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Temporal evolution of the velocity distribution in systems described by the Vlasov equation; Radiation Belts: Analytical and computational results

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Abstract. An interesting problem in plasma physics, when approached from the point of view of Statistical Mechanics is to obtain properties of collisionless plasmas, which are described by the Vlasov equation. Through what we call the Ehrenfest procedure, which uses statistical mechanical relations we obtain expectation value relations for arbitrary observables, which allows us to study the dynamics of the Earth's Outer Radiation Belt. Focusing on the velocity fluctuations, the width of the distribution function and the pitch angle, a computer simulation was performed to describe the system in order to compare and test the Ehrenfest approach. Our results show that the change in the average width of the distribution follows the analytical relation. However, for the velocity fluctuation results are not conclusive yet and require more exploration. It remains as future work to verify the relation for the pitch angle.

Keywords. Statistical Mechanics, Space Plasma Physics, Ehrenfest Procedure, Radiation Belts.

1. Introduction

Throughout our lives we have witnessed the interaction between the planet we inhabit and our star. The Earth's magnetosphere is one of its main consequences, originated by the interaction of the Earth's magnetic field and the solar wind. It is well known that the magnetosphere is highly sensitive to the activity of the Sun, which gives rise to many natural phenomena that intervene in our daily lives, such as geomagnetic storms, responsible for affecting navigation instruments, an. Many effects of this interaction are permanent, for example the radiation belts, composed of charged particles that were trapped in the magnetic field, whose variability in the outer belt is intimately related to solar activity and solar wind. Most of these phenomena are mediated by space plasmas, in particular the solar wind and regions of the magnetosphere, among others, are important examples of *non-collisional plasmas*, in which the presence of long-range interactions gives rise to stationary states (but not thermodynamic equilibrium) described by non-Maxwellian distributions such as the Kappa distribution (Viñas *et al.* 2015).

The dynamics of this system can be described from the point of view of nonequilibrium Statistical Mechanics, through the Vlasov equation (Bellan 2006).

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}} f + \frac{q}{m}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v} \times \boldsymbol{B}\right) \cdot \nabla_{\boldsymbol{v}} f = 0. \tag{1.1}$$

This equation is the Liuoville theorem when the Hamiltonian describes an electromagnetic interaction, and the force on each particle is given by the Lorentz force. Whatever the nature and form of the fields $\boldsymbol{E}$ and $\boldsymbol{B}$, it is possible to derive relations for the expectation values of time-dependent observables from the Vlasov equation, using what we will call the Ehrenfest procedure (as classical analog of Ehrenfest theorem in Quantum Mechanics) (Davis & González 2015), (Davis & Gutiérrez 2012).

$$\frac{\partial}{\partial t}\big\langle w\big\rangle_t = \Big\langle \frac{\partial w}{\partial t}\Big\rangle_t + \Big\langle \boldsymbol{v}\cdot\partial_{\boldsymbol{x}} w\Big\rangle_t + \Big\langle \frac{\boldsymbol{F}}{m}\cdot\partial_{\boldsymbol{v}} w\Big\rangle_t, \tag{1.2}$$

where $w = w(\boldsymbol{x},\boldsymbol{v})$ is an arbitrary, differentiable function of position and velocity.

An example of the consequences of this differential equation is an expression for the temporal evolution of the velocity fluctuations, which in our case of interest to model, has the form (González *et al.* 2018)

$$\frac{\partial}{\partial t}\big\langle(\delta\mathbf{v})^2\big\rangle_t = \frac{-2q}{mc}\Big\{\big\langle\mathbf{v}\times\mathbf{B}\big\rangle_t\cdot\big\langle\mathbf{v}\big\rangle_t\Big\}. \tag{1.3}$$

Here if we make the observable $w = \delta(\boldsymbol{v} - \boldsymbol{v_0})$ we then get an expression for the temporal evolution of the logarithm of the velocity distribution function in a component,

$$\frac{\partial}{\partial t}\ln P(v_i|t) = \frac{-2q}{mc}\Big\{(\mathbf{v}\times\big\langle\mathbf{B}\big\rangle)_i\cdot\frac{\partial}{\partial\mathbf{v}}\ln P(v_i|t)\Big\}. \tag{1.4}$$

Finally as the last analytical result, if we consider $w = \frac{\mathbf{B}\cdot\mathbf{v}}{|B||v|} = \cos\theta$ searching information about the collective behavior of the pitch angle θ, we obtain an expression for the temporal evolution of the average value of $\cos\theta$,

$$\frac{\partial}{\partial t}\big\langle\cos\theta\big\rangle = \big\langle\mathbf{v}\cdot\big(\hat{\mathbf{v}}\cdot\nabla_{\mathbf{r}}\big)\hat{\mathbf{B}}\big\rangle. \tag{1.5}$$

In this work, we seek to verify, by means of a computer simulation, the analytic relations previously shown in the outer radiation belt.

2. About the simulation

Considering the application of our theoretical results for the study of energetic particles in the Earth's outer radiation belt, we performed a test particle simulation of non interacting electrons trapped in a dipole magnetic field. As initial conditions we considered a Maxwellian distribution with a thermal velocity corresponding to an energy in the order of a few keV, and the particles were injected at a the magnetic Equator at a radial distance of 4 R_E, being R_E the Earth radius. The simulation was run for 80000 time steps in units of R_E/c, where c is the speed of light. To obtain the velocity and position of the particle, the Boris algorithm was used, which by construction based in Liouville's theorem, preserves energy without being simplectic (Hong Qin *et al.* 2013).

3. Results

The first result to show corresponds to the trajectory of a particle and its corresponding energy in the time of flight (See Fig. 1).

In order to test Eq. (1.3), the velocity and magnetic field data were retrieved at each time step and were treated in order to plot both sides of the equation.

To analyze the evolution of the velocity distribution by component, we built histograms for each time step in each component, noting the most significant differences in the start, middle and end of the dynamics. The expectation (average) values of one of the components of velocity in time was also plotted.

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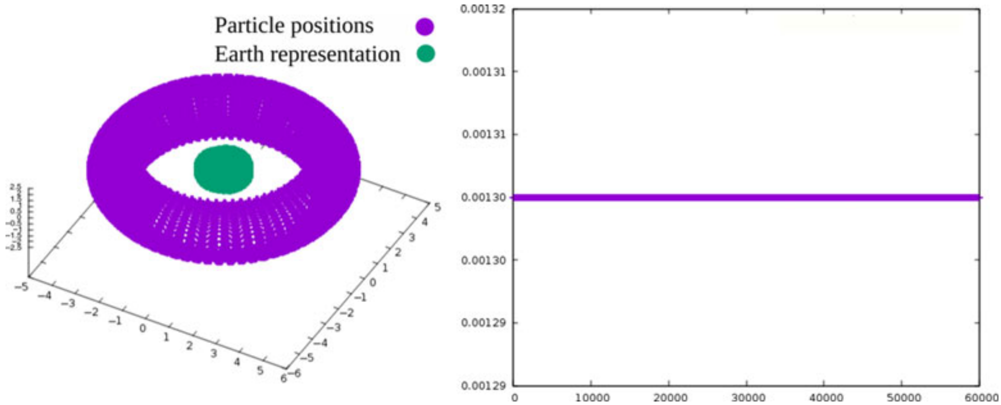

Figure 1. Left panel, trajectory of a given particle. Right panel, energy evolution during the dynamics.

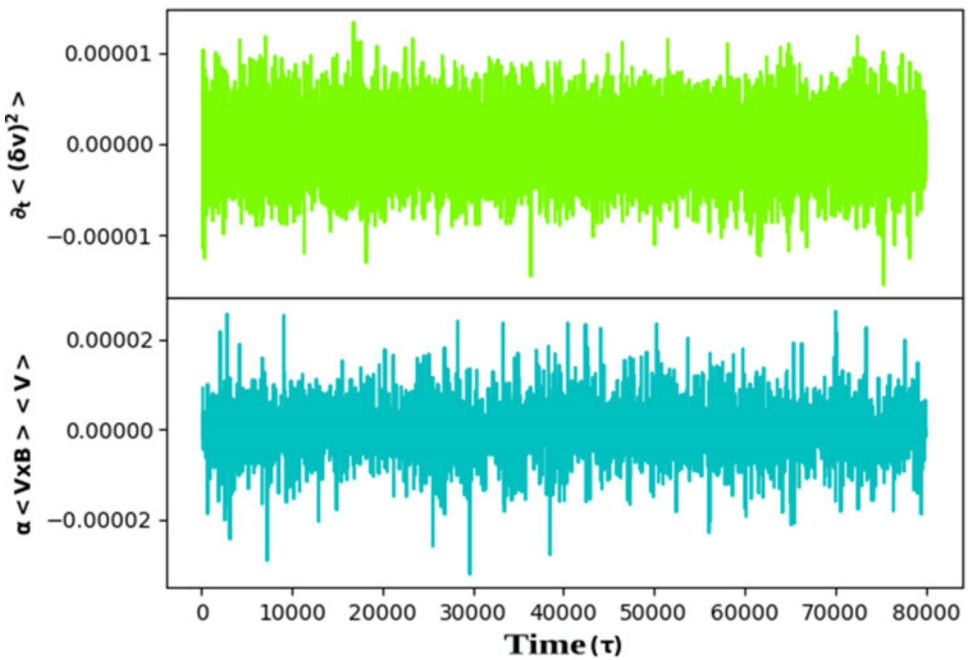

Figure 2. Temporal evolution of the left and right-hand sides of the relation 1.3, using data extracted from the simulation.

4. Summary

From Fig. 2, effects such as noise in the curve and the small number of particles, which generates a statistically limited sample, do not allow us yet to affirm that there is a statistically significant correlation for relation (1.3). We believe that, since the movement is not ergodic, following periodic orbits, it may be necessary to consider more particles to sweep away the effect of initial conditions. From relation (1.4) it is clear that we expected a variation in the average width of the distribution, which is confirmed by the results of the simulation represent in Fig. 3. It remains as a future work to calculate the variation in the average for each time step and see its correlation with Fig. 4. About relation (1.5), our results suggest a good agreement between theory and numerical simulations, but in

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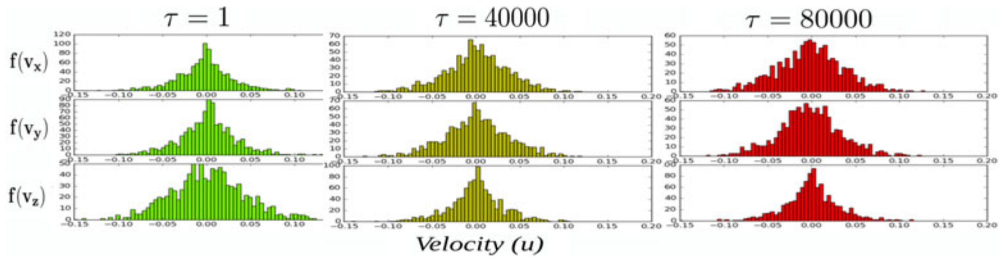

Figure 3. Evolution of the velocity distribution by component extracted from the simulation.

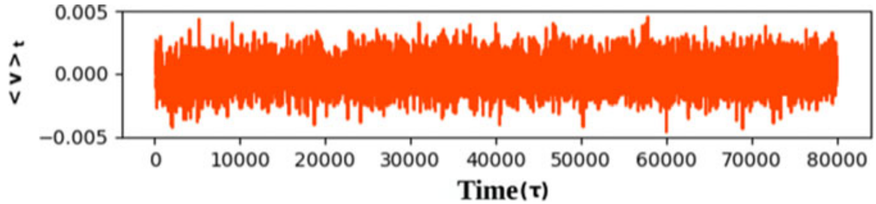

Figure 4. Temporal evolution of the expectation value of the velocity during dynamics.

order to make stronger conclusions further analysis is needed. In general, despite being a work in progress, from the relations derived from the Ehrenfest procedure to the Vlasov equation we can, as a first approximation, rescue information from the system and see from the results of the simulation, as these show expected behaviors of the dynamics, which is a good indication to consider the Ehrenfest procedure as a useful tool to address widely studied plasma systems as is the case of the outer radiation belt.

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