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Merging of spinning binary black holes in globular clusters

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Abstract. Based on our current high resolution direct N-body modelling of the Milky Way typical Star Cluster systems dynamical evolution we try to numerically estimate the influence of individual spin values and orientations on gravitational wave (GW) waveforms and observed time-frequency maps during multiple cycles for binary black hole (BBH) mergers. In our up to date N-body dynamical simulations we use the high order relativistic post-Newtonian corrections for the BH binary particles (3.5 post-Newtonian (PN) terms including spin-spin and spin-orbit terms). In the current work, we present the GW waveforms catalogue which covers the large parameter space in mass ratios 0.05 - 0.82 and extreme possible individual spin cases.

Keywords. black hole physics, gravitational waves, methods: n-body simulations

1. Introduction

Currently, over 50 gravitational events from compact binaries were reported by the LIGO-Virgo-Kagra consortium (Abbott B. P. 2019; Abbott R. 2021; The LIGO Scientific Collaboration et al. 2021a)†. We see Gravitational Waves (GW) as a new and very powerful informational channel. The current GW observations contain the Binary Black Hole (BBH) systems key orbital parameters, such as mass, semi-major axis, eccentricity and even the possible spins of the BH's. The next 3G generation of ground-based observatories (Punturo et al. 2010; Abbott et al. 2017) will have the opportunity to work with GWs during multiple cycles. It can significantly improve the estimations of individual component parameters of BH's.

† Just before the conference started the new catalogue GWTC-3 was presented (The LIGO Scientific Collaboration et al. 2021b).

2. Binary black hole initial models

To be more physically motivated, we took initial physical parameters (such as individual masses, initial separation and eccentricity) for 16 BBHs from N-body (Arca-Sedda et al. 2021) (with Id 1-4 in Table 1) and MOCCA (Maliszewski et al. 2021) simulations (with Id 5-16 in Table 1). Gravitational wave transient catalogues GWTC-1 (Abbott B. P. 2019), GWTC-2 (Abbott R. 2021) and GWTC-2.1 (The LIGO Scientific Collaboration et al. 2021a) contain systems with more than 50 % credibility of non-zero individual BH spin which allows us to investigate such systems. Initial binary orbit lies on XY plane, and each BH has individual spin:

$$|\boldsymbol{S}_{\mathbf{0,1}}| = \chi_{0,1}\frac{Gm_{0,1}^2}{c}, \tag{2.1}$$

where G is gravitational constant, $m_{0,1}$ is individual BH mass, and $\chi_{0,1} \in [0,1]$ is dimensionless spin magnitude (shortly "spin" throughout this paper). The dimensionless spin magnitude was limited by values 0, 1 or -1 independently for two BHs in three directions x, y, z. The total number of relative spin combinations for two BHs is 49. Effective inspiral spin parameter (Damour 2001) measures the mass-averaged spin along the orbital momentum axis, that LIGO can infer from gravitational waveform:

$$\chi_{\rm eff} = \frac{m_0\chi_0\cos\theta_0 + m_1\chi_1\cos\theta_1}{M}, \tag{2.2}$$

where $M = m_0 + m_1$ is total BBH mass, $\theta_{0,1}$ is angle between individual spin vector $\boldsymbol{S}_{0,1}$ and orbital angular momentum vector $\boldsymbol{L}$. Another value obtained from the observations is chirp mass (Cutler & Flanagan 1994):

$$\mathcal{M} = (m_0 m_1)^{3/5}(m_0 + m_1)^{-1/5}. \tag{2.3}$$

We also determine mass ratio as $q = m_1/m_0$, where $m_1 < m_0$.

For numerical dynamical evolution we used N-body ϕ-GPU code†, reducing number of particles to two BHs (Berczik et al. 2011). The acceleration for BH particle with post-Newtonian terms up to 3.5PN, spin-orbit (denoted SO) and spin-spin (denoted SS) terms (Blanchet 2006; Faye et al. 2006; Tagoshi et al. 2001; Buonanno et al. 2003) can be written in form:

$$\begin{aligned}\frac{d\boldsymbol{v}}{dt} = \boldsymbol{a}_{\rm N} + \frac{\boldsymbol{a}_{\rm 1PN}}{c^2} + \frac{\boldsymbol{a}_{\rm 1.5PN,SO}}{c^2} + \frac{\boldsymbol{a}_{\rm 2PN}}{c^4} + \frac{\boldsymbol{a}_{\rm 2PN,SS}}{c^4} + \frac{\boldsymbol{a}_{\rm 2.5PN}}{c^5} + \frac{\boldsymbol{a}_{\rm 2.5PN,SO}}{c^5} \\ + \frac{\boldsymbol{a}_{\rm 3PN}}{c^6} + \frac{\boldsymbol{a}_{\rm 3.5PN}}{c^7} + \mathcal{O}\left(\frac{1}{c^8}\right),\end{aligned} \tag{2.4}$$

where $\boldsymbol{a}_{\rm N}$ is Newtonian acceleration, $\boldsymbol{a}_{\rm 1PN,2PN,3PN}$ are conservative terms, $\boldsymbol{a}_{\rm 2.5PN,3.5PN}$ are dissipative terms, that respond to emission of GW. For the simple waveform calculation, we used the GW quadrupole term expression and obtained h_+ and $h_\times$ polarisation strains from h^{ij} tensor (Kidder 1995; Cutler 1998):

$$h^{ij} \approx \frac{4G\mu}{Dc^4}\left[v^i v^j - \frac{GM}{r}n^i n^j\right], \tag{2.5}$$

where $\mu = m_0 m_1/(m_0 + m_1)$ is reduced binary mass, D is luminosity distance, c is light velocity, v^i and n^i are the relative velocity and normalized position vectors in this reference frame respectively.

3. Evolution of Spinning Black Holes

We obtained merging time $T_{\rm merge}$ for 16 BBHs (column (9) at Table 1) from simulations with low resolution in time. Spin post-Newtonian terms were neglected. BBHs were

† `ftp://ftp.mao.kiev.ua/pub/berczik/phi-GPU/`

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Table 1. Parameters for simulated BBHs.

Id	M	$\mathcal{M}$	m_0	m_1	q	a	e	$T_{\rm merge}$	Reference
	$\mathbf{M}_\odot$	$\mathbf{M}_\odot$	$\mathbf{M}_\odot$	$\mathbf{M}_\odot$		$\mathbf{R}_\odot$		**yr**	
(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**	**(10)**
1	349	62.3	328	21	0.064	1.21	0.410	6.9×10^1	[1]
2	329	62.3	307	22	0.072	0.70	0.085	1.6×10^1	[1]
3	355	70.7	329	26	0.079	697.00	0.997	4.10948×10^5	[1]
4	307	60.4	285	22	0.077	36.70	0.955	4.1326×10^4	[1]
5	443.181	68.2	422.640	20.541	0.049	25.0410	0.95592	3.7787×10^3	[2]
6	595.418	168.9	510.350	85.068	0.167	633.5100	0.99914	3.340×10^2	[2]
7	143.036	41.4	121.800	21.236	0.174	24.7450	0.97453	6.2124×10^3	[2]
8	256.010	79.9	211.460	44.550	0.211	284.3800	0.99800	2.7372×10^3	[2]
9	349.159	113.7	282.670	66.489	0.235	562.5900	0.99886	2.1929×10^3	[2]
10	211.460	70.4	169.230	42.230	0.250	152.0200	0.99520	7.3900×10^3	[2]
11	216.321	81.6	157.980	58.341	0.369	28.3510	0.97000	3.3422×10^3	[2]
12	282.672	107.2	205.310	77.362	0.377	133.5300	0.99864	1.85×10^1	[2]
13	157.778	60.7	112.940	44.838	0.397	95.7140	0.99851	3.78×10^1	[2]
14	134.505	54.4	89.955	44.550	0.495	303.4000	0.99949	1.375×10^2	[2]
15	106.500	45.4	63.270	43.230	0.683	0.4666	0.16982	2.21×10^1	[2]
16	144.994	62.7	79.783	65.211	0.817	132.9000	0.99891	5.05×10^1	[2]

NOTE: Columns from left to right contain the following information: (1) - identifier for BBH; (2) - BBH total mass; (3) - BBH chirp mass; (4) and (5) - primary and secondary BH individual masses; (6) - mass ratio; (7) - initial binary separation; (8) - initial binary eccentricity; (9) - estimated merging time; (10) - source of the initial BBH data, where [1] is Arca-Sedda et al. (2021) and [2] is Maliszewski et al. (2021).

merged if separation fall below $5R_{\rm Sch}$, where $R_{\rm Sch}=2GM/c^2$ is Schwarzschild radius. Simulations were rerun several times with higher time resolution each time for reaching the moment when separation fall below $\sim 1000R_{\rm Sch}$, this gives us $16\times 3=48$ runs. From this point we turned on spin-spin and spin-orbit terms and simulated 16 binaries with 49 relative spin combinations with time resolution enough to have 100 points in each orbit, which gives us $16\times 49=784$ runs. In summary, the number of simulations with low and high time resolutions is 832.

3.1. *GW waveform and time frequency picture*

For the BBH set the minimum merging time $T_{\rm merge}$ was observed for systems with spin direction opposite to orbital angular momentum: $\boldsymbol{S}_{0,1}\uparrow\downarrow\boldsymbol{L}$; the maximum merging time $T_{\rm merge}$ was observed for systems with same spin direction and orbital angular momentum: $\boldsymbol{S}_{0,1}\uparrow\uparrow\boldsymbol{L}$; which demonstrates known "hang up" effect† (Campanelli et al. 2006). A system with zero $\boldsymbol{S}_{0,1}=0$ and antialigned spins $\boldsymbol{S}_{0,1}\swarrow\uparrow\boldsymbol{L}$ has an average merging time. The difference between maximum and minimum merging time is less than $\sim 10\%$ accounting just timescale from high time resolution runs.

The study aim was to understand how we can catch systems with non-zero individual spins. For each binary with different spin combinations, we obtained waveforms and time-frequency pictures for h_+ and $h_\times$ polarisations. The initial orbital angular moment $\boldsymbol{L}$ has a z-direction.

For purposes of illustration, we describe results for BBH Id 1 from Table 1. Resulting waveforms can be distinguish into two parts for evolution during multiple GW cycles. We see the continuously increasing waveforms till merging if a spin vector, at least of the more massive BH, has precisely the same or opposite direction than orbital angular momentum. But at the same time these systems have maximum value for effective spin parameter $\chi_{\rm eff}=1$ in case of $\boldsymbol{S}_{0,1}\uparrow\uparrow\boldsymbol{L}$ and $\chi_{\rm eff}=-1$ in case of $\boldsymbol{S}_{0,1}\uparrow\downarrow\boldsymbol{L}$. This means that such systems can be easily specified from observations.

† Video illustration of this effect at the last 100 sec of merging for BBH Id 1 from Table 1 can be found by the link: `https://youtu.be/D0luUTebBzk`

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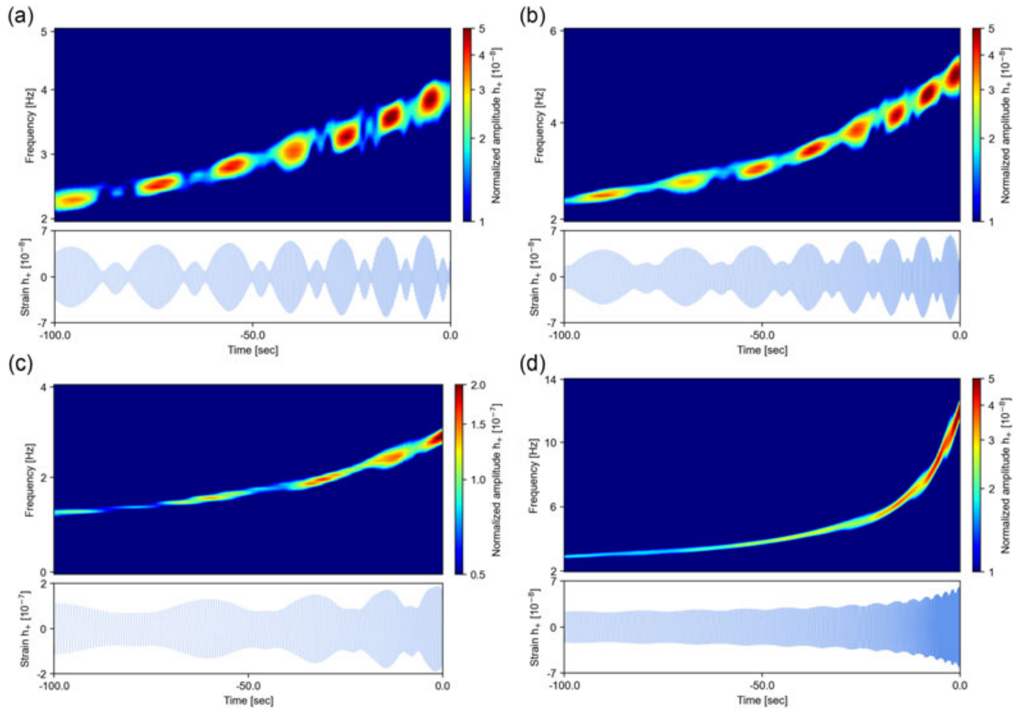

Figure 1. Time-frequency representation (top) of the strain data (bottom) for gravitational waveforms of h_+ polarisation from BBHs (Table 1): (a) - Id 5 ($q = 0.049$), (b) - Id 1 ($q = 0.064$), (c) - Id 6 ($q = 0.167$), (d) - Id 7 ($q = 0.174$). Data are depicted for the last 100 sec of merging. Individual dimensionless spin parameters are $\chi_0 = \chi_1 = [-1, 0, 0]$.

Other systems with antialigned spin vectors show continuously increase waveforms with bumps, we named this effect "waveforms beating". It's caused by more complex black holes orbits caused by the precession of the spin vectors. Also, this beating is observed in the time-frequency picture, where we see spotted-like-signal. It means that this type of system can also be specified from observations. It should be mentioned that such type of signal possibly can be caused by very eccentric binaries (in Keplerian sense) Romero-Shaw et al. (2021).

The remaining type is the systems with zero spin for more massive BH that have continuously increasing waveforms and $\chi_{\text{eff}} \approx 0$, which make them most hardly specifying from observations.

The described results are for a system with an extreme mass ratio $q = 0.064$. Systems with $q \approx 1$ do not show beating in the waveforms which make spin detections harder. We compared waveforms and time-frequency pictures for all our 16 binaries with mass ratios from 0.049 to 0.817 and found that from mass ratio 0.2 it is hard to observe waveforms beating (Fig. 1). Signal start to be more continuous at the time-frequency map with increasing mass ratio.

3.2. *Comparison with GWTC catalogues*

We plotted individual masses distribution for our BBH from Table 1 and observed merging BBH from GWTC-1, 2, 2.1 catalogues to find comparable systems. Three simulated systems Id 7, Id 15, Id 16 coincided in parameter space with observed events, and Table 2 contains combined literature data for them. Event GW190403_051519 has the

Table 2. Comparison of simulated and observed BBHs.

Id (1)	M, $\mathbf{M}_\odot$ **(2)**	q **(3)**	$\boldsymbol{\chi}_{\rm eff}$ **(4)**	$\boldsymbol{\chi}_{0,1}$ **(5)**
7	143.036	0.1743510		
GW190929_012149	$104.3^{+34.9}_{-25.2}$	$0.298^{+0.180}_{-0.613}$	$0.01^{+0.34}_{-0.33}$	
GW190403_051519	$110.5^{+30.6}_{-24.2}$	$0.251^{+0.138}_{-0.582}$	$0.70^{+0.15}_{-0.27}$	$\chi_1 = 0.92^{+0:07}_{-0.22}$
15	106.500	0.6832620		
GW190519_153544	$106.6^{+13.5}_{-14.8}$	$0.614^{+0.230}_{-0.340}$	$0.31^{+0.20}_{-0.22}$	
GW190701_203306	$94.3^{+12.1}_{-9.5}$	$0.757^{+0.319}_{-0.321}$	$-0.07^{+0.23}_{-0.29}$	
GW190706_222641	$104.1^{+20.2}_{-13.9}$	$0.570^{+0.265}_{-0.469}$	$0.28^{+0.26}_{-0.29}$	
16	144.994	0.8173550		
GW190521	$163.9^{+39.2}_{-23.5}$	$0.724^{+0.354}_{-0.476}$	$0.03^{+0.32}_{-0.39}$	$\chi_{i=\{1,2\}} > 0.8$ with 58% credibility.
GW190426_190642	$184.4^{+41.7}_{-36.6}$	$0.717^{+0.427}_{-0.542}$	$0.19^{+0.43}_{-0.40}$	

NOTE: Columns from left to right contain the following information: (1) - the identifier for BH pairs, where bold text denoted events from GWTC-2.1 (The LIGO Scientific Collaboration et al. 2021a), other events from GWTC-2 (Abbott R. 2021); (2) - BBH total mass; (3) - mass ratio; (4) - observed effective spin; (5) - observed individual spin.

highest credibility of non-zero effective spin, which can mean that at least more massive BH should have non-zero individual spin. Estimated individual spin for the more massive BH should be near the maximum value ~ 1. Another event GW190521 shows a non-zero individual spin. But high mass ratio $q \sim 0.7$ did not allow analysing the waveforms and time-frequency picture.

Also, we tried to analyse presented strain data by eyes and to find any type of beating at the waveforms. Events GW151226, GW170608, GW190412, GW190707_093326, GW190728_064510 were chosen as more promising. Only event GW190412 have a suitable parameters: possibility of non-zero effective spin $\chi_{\rm eff} = 0.25^{+0.08}_{-0.11}$ and asymmetric mass ratio $q = 0.28^{+0.12}_{-0.06}$. Signal was observed during just several gravitational waves cycles (last 0.5 sec) which is very short to specify beating in waveforms. The next 3G generation of gravitational wave telescopes (Einstein Telescope (Punturo et al. 2010), Cosmic Explorer (Abbott et al. 2017)) will capture several thousands of cycles and we will have the opportunity to see the waveforms beating.

4. Conclusions

The main conclusions of this study can be summarized as follows.

(i) We obtained merging time $T_{\rm merge}$, h_+ and $h_\times$ polarization waveforms and time-frequency maps for a set of BBHs with mass ratio $q = 0.064 - 0.82$ and 49 spin combinations. The “hang-up effect” was observed: minimum merging time was observed for systems with spin direction opposite to orbital angular momentum; the maximum merging time was observed for systems with the same spin direction and orbital angular momentum.

(ii) A system with beating in waveforms and spotted-like-signal at time-frequency picture should have non-zero individual spin $\chi_{1,2}$ at least for a more massive BH and non-symmetric mass ratio q. The transition point of the possibility to observe this effect is mass ratio $q \sim 0.2$. This effect can be possibly observed with the next 3G generation of gravitational wave telescopes.

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(iii) A system with non-zero effective spin χ_{eff} should have non-zero spin $\chi_{1,2}$ at least for a more massive BH.

(iv) A system with zero spin for a more massive BH is most hardly specified from observations.

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