INVESTIGATION ON REFRACTION IN TRIGONOMETRICAL LEVELING TRAVERSES

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Abstract: When investigating refraction in trigonometrical traverses, additional observations for vertical angles have made it possible to evaluate the coefficient of refraction for each station. The improvement in elevation accuracy was substantial in traverses with longer distances (about 4 km) between stations.

1. DOUBLE TRAVERSE FOR EVALUATION OF REFRACTION

At the IAG Symposia in Stockholm 1974 and in Wageningen 1977 one of the authors presented a method for evaluation of refraction by the adjustment of three-dimensional nets and illustrated an application of such nets for the determination of crustal movements in the western Carpathians. He outlined further investigation on refraction in trigonometric leveling traverses (Hradilek, 1974, 1977).
To obtain more information about refraction, trigonometric leveling traverses were designed as double traverses, i.e. two vertical angles were observed both foresight and backsight at most of the stations (Fig.1). Additional observations for vertical angles made it possible to calculate one coefficient of refraction for each station.

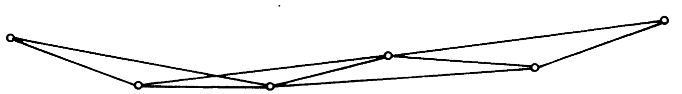

Figure 1. A double traverse.

E. Tengström and G. Teleki (eds.), Refractional Influences in Astrometry and Geodesy, 195–201.

2. FIELD EXPERIMENTS

2.1. A double traverse with distances of about 200 – 300 m between the neighboring stations was observed by forced centering equipment along the highway Praha-Brno (Fig.2).

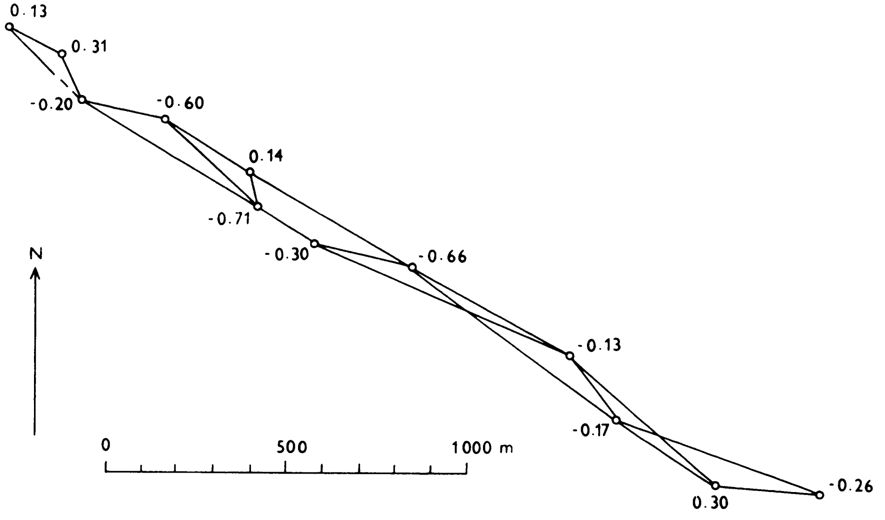

Figure 2. The double traverse situated along the highway Praha-Brno.The numerical quantities indicate the values of coefficients of refraction calculated at each station.

Following refraction models were used for evaluation of the traverse:

a) a constant value $k = 0.13$ for all stations,
b) one unknown value k for all stations,
c) one unknown value k_i for each station P_i.

In Table 1 are given the results A, B of the adjustments ; A being an average value of the standard deviation of adjusted trigonometric elevations, B denotes the mean square difference between the trigonometric and spirit leveling elevations.

Table 1. The mean square difference between the elevations determined by double traverse and by spirit leveling along the highway Praha-Brno.

Refraction model	a	b	c
Results A	6.5 mm	4.2	4.3
Results B	2.5	3.0	3.8

The coefficients of refraction calculated under the procedure c) are given in Fig.2.

The results of the adjustment indicate:

i) the choices of the refraction model a), b), c), respectively, are immaterial in trigonometric leveling traverses with distances of about 300 m between the neighboring stations,

ii) the precision of trigonometric leveling is comparable to that of a lower order spirit leveling.

2.2. A double traverse with distances of about 500 - 1000 m between the neighboring stations was observed by forced centering equipment in the Žiar Valley (western Carpathians, Fig.3). The values of refraction coefficients calculated under the assumption c) are given in Fig.3. The largest elevation difference in the traverse reached 900 m and was checked by the results of a three-dimensional triangulation with a discrepancy of 30 mm.

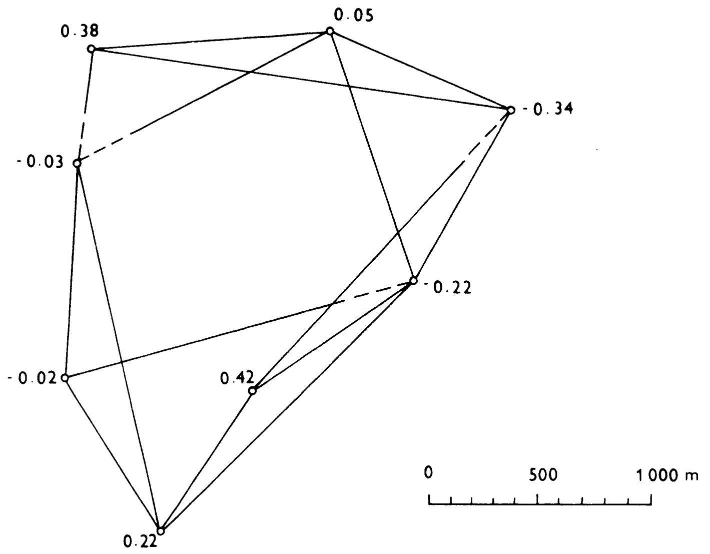

Figure 3. The double traverse observed in the Žiar Valley. The numerical quantities indicate the values of the coefficients of refraction calculated at each station.

2.3. A double traverse with distances of about 4 km between the neighboring stations was designed in the western Carpathians (Fig.4). The values observed were taken from the observation material of a three-dimensional network. Eight computing procedures, denoted as d),...k) were used for the elaboration of the traverse:

d) The adjustment of a single (i.e. normal) traverse with vertical angles observed (not simultaneously) between the neighboring stations only. The refraction model was chosen

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according to b), the deflections of the vertical were neglected. The elevation of the point No 16 was taken as a fixed value into the adjustment.

e) The same procedure as in d) except two fixed points No 16 and No 4.

f) and g) The same procedure as under d) and e), respectively, except the deflections of the vertical. The latter had been known before the adjustment and reduced the vertical angles to the spheroid.

h) The adjustment of the double traverse (Fig. 4) with all the lines of sight observed for vertical angles (not simultaneous observations). All the vertical angles were reduced to the spheroid except that at the station 13. The components of deflections of the vertical at the point No 13 were introduced as unknown parameters and calculated by the adjustment of the traverse. The refraction model was chosen according to the assumption c). The elevation of point No 16 was considered a fixed value.

i) The same procedure as under h) except two fixed elevations of points No 16 and No 4.

j) and k) The same procedure as under h) and i), respectively, except the deflection of the vertical at the point No 13 The deflection had been known before the adjustment, and the vertical angles at all stations were reduced to the spheroid.

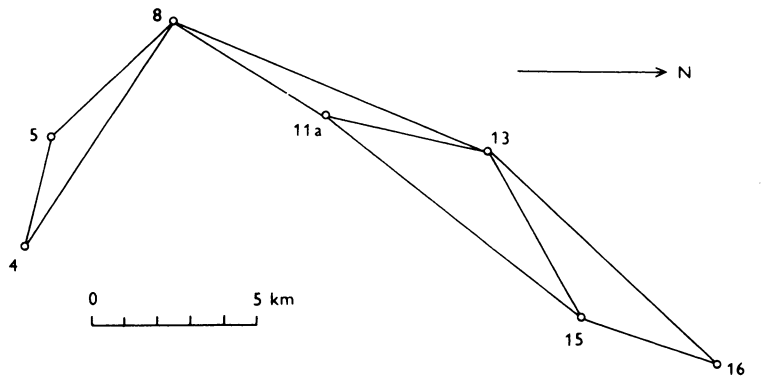

Figure 4. The double traverse designed in the western Carpathians.

The results of the adjustment procedures d),...k) are given in Tables 2 and 3. They were checked by a three-dimensional network surrounding the traverse. The accuracy of the network was estimated to be about 20 mm in elevations and 1.5 seconds of arc in deflections of the vertical. The results given in Table 3 indicate the well known distinction between the traverses with one and two fixed end points, respectively.

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Table 2. The coefficients of refraction calculated by the adjustment of the double traverse designed in the western Carpathians.

Point	Adjustment procedures d,...k. Refraction models (b),(c)							
No	d (b)	e (b)	f (b)	g (b)	h (c)	i (c)	j (c)	k (c)
4	0.114	0.113	0.131	0.130	0.126	0.124	0.126	0.123
5	0.114	0.113	0.131	0.130	0.131	0.131	0.132	0.131
8	0.114	0.113	0.131	0.130	0.139	0.139	0.139	0.140
11a	0.114	0.113	0.131	0.130	0.127	0.122	0.125	0.123
13	0.114	0.113	0.131	0.130	0.131	0.130	0.130	0.128
15	0.114	0.113	0.131	0.130	0.117	0.123	0.119	0.121
16	0.114	0.113	0.131	0.130	0.175	0.179	0.176	0.179

Table 3. The differences between the elevations determined by the double traverse and by the three-dimensional triangulation in the western Carpathians (MSE denotes the mean square difference).

Point	Adjustment procedure d),...k)							
No	d	e	f	g	h	i	j	k
4	145 mm	0	93	0	61	0	48	0
5	117	20	51	36	26	33	12	31
8	97	17	36	37	32	21	18	16
11a	49	49	75	13	57	3	41	10
13	80	10	34	11	15	20	8	16
15	34	18	15	5	26	27	25	31
16	0	0	0	0	0	0	0	0
MSE	88	22	53	21	37	20	27	19

The stronger design of the double traverse diminish this distinction significantly.The deflections of the vertical calculated by the procedures h) and i) at the station No 13 differed by 2 seconds of arc from its check value.

3. CONCLUSIONS

The field experiments proved the possibility of estimating refraction at each station of a double traverse. All the three traverses discussed gave reasonable results with respect to both the values of the coefficients of refraction and the differences between the trigonometric elevations and their check values determined by spirit leveling and three-dimensional triangulation,respectively.However,the results obtained for shorter lines of sight were independent of the choice of the refraction model, and indicate the double traverses have not a major significance for practical applications, except the traverses with longer lines of sight (over 3 km) which are designed in mountain regions without an adjustment to the spirit leveling.

References
Hradilek,L.:1974,Proceedings IAG Symp.Stockholm,Vol.5.
Hradilek,L.:1977,Proceedings IAG Symp.Wageningen,pp.185-190.

DISCUSSION

K. Poder: Thank you very much, professor Hradilek. You have really introduced a lot of new information to the geodetic community. If I may slightly disagree with you, I would say you are happy getting rid of these refraction coefficients, but you must not forget that at the same time you actually get rid of it, you could possible also determine it, and I am quite sure, that the trigonometric network will be very useful to get a better model of the refraction or the refractive index of the air in general. You get a lot of information of its derivatives and this will possibly be useful. But again, I think it is only a matter that instead of just eliminating, you get the information out after all.

L. Hradilek: By mathematical procedures, we can either determine or eliminate coefficients of refraction with the same results in elevations. The elimination of refraction seems to be more convenient with respect to a decrease in computing time. In mountainous regions, the evaluation of refraction, as well as of elevations, is substantially supported by ranging of very inclined distances.

D.G. Currie: I think, that in astronomy we recognize that there is a normal refraction and an anomalous refraction. The normal refraction is what the tables are made to work with. It seems part of what you have just mentioned is connected with trying to make a distinction. Is the data perhaps not so much to determine more coefficients but to evaluate how often the models can be used and of what accuracy, and how much has to be left open because it is out of control of a given class of models. In other words, there is ultimately certainly in our case a variability, that we might never expect to have in the model. And one thing, that I think is important to try in addresses, is how much is the model and how much of it can you not work with.

L. Hradilek: Our refraction models are mostly based on the zero hypothesis assuming a different coefficient of refraction for each observation station and the same refraction for all lines of sight radiating the station (under certain conditions concerning the observation procedure and the height of the observation station above the ground). The zero hypothesis is tested by the actual changes in refraction, which are determined by the measurements of vertical angles. If the test fails, an alternative hypothesis is designed considering the actual changes in refraction for each individual line of sight. Primarily, we are interested in determining elevations and in eliminating the influence of refraction on vertical angles. The value of refraction itself has not so large importance for us. However, when elaborating 7 networks in mountainous and hilly regions, the coefficients of refraction were determined according to our zero hypothesis at all 130 stations; the coefficients of refraction were evaluated within the limits 0.067 - 0.214, the average value was 0.1302. The zero hypothesis failed at several points of another network designed in the plane area of southern Slovakia (with changes in vertical angles attaining 180").