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HOLOMORPHIC MAPPING INTO ALGEBRAIC VARIETIES OF GENERAL TYPE, II

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This announcement is a continuation of Hu [3]. Our results improve Theorem 1 of [3], but the latter is needed in the proof of the former.

Let $f: M \to N$ be a holomorphic mapping from a connected complex manifold M of dimension m to a projective algebraic manifold N of dimension n. Assume that M possess a parabolic exhaustion τ and denote

$$\upsilon = dd^c\tau,\ \sigma = d^c \log \tau \wedge (dd^c \log \tau)^{m-1},$$
$$A(t;\zeta) = t^{2-2m}\int_{M[t]} \zeta \wedge \upsilon^{m-1},\ T(r,s;\zeta) = \int_s^r \frac{A(t;\zeta)}{t} dt,$$

where ζ is a form of bidegree (1,1) on M and $M[t] = \{x \in M: \tau(x) \leq t^2\}$. Suppose throughout that L is a positive holomorphic line bundle over N with a hermitian metric ρ along the fibers of L such that the Chern form $c(L,\rho) > 0$. The characteristic function of f for (L,ρ) is defined by

$$T(r,s) = T(r,s; f^*(c(L,\rho))).$$

Let $\mathrm{Ric}_\tau(r,s)$ be the Ricci function of τ. We obtain that

THEOREM 1. *Let N be of general type. If M is a Stein, covering parabolic space of $\boldsymbol{C}^m$ and if rank $f = \min(m,n)$, then there exist positive constants c_1 and c_2 such that*

$$c_1 T(r,s) \leq \mathrm{Ric}_\tau(r,s) + c_2 \log r$$

with the exception of a set of values (r) of finite measure.

COROLLARY 2. *If N is of general type, any non-degenerate holomorphic mappings f: $\boldsymbol{C}^m \to N$ is necessarily rational.*

In fact, we will prove a more general result than Theorem 1 (see Theorem 4). For this, we need some facts about hermitian geometry, dual classification map, associated maps and covering space.

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a) Hermitian Geometry

Let V be a complex vector space of dimension $n+1$. Then V^* is the dual vector space, $\bigwedge_k V$ is the exterior product. The Grassmann cone in $\bigwedge_{k+1} V$ is defined by $\tilde{G}_k(V) = \{a_0 \wedge \cdots \wedge a_k : a_i \in V\}$ with $\tilde{G}_0(V) = V$ and $\tilde{G}_n(V) \approx \boldsymbol{C}$. If $0 \neq x \in V$, let $\boldsymbol{P}(x) = \boldsymbol{C}x$ be the complex line spanned by x. If $A \subseteq V$, define $\boldsymbol{P}(A) = \{\boldsymbol{P}(x) : 0 \neq x \in A\}$. Then $\boldsymbol{P}(V)$ is the complex projective space associated to V. A holomorphic map $\boldsymbol{P}\colon V - \{0\} \to \boldsymbol{P}(V)$ is defined. The same symbol $\boldsymbol{P}$ is used for all vector spaces. Take an integer k with $0 \leq k \leq n$. The Grassmann manifold $G_k(V) = \boldsymbol{P}(\tilde{G}_k(V))$ of order k is a connected, smooth, compact submanifold of $\boldsymbol{P}(\bigwedge_{k+1} V)$. Take $a \in G_k(V)$. Then $\tilde{a} = a_0 \wedge \cdots \wedge a_k \neq 0$ exists such that $P(\tilde{a}) = a$. A $(k+1)$-dimensional linear subspace $E(a) = \boldsymbol{C}a_0 + \cdots + \boldsymbol{C}a_k$ is associated to a, independent of the choice of a. The associated projective space $\ddot{E}(a) = \boldsymbol{P}(E(a))$ is smoothly imbedded into $\boldsymbol{P}(V)$ and called a k-plane.

Take $a \in G_k(V^*)$. Then $\alpha = \alpha_0 \wedge \cdots \wedge \alpha_k \neq 0$ exists such that $\boldsymbol{P}(\alpha) = a$. A $(k+1)$-codimensional linear subspace

$$E[a] = \bigcap_{j=0}^{k} \alpha_j^{-1}(0)$$

and a $(n-k-1)$-plane $\ddot{E}[a] = \boldsymbol{P}(E[a])$ are associated to a. The biholomorphic dualism map $\delta\colon G_k(V) \to G_{n-k-1}(V^*)$ is defined by $E[\delta(a)] = E(a)$.

The trivial bundle $G_k(V) \times V$ contains the tautological bundle

$$S_k(V) = \{(a, x) \in G_k(V) \times V : x \in E(a)\}$$

as a holomorphic subbundle. The quotient bundle $Q_k(V)$ exists and the classifying sequence

$$0 \to S_k(V) \to G_k(V) \times V \to Q_k(V) \to 0 \tag{1}$$

is obtained. If $q = n-k-1$, then (1) is the pullback of

$$0 \to Q_q(V^*)^* \to G_q(V^*) \times V \to S_q(V^*)^* \to 0 \tag{2}$$

under the dualism $\delta\colon G_k(V) \to G_q(V^*)$.

Let l be a hermitian metric on V. Then l induces hermitian metrics l along the fibers of $Q_q(V^*)^*$, $G_q(V^*) \times V$ and $S_q(V^*)^*$ and Fubini-Kaehler forms $\Omega_q > 0$ on $G_q(V^*)$. Then

$$c(S_0(V^*)^*, l) = \Omega_0,$$
$$\mathrm{Ric}(\Omega_0^n) = -(n+1)\Omega_0,$$

b) Dual classification map

A holomorphic vector bundle homomorphism $\xi: N \times V \to E$ is said to be ample at $x \in N$, if $\xi(\{x\} \times V) = E_x$, where E is a holomorphic vector bundle over N. The set N_∞ of all $x \in N$ such that ξ is ample at x is open. Also $N - N_\infty$ is analytic. Then ξ is said to be an amplification if $N = N_\infty$, semi-amplification if $N - N_\infty$ is thin (see Stoll [5]).

Abbreviate the tensor product $L^{\otimes p}$ by L^p. We say that L is ample if there exists some p such that a basis of sections $(s_0, \cdots, s_k)$ of $H^0(N, L^p)$ generates L^p at every point (i.e., the evaluation map $e: N \times H^0(N, L^p) \to L^p$ defined by $e(x, s) = s(x)$ is an amplification), and give a projective imbedding

$$(s_0, \cdots, s_k): N \to \boldsymbol{P}(H^0(N, L^p)) . \tag{3}$$

We say that L is very ample if we can take $p = 1$ in the above condition (see S. Lang [4]). Let L be ample. We have a projective imbedding (3). Hence we can take a complex vector subspace V of $H^0(N, L^p)$ with dim $V = n + 1$ such that the evaluation map $e: N \times V \to L^p$ is an amplification (see Stoll [5], Lemma 16.1, Proposition A16). Let S be the kernel of e. An exact sequence

$$0 \to S \to N \times V \to L^p \to 0 \tag{4}$$

is defined. Here S has fiber dimension n. If $x \in N$, one and only one $\varphi(x) \in \boldsymbol{P}(V^*)$ and $\varphi_0(x) \in G_{n-1}(V)$ exist such that

$$E[\varphi(x)] = S_x = E(\varphi_0(x)) .$$

The maps $\varphi_0: N \to G_{n-1}(V)$ and $\varphi; N \to \boldsymbol{P}(V^*)$ are called the classification map and the dual classification map respectively, which are holomorphic. If δ is the dualism, then $\varphi = \delta \circ \varphi_0$. The classification map φ_0 pulls back (1) to (4) for $k = n - 1$. Hence

$$L^p = (\varphi_0)^*(Q_{n-1}(V)) = \varphi^*(S_0(V^*)^*) .$$

Let l be a hermitian metric on V. Then l induces hermitian metrics l along the fibers of S, $N \times V$ and L by (4) and along $Q_0(V^*)^*$, $\boldsymbol{P}(V^*) \times V$ and $S_0(V^*)^*$ by (2) for $q = 0$. Hence

$$\begin{aligned} &pc(L, l) = c(L^p, l^p) = \varphi^*(c(S_0(V^*)^*, l)) = \varphi^*(\Omega_0) , \\ &\mathrm{Ric}\, c(L^p, l^p)^n = \mathrm{Ric}\, \varphi^*(\Omega_0^n) = \varphi^*(\mathrm{Ric}\, \Omega_0^n) = -(n+1)c(L^p, l^p) . \end{aligned} \tag{5}$$

c) Associated maps

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Now we consider the holomorphic map $\varphi_f = \varphi \circ f \colon M \to \boldsymbol{P}(V^*)$, where $\varphi \colon N \to \boldsymbol{P}(V^*)$ is the dual classification map in b). Let L_f is the pullback $\varphi_f^*(S_0(V^*)^*)$ of the hyperplane section bundle $S^0(V^*)^*$ on $\boldsymbol{P}(V^*)$. Take a holomorphic form B of bidegree $(m-1, 0)$ on M. We can define the k^{th} representation section F_k of φ_f of the holomorphic vector bundle

$$L_f[k] = (M \times (\bigwedge_{k+1} V^*)) \otimes (L_f)^{k+1} \otimes (K_M)^{k(k+1)/2}$$

by means of the B-derivative, where K_M is the canonical bundle of M. Here

$$F_0 \colon M \to L_f[0] = (M \times V^*) \otimes L_f$$

but $F_k \equiv 0$ if $k > n$. If $F_k \equiv 0$, then $F_{k+1} \equiv 0$. Hence an integer l_f exists uniquely such that $F_k \not\equiv 0$ if $0 \leq k \leq l_f$ and $F_k \equiv 0$ if $k > l_f$. We call l_f the generality index of φ_f for B. The map φ_f is said to be general for B if $l_f = n$ (see Stoll [6]). If M admits m analytically independent holomorphic functions, then for any finite sets of meromorphic maps defined on M, there exists a holomorphic form B of degree $m-1$ on M such that the generality index of each of these maps φ_f for B equals the dimension of the smallest projective plane containing the image of φ_f (see Stoll [7], Theorem 7.11).

For each k with $0 \leq k \leq l_f$, the k^{th} associated map

$$f_k = \boldsymbol{P} \circ F_k \colon M \to G_k(V^*)$$

of φ_f is defined with $f_0 = \varphi_f$, and is holomorphhic. Define

$$L_f[-1] = M \times \boldsymbol{C}$$

and let F_{-1} be the trivial section defined by $F_{-1}(z) = (z, 1)$. Denote the divisor of F_k by μ_{F_k}. Then $\mu_{F_{-1}} = \mu_{F_0} = 0$. For $0 \leq k \leq l_f$, teh k^{th} stationary divisor

$$D_{f_k} = \mu_{F_{k-1}} - 2\mu_{F_k} + \mu_{k+1} \geq 0 \tag{6}$$

is non-negative (effective).

Define

$$H_k = m i_{m-1} f_k^*(\Omega_k) \wedge B \wedge \bar{B} \geq 0$$

with $H_k = 0$ if $k < 0$ or if $k \geq l_f$. For $0 \leq k < l_f$, we have the identity

$$\operatorname{Ric} H_k = f_{k-1}^*(\Omega_{k-1}) - 2 f_k^*(\Omega_k) + f_{k+1}^*(\Omega_{k+1}). \tag{7}$$

Since M is a parabolic manifold, the open set

$$M^{+} = \{x \in M : \upsilon(x) > 0\}$$

is not empty. On M^{+}, an on-negative function h_k is defined by $H_k = h_k^2 \upsilon^m$. Abbreviate

$$T_k(r, s) = T(r, s; f_k^*(\Omega_k)) .$$

Then for almost all s, r with $0 < s < r$, we have the Plücker Difference Formula

$$\begin{aligned} N(r, s; D_{f_k}) + T_{k-1}(r, s) - 2T_k(r, s) + T_{k+1}(r, s) \\ = B(r, s; h_k^2) + \mathrm{Ric}_\tau (r, s) , \end{aligned} \tag{8}$$

where $T_k(r, s) = 0$ if $k < 0$ or $k \geq l_f$, and

$$B(t, h) = \frac{1}{2} \int_{\partial M[t]} (\log h)\sigma, \ B(r, s; h) = B(r, h) - B(s, h) ,$$

$$N(r, s; D) = \int_s^r n(t, D) \frac{dt}{t}, \ n(t, D) = t^{2-2m} \int_{D \cap M[t]} \upsilon^{m-1} .$$

The exhaustion τ is said to majorize the holomorphic form B of degree $m - 1$, if for every $r > 0$ there exists a constant $c \geq 1$ such that

$$0 \leq m i_{m-1} B \wedge \bar{B} \leq c \upsilon^{m-1} \qquad \text{on } M[r],$$

where

$$i_{m-1} = (-1)^{(m-1)(m-2)/2} \left(\frac{\sqrt{-1}}{2\pi} \right)^{m-1} (m-1)\,!$$

The infimum of all these constants is called $Y_0(r)$. Then $Y_0(r) \geq 1$, and increases. Define

$$Y(r) = \lim_{r<t\to r} Y_0(t) .$$

Then $Y(r) \geq Y_0(r) \geq 1$. The increasing function Y is called the majorant associated to τ and B. If $r > 0$, then

$$\begin{aligned} m\, i_{m-1} B \wedge \bar{B} &\leq Y(r) \upsilon^{m-1} \qquad &\text{on } M[r] \\ m\, i_{m-1} B \wedge \bar{B} &\leq (Y \circ \sqrt{\tau}) \upsilon^{m-1} \qquad &\text{on } M. \end{aligned}$$

If $m = 1$, that is, if M is an open parabolic Riemann surface, we take $B = 1$, then $m\ i_{m-1}\ B \wedge \bar{B} = \upsilon^{m-1}$ and τ majorizes B with $Y \equiv 1$. From now, we assume that τ majorizes B with Y. We use the notation

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$$\|_\varepsilon\, a(r) \leq b(r)$$

to mean that the stated inequality holds except on an open set $I \subset R^+$ such that $\int_I r^\varepsilon dr < \infty$ for $\varepsilon > 0$. We have

$$(9) \qquad \|_\varepsilon\, B(r, h_k^2) \leq \frac{\mathfrak{c}}{2}(1+\varepsilon)^2(\log T_k(r, s) + \log Y(r)) + \frac{\mathfrak{c}}{2}\varepsilon \log r,$$

$$(10) \qquad \|_\varepsilon\, T_k(r, s) \leq 3^k T_0(r, s) + \frac{3^k - 1}{2}(\log Y(r) + \mathrm{Ric}_\tau(r, s) + \varepsilon\mathfrak{c} \log r),$$

where the constant $\mathfrak{c}$ is the volume of $\partial M[r]$ (see Stoll [6], Proposition 6.14 6.15). (9) and (10) imply

$$(11) \qquad \|_\varepsilon\, B(r, s; h_k^2) \leq \frac{\mathfrak{c}}{2}(1+\varepsilon)^3(\log T_0(r, s) + \log Y(r) + \log^+ \mathrm{Ric}_\tau(r, s)) + \varepsilon\mathfrak{c} \log r.$$

d) Covering space

If (M, τ) is a covering parabolic space of (C^m, τ_0) where $\tau_0(z) = |z|^2$, then there is a proper surjective holomorphic map

$$\beta = (\beta_1, \cdots, \beta_m) \colon M \to C^m$$

such that $\tau = \tau_0 \circ \beta = |\beta|^2$. The divisor of $d\beta_1 \wedge \cdots \wedge d\beta_m \not\equiv 0$ is called the branching divisor of β and denoted by D_β. Then

$$(12) \qquad \mathrm{Ric}_\tau(r, s) = N(r, s; D_\beta) \geq 0.$$

Define $S = \operatorname{supp} D_\beta$. Then $\beta(S)$ is an analytic subset of C^m. Let S_0 be the $(m-1)$-dimensional component of $\beta(S)$. If S_0 is affine algebraic of degree d, then we have

$$(13) \qquad \mathrm{Ric}_\tau(r, s) = N(r, s; D_\beta) \leq d\mathfrak{c} \log \frac{r}{s}$$

for $0 < s < r$. If β is biholomorphic, $\mathrm{Ric}_\tau(r, s) \equiv 0$.

If φ_f is linearly non-degenerate, then there is a holomorphic form $\hat{B}$ of bidegree $(m-1, 0)$ on C^m whose coefficients are polynomials of at most degree $n-1$, such that φ_f is general for $B = \beta^*(\hat{B})$. Hence there is a constant $c > 0$ such that τ majorizes B with

$$(14) \qquad Y(r) \leq 1 + cr^{2n-2} \qquad \text{for } r \geq 1.$$

see Stoll [6].

If $M = C^m$ and $\varphi_f(C^m)$ does not be contained in any hyperplanes of $P(V^*)$, there is a holomorphic form B of degree $m-1$ on C^m whose coefficients are constants such that φ_f is general for B and such that τ_0 majorizes B with

$$(14)' \qquad Y(r) \leq c$$

for a constant c.

e) Main results

Let ψ be a positive form of class C^∞ and bidegree $(1, 1)$ on N such that

$$(15) \qquad \overline{\lim_{r\to\infty}} \log T(r, s; f^*(\psi))/T(r, s) = 0$$

Define

$$\ddot{\psi}_f = m\, i_{m-1} f^*(\psi) \wedge B \wedge \overline{B},\ e_f = f^*(\mathrm{Ric}\, \psi^n) - n\, \mathrm{Ric}\, \ddot{\psi}_f$$

and define η by $\ddot{\psi}_f = \eta f^*(\psi) \wedge v^{m-1}$. Let

$$E_f(r, s) = T(r, s; e_f) + nB(r, s; \eta).$$

In [3], we proved that

THEOREM A. *Let N be of general type. If there exists an effective Jacobian section of f and if* rank $f = \min(m, n)$, *then exist positive constants c_1 and c_2 such that*

$$(16) \qquad \|_\varepsilon\, c_1 T(r, s) \leq n\, \mathrm{Ric}_\tau(r, s) + E_f(r, s) - nN(r, s; D_f) + c_2 \varepsilon \log r,$$

where D_f is the divisor of $\ddot{\psi}_f$.

Abbreviate

$$n_k(t) = n(t, \mu_{F_k}),\ N_k(r, s) = N(r, s; \mu_{F_k})$$

for the k^{th} representation section F_k of φ_f. We have

THEOREM 3. *Let L be an ample, positive holomorphic line bundle over N with the projective imbedding* (3). *Assume that B is a holomorphic form of bidegree $(m-1, 0)$ on M such that τ majorizes B with Y and such that φ_f is general for B. Then for $\psi = pc(L, l)$, we have*

$$(17) \qquad \begin{aligned} &\|_\varepsilon\, E_f(r, s) - nN(r, s; D_f) \leq -N_n(r, s) \\ &\quad + \frac{nc}{2} \log Y(r) + \frac{n(n-1)}{2} Q_\varepsilon(r) - nB(s, \eta), \end{aligned}$$

where

$$(18)\qquad Q_\varepsilon(r) = \frac{c}{2}(1+\varepsilon)^3(\log T(r,s) + \log Y(r) + \log^+ \mathrm{Ric}_\tau(r,s)) + \mathrm{Ric}_\tau(r,s) + 2c\varepsilon \log r.$$

Proof. Note that

$$(19)\qquad T_0(r,s) = T(r,s;f^*(\psi)) = pT(r,s;f^*(c(L,l))) = pT(r,s) + 0(1)$$

See Stoll [5], Theorem 12.5. Hence (6), (8) and (11) imply

$$(20)\qquad \|_\varepsilon\ N_{k-1}(r,s) + T_{k-1}(r,s) - 2(N_k(r,s) + T_k(r,s)) + N_{k+1}(r,s) + T_{k+1}(r,s) \le Q_\varepsilon(r)$$

Multiply (20) by $(n-k)$ and add these for $k = 1, \cdots, n-1$. We get

$$(21)\qquad \|_\varepsilon\ (n-1)T_0(r,s) - nT_1(r,s) \le nN_1(r,s) - N_n(r,s) + \frac{n(n-1)}{2}Q_\varepsilon(r)\,.$$

Now $\ddot{\psi}_f = H_0$, (5) and (7) imply

$$f^*(\mathrm{Ric}\,\psi^n) - n\,\mathrm{Ric}\,\ddot{\psi}_f = -(n+1)f_0^*(\Omega_0) - n\,\mathrm{Ric}\,H_0 = (n-1)f_0^*(\Omega_0) - nf_1^*(\Omega_1)\,,$$

which yields

$$(22)\qquad T(r,s;e_f) = (n-1)T_0(r,s) - nT_1(r,s)\,.$$

Since τ majorizes B with Y, we obtain

$$\eta f^*(\psi) \wedge \upsilon^{m-1} = \ddot{\psi}_f \le (Y \circ \sqrt{\tau})f^*(\psi) \wedge \upsilon^{m-1}\,,$$

which implies $\eta \le Y \circ \sqrt{\tau}$. Also we have

$$(23)\qquad N(r,s;D_f) = N(r,s;D_{f_0}) = N_1(r,s)$$

by (6) and the definition of D_f ad D_{f_0} for $\psi = c(L^p, l^p)$. So (17) follows from (21)–(23). Q.E.D.

Take $\psi = c(L^p, l^p)$ in Theorem A. Then (15) follows from (19). Hence Theorem A and 3 imply

THEOREM 4. *Let N be of general type. Let B be a holomorphic form of bidegree $(m-1,0)$ on M such that τ majorizes B with Y and such that*

φ_f is general for B. If there exists an effective Jacobian section of f and if rank $f = \min(m, n)$, then exist positive constants c_1 and c_2 such that

$$\begin{aligned}(24)\qquad &\|_\varepsilon\; N_n(r, s) + c_1 T(r, s) \leq \frac{n(n+1)}{2} \operatorname{Ric}_\tau(r, s)\\ &+ \frac{n(n+1)c}{4}(1+\varepsilon)^3(\log Y(r) + \log^+ \operatorname{Ric}_\tau(r, s)) + c_2\varepsilon \log r\,.\end{aligned}$$

If M is Stein and rank $f = \min(m, n)$, effective Jacobian sections exist by Stoll [5], Theorem 14.1, 14.2. Hence (12), (14) and Theorem 4 imply Theorem 1.

Abbreviate

$$A(t) = A(t; f^*(c(L, \rho)))$$

and define

$$R_\tau = \lim_{r\to\infty} \frac{\operatorname{Ric}_\tau(r, s)}{\log r}, \qquad Y_B = \lim_{r\to\infty} \frac{\log Y(r)}{\log r}.$$

Hence Theorem 4 with $\varepsilon \to 0$ implies.

$$(25)\qquad n_n(\infty) + c_1 A(\infty) \leq \frac{n(n+1)}{2} R_\tau + \frac{n(n+1)c}{4} Y_B\,.$$

f) Green-Griffiths' Conjecture

If M is an irreducible, affine algebraic variety with $A(\infty) < \infty$, then f is rational (Griffiths-King [2], Proposition 5.9, Carlson-Griffiths [1], Proposition 6.20 and Stoll [5], Theorem 20.6). Hence (14) and (25) imply Corollary 2.

If $M = \boldsymbol{C}^m$, then (14)′ and (25) yield $A(\infty) = 0$, which implies that

Corollary 5. *If N is of general type, then the image of any holomorphic map $f\colon \boldsymbol{C}^m \to N$ with* rank $f = \min(m, n)$ *is contained in a proper subvariety.*

Proof. If not, then φ_f is linearly non-degenerate. Hence there is a holomorphic form B of degree $m-1$ on $\boldsymbol{C}^m$ such that φ_f is general for B and such that τ_0 majorizes B with (14)′. Since rank $f = \min(m, n)$ and $c(L, \rho) > 0$,

$$A(\infty) = \lim_{r\to\infty} A(r) > 0$$

which contradicts $A(\infty) = 0$. Q.E.D.

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Corollary 5 implies the following

GREEN-GRIFFITHS' CONJECTURE. *Let N be of general type (or pseudo canonical). Let $f: \boldsymbol{C} \rightarrow N$ be holomoprhic non-constant. Then the image of f is contained in a proper subvariety.*

For more detail, see S. Lang [4].

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