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Whewell's Puzzle.—

POSTSCRIPT.

Mr J. F. Cameron of Gonville and Caius College, Cambridge, has suggested the use of factorial functions such as $\underline{|n}$ or $n!$ to solve Whewell's problem. I had excluded them as beyond the bounds of *ordinary* arithmetic. I append half a score of my attempts with the factorials; they could be easily increased.

$$2 = \underline{|\sqrt{9}} - \sqrt{9} - \frac{9}{9}$$

$$4 = \underline{|\sqrt{9}} - \sqrt{9} + \frac{9}{9}$$

$$6 = \underline{|\sqrt{9}} + \frac{9}{9} - \cdot\dot{9}$$

$$8 = \underline{|\sqrt{9}} + \sqrt{9} - \frac{9}{9}$$

$$10 = \underline{|\sqrt{9}} + \sqrt{9} + \frac{9}{9}$$

$$14 = \underline{|\sqrt{9}} + 9 - \frac{9}{9}$$

$$15 = \underline{|\sqrt{9}} + \sqrt{9} + \sqrt{9} + \sqrt{9}$$

$$16 = \underline{|\sqrt{9}} + 9 + \frac{9}{9}$$

$$33 = \underline{|\sqrt{9}} + 9 + 9 + 9$$

$$36 = \underline{|\sqrt{9}} + 9\sqrt{9} + \sqrt{9}$$

Mr Cameron solves the "intractable" 38 very beautifully (I change his notation):

$$38 = \underline{|\sqrt{9}} \times \underline{|\sqrt{9}} + \cdot\dot{9} + \cdot\dot{9}$$

He gives two other solutions by employing the Gamma functions (see Professor G. A. Gibson's *Calculus*, p. 349).

$$\begin{aligned} 38 &= (\sqrt{9})^{\sqrt{9}} + 9 + \Gamma(\sqrt{9}) \\ &= 9\sqrt{9} + 9 + \Gamma(\sqrt{9}) \end{aligned}$$

Since this postscript was written another solution for 38 has arrived from Mr L. C. Mathewson of the University of Illinois, U.S.A,

$$38 = \left|\underline{\sqrt{9}}\right. \times \left|\underline{\sqrt{9}}\right. + \frac{\left|\underline{\sqrt{9}}\right.}{\sqrt{9}}$$

He adds that "other similar forms can be derived readily."

In the *Educational Times* of 1st January 1913 (Vol. 66, new series, No. 621), Mr W. W. Rouse Ball gives an account of "the problem of the expression of the consecutive integers from 1 upwards, as far as practicable, by the use of four 4's, using only the ordinary arithmetic and algebraic notation."

Somewhere about 1905 the Rev. W. A. Whitworth proposed the following question:

"Express all the numbers from 1 to 140 by four nines. Algebraical symbols and decimal points may be used, but the expression for each number must contain the four nines, *and no other figure.* Also express the same series of numbers by four fours."

Two sets of solutions are published in the *Mathematical Reprint* (Vol. VII., new series, 1905, pp. 43–46), the first by Lt.-Col. Allan Cunningham, R.E., and the second by Rev. T. Wiggins, B.A., for the problem of four nines. Only one solution is given in each case.

Col. Cunningham states that in this problem "the chief difficulty appears to be to find the simplest expression for each number." In his solution "an attempt has been made to use the simplest arithmetical and algebraical symbols possible in each instance. Thus the factorial sign has not been used when simpler symbols sufficed, and the gamma-function symbol has been used only five times."

An editorial note (*Reprint*, Vol. VII., p. 46) states that "the problem of expressing all numbers up to 200 by three fours was proposed some years ago in the *Royal Engineer Journal* by Major W. H. Turton, R.E.; his solution was published in that journal, January 1892."

J. S. Mackay