

Then $B\widehat{C}E = C\widehat{B}O = O\widehat{B}A = C\widehat{E}B$.

$\therefore$ $BE = BC$, and the triangle BEC is similar to the triangle OBC.

Then on a certain scale the velocity of P when in AB is represented by BE, and on the same scale the velocity of P when in BC is represented by BC; then on a certain scale the change of P's velocity at B is represented by EC.

Hence the magnitude of the change is

$$V \cdot \frac{EC}{BE} = V\frac{BC}{OB},$$

its direction BO.

The time P takes to move from B to C is $= BC \div V$.

Dividing the change of velocity by this time, which is the interval between two successive changes in P's velocity, we get

$$V\frac{BC}{OB} \div \frac{BC}{V} = \frac{V^2}{OB}.$$

Now suppose the number of sides in the polygon to increase indefinitely, while V and OB remain the same, and the motion tends towards that of a point moving with uniform speed V in the circumference of a circle of radius $R = OB$. And in the limit the quantity $\dfrac{V^2}{R}$ becomes the acceleration of P in this motion, the direction being inwards along the radius vector of P.

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Feuerbach's Theorem.

Generally $\sum a^2 (b^2 + c^2 - a^2)(b - c)^2$ is divisible by

$$\sum (b + c - a)(b - c)^2,$$

the quotient being abc.

Let a, b, c be the sides of a triangle ABC; D, E, F their middle points. The tangent from D to the in-circle is equal in length to