Advances in Plasma Astrophysics
Proceedings IAU Symposium No. 274, 2010
A. Bonanno, E. de Gouveia Dal Pino & A. G. Kosovichev, eds.

doi:10.1017/S1743921311006612

Large eddy simulations in plasma astrophysics. Weakly compressible turbulence in local interstellar medium

A. A. Chernyshov, K. V. Karelsky and A. S. Petrosyan

Theoretical section, Space Research Institute of the Russian Academy of Sciences, Profsoyuznaya 84/32, 117997, Moscow, Russia
email: apetrosy@iki.rssi.ru

Abstract. We apply large eddy simulation technique to carry out three-dimensional numerical simulation of compressible magnetohydrodynamic turbulence in conditions relevant local interstellar medium. According to large eddy simulation method, the large-scale part of the flow is computed directly and only small-scale structures of turbulence are modeled. The small-scale motion is eliminated from the initial system of equations of motion by filtering procedures and their effect is taken into account by special closures referred to as the subgrid-scale models. Establishment of weakly compressible limit with Kolmogorov-like density fluctuations spectrum is shown in present work. We use our computations results to study dynamics of the turbulent plasma beta and anisotropic properties of the magnetoplasma fluctuations in the local interstellar medium.

Keywords. turbulence, ISM: kinematics and dynamics, methods: numerical

1. Introduction

Turbulence represents one of the most important phenomena, both in astrophysical and in laboratory plasmas. There is increasing evidence of the key role played by turbulence within different physical processes taking place in magnetofluids, like transport phenomena or the nonlinear dynamics of such complex systems. The presence of velocity and magnetic field fluctuations in a wide range of space and time scales has been directly detected in the interplanetary medium, while there are strong indications of their presence also in the solar corona. In contrast, large-eddy simulation (LES) is a multiscale computational modeling approach that offers a more comprehensive capturing of unsteady turbulent flow. So far, LES have been mainly associated with flows of modest complexity and primarily of academic interest. However, LES hold much promise for becoming the future research and development strategy for astrophysical applications in which turbulent flow is of pivotal importance. Some first important developments in this direction in which LES is being applied to astrophysical applications have recently emerged by development LES model for compressible MHD. Turbulent flows are inherently unsteady in this model and LES capture their major properties. The emergence of LES follows the rapidly growing computer capabilities that now allows for describing, not only the average behavior, but also most of the time evolution of the solar flow dynamics.

2. Formulation of Large Eddy Simulation

For study of compressible MHD turbulence in interstellar, medium we use large eddy simulation (LES) method (Chernyshov *et al.* 2007, Chernyshov *et al.* 2008). The filtering procedure is applied to the initial equations in the LES method. Each physical parameter is expanded into large- and small-scale components. The effects on large scales are calculated directly and those on small scales are modeled. The filtered part $\bar{f}(x_i)$ is defined as follows:

$$\bar{f}(x_i) = \int_{\Theta} f(\acute{x_i})\xi(x_i, \acute{x_i}; \bar{\triangle})d\acute{x_i}, \tag{2.1}$$

where ξ is the filter function satisfying the normalization property, Θ is the domain, $\bar{\triangle}$ is the filter-width and $x_j = (x, y, z)$ are axes of Cartesian coordinate system.

In order to simplify the resulting equations describing turbulent MHD flow with variable density, it is convenient to use the Favre filtering (it is also called mass-weighted filtering) to avoid additional subgrid-scale (SGS) terms. Therefore, Favre filtering will be used further. The mass-weighted filtration is used for all parameters of charged fluid flow except for the pressure and the magnetic field. Mass-weighted filtering is determined as follows: $\tilde{f} = \overline{\rho f}/\bar{\rho}$. To denote the filtration in this relation we use two symbols, viz. the overbar denotes the ordinary filtration and the tilde specifies the mass-weighted filtration. Using the mass-weighted filtration operation, we rewrite the compressible MHD equations as:

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho}\tilde{u_j}}{\partial x_j} = 0; \tag{2.2}$$

$$\frac{\partial \bar{\rho}\tilde{u_i}}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{\rho}\tilde{u_i}\tilde{u_j} + \bar{p}\delta_{ij} - \frac{1}{Re}\,\tilde{\sigma_{ij}} + \frac{\bar{B}^2}{2M_a^2}\delta_{ij} - \frac{1}{2M_a^2}\,\bar{B_j}\bar{B_i}\right) = -\frac{\partial \tau^u_{ji}}{\partial x_j}; \tag{2.3}$$

$$\frac{\partial \bar{B_i}}{\partial t} + \frac{\partial}{\partial x_j}\left(\tilde{u_j}\bar{B_i} - \tilde{u_i}\bar{B_j}\right) - \frac{1}{Re_m}\frac{\partial^2 \bar{B_i}}{\partial x_j^2} = -\frac{\partial \tau^b_{ji}}{\partial x_j}, \tag{2.4}$$

The filtered nondivergent (solenoidal) property of magnetic field is $\partial \bar{B_j}/\partial x_j = 0$.

Here, ρ is the density; p is the pressure; u_j is the velocity in the direction x_j; B_j is the magnetic field in the direction x_j; $\sigma_{ij} = 2\mu S_{ij} - \frac{2}{3}\mu S_{kk}\delta_{ij}$ is the viscous stress tensor; $S_{ij} = 1/2\left(\partial u_i/\partial x_j + \partial u_j/\partial x_i\right)$ is the strain rate tensor; μ is the coefficient of molecular viscosity; η is the coefficient of magnetic diffusivity; δ_{ij} is the the Kronecker delta.

The right-hand sides terms in equations (2.3) - (2.4) designate influence of subgrid terms on the filtered part: $\tau^u_{ij} = \bar{\rho}\left(\widetilde{u_i u_j} - \tilde{u_i}\tilde{u_j}\right) - \frac{1}{M_a^2}\left(\overline{B_i B_j} - \bar{B_i}\bar{B_j}\right)$; and $\tau^b_{ij} = \left(\overline{u_i B_j} - \tilde{u_i}\bar{B_j}\right) - \left(\overline{B_i u_j} - \bar{B_i}\tilde{u_j}\right)$.

To close the system of MHD equations, we assume that the relation between density and pressure is polytropic and has the following form: $p = \rho^\gamma$, γ is a polytropic index.

The effect of the subgrid terms which appear in the right-hand side of the magnetohydrodynamics equations (2.3) - (2.4) on the filtered part is modelled by the SGS terms. We use Smagorinsky model for compressible MHD case for subgrid-scale parameterization. The Smagorinsky model for compressible MHD turbulence showed accurate results under various range of similarity numbers (Chernyshov *et al.* 2007):

$$\tau^u_{ij} - \frac{1}{3}\tau^u_{kk}\delta_{ij} = -2C_1\bar{\rho}\bar{\triangle}^2|\tilde{S^u}|\left(\tilde{S_{ij}} - \frac{1}{3}\tilde{S_{kk}}\delta_{ij}\right), \tag{2.5}$$

$$\tau^b_{ij} = -2D_1\bar{\triangle}^2|\bar{j}|\bar{J_{ij}}, \tag{2.6}$$

$$\tau^u_{kk} = 2Y_1\bar{\rho}\bar{\triangle}^2|\tilde{S^u}|^2 \tag{2.7}$$

The parameters C_1, Y_1 and D_1 in equations (2.5) - (2.7) are model constants, their

https://doi.org/10.1017/S1743921311006612 Published online by Cambridge University Press

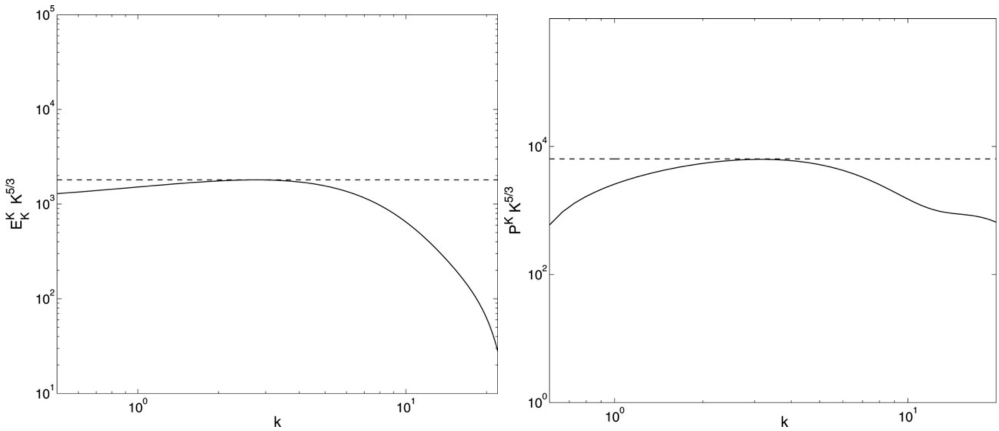

Figure 1. Normalized and smoothed spectrum of kinetic energy E^k_k (left) and ensity fluctuations P^k (right), multiplied by $k^{5/3}$. Notice that the spectrum is close to $\sim k^{-3}$ in a forward cascade regime of decaying turbulence. However, there is well-defined inertial Kolmogorov-like range of $k^{-5/3}$ that confirms observation data.

values being self-consistently computed during run time with the help of the dynamic procedure.

We perform three-dimensional numerical simulation of decaying compressible MHD turbulence for study of interstellar turbulence. The numerical code of the fourth order accuracy for MHD equations in the conservative form is used in our work. The skew-symmetric form for nonlinear terms is applied to reduce error of discretization when finite difference scheme is employed for modeling of turbulent flow. The third order low-storage Runge-Kutta method is applied for time integration. The explicit LES method is used in this work. To separate the turbulent flow into large and small eddy components, Gaussian filter of the fourth order of accuracy is applied. Periodic boundary conditions for all the three dimensions are applied. The simulation domain is a cube with dimensions of $\pi \times \pi \times \pi$. Initial hydrodynamic turbulent Reynolds number is chosen $Re \approx 2000$ and magnetic Reynolds number is chosen $Re_m \approx 200$. The initial isotropic turbulent spectrum was chosen for kinetic and magnetic energies in Fourier space to be close to k^{-2} with random amplitudes and phases in all three directions. The choice of such spectrum as initial conditions is due to velocity perturbations with an initial power spectrum in Fourier space similar to that of developed turbulence.

3. Results

Compressible MHD turbulence evolves under the effect of nonlinear interactions in which larger eddies transfer energy to smaller ones through forward turbulent energy cascade. Notwithstanding the fact that supersonic flows with high value of large-scale Mach numbers are characterized in interstellar medium, nevertheless, there are subsonic fluctuations of weakly compressible components of interstellar medium. These weakly compressible subsonic fluctuations are responsible for emergence of a Kolmogorov-type spectrum in interstellar turbulence which is observed from experimental data. It is shown that density fluctuations are a passive scalar in a velocity field in weakly compressible magnetohydrodynamic turbulence and demonstrate Kolmogorov-like spectrum in a dissipative range of the energy cascade (Fig. 1). The spectral indexes of density fluctuations

https://doi.org/10.1017/S1743921311006612 Published online by Cambridge University Press

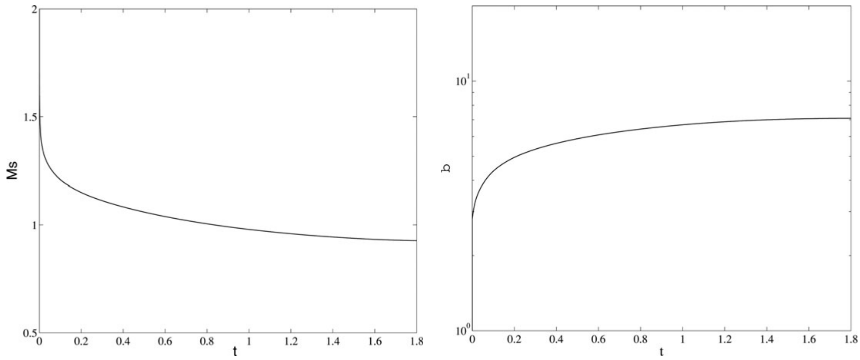

Figure 2. Decay of turbulent small-scale Mach number $\breve{M}_s$ with time(left). A transition from a supersonic $\breve{M}_s > 1$ to a subsonic $\breve{M}_s < 1$ can be observed. Time dynamics of the turbulent plasma beta $\breve{\beta}$ in compressible MHD turbulence (right). The MHD plasma is strongly magnetized initially and then, as the turbulence evolves, the plasma becomes less magnetized.

and kinetic energy are almost coincident. Notice that the range with Kolmogorov-like spectrum exists the same as kinetic energy spectrum, with the same wave numbers $2 \leqslant k \leqslant 5$. On the whole, the density fluctuation spectrum demonstrates similar behaviour in Fourier space, as kinetic energy spectrum. Consequently, we infer that density fluctuations are passive scalar in weakly compressible subsonic turbulent flow. Furthermore, theoretical models of turbulence support that any physical characteristic of flow, that propagates passively in large-scale or ambient velocity component of the background turbulence, demonstrates similar spectrum.

It is shown in Fig. 2(left), that the turbulent sonic Mach number decreases significantly from a supersonic turbulent regime ($\breve{M}_s > 1$), where the medium is strongly compressible, to a subsonic value of Mach number ($\breve{M}_s < 1$), describing weakly compressible flow. This fact indicates that turbulent cascades associated with the nonlinear interactions in combination with the dissipative effects at the small scales predominantly cause the supersonic MHD plasma fluctuations to damp strongly leaving primarily subsonic fluctuations in the MHD fluid.

In the interstellar medium, the transition of MHD turbulent flow from a strongly compressible to a weakly compressible state not only transforms the characteristic supersonic motion into subsonic motion, but also attenuates plasma magnetization, which is shown in Fig. 2(right) because plasma beta $\breve{\beta}$ increases with time, thus, role of magnetic energy decreases in comparison with plasma pressure. Fig. 2(right) demonstrates that the thermal pressure do not exceeds the magnetic energy (that is $\breve{\beta} \leqslant 1$) in initial time interval in fully compressible magnetohydrodynamic flow. Plasma particles coupled to the magnetic field lines are expelled from their gyro orbits due to increase of plasma pressure role in comparison with magnetic energy. This fact leads ultimately to a reduced plasma magnetization and hence plasma fluctuations, and transit into $\breve{\beta} > 1$ regime and subsonic weakly compressible flow. Besides, the gradual increase of the turbulent plasma beta $\breve{\beta}$ leads to change of speed of turbulent cascade in subsonic regime of the compressible MHD flow. The high plasma beta $\breve{\beta}$ state implies that the shear Alfvenic modes propagate more slowly than sound waves. When magnetized compressible plasma decreases and the turbulent plasma pressure evolves to exceed the turbulent magnetic energy, the perturbations are essentially non-magnetized, that is, the situation is hydrodynamic-like.

Mixing properties of compressible turbulence in the local interstellar medium predicted by LES method are important to understand effects of radio waves propagation and their scattering in observations data.

References

Chernyshov A., Karelsky K., & Petrosyan A. 2007, *Phys. Fluids*, 19, 5, 055106
Chernyshov A., Karelsky K., & Petrosyan A. 2008, *Phys. Fluids*, 20, 8, 085106