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Hubble constant, lensing, and time delay in TeVeS

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Abstract. The Hubble constant can be determined from the time delay of gravitationally lensed systems. We adopt TeVeS as the relativistic version of Modified Newtonian Dynamics to study gravitational lensing phenomena and evaluate the Hubble constant from the derived time-delay formula. We test our method on observed quasar lensing published in the literature. Three candidates are suitable for our study, HE 2149−2745, FBQ J0951+2635, and SBS 0909+532.

Keywords. dark matter, gravitation, gravitational lensing, quasars: individual (HE 2149−2745, FBQ J0951+2635, SBS 0909+532)

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1. Introduction

The value of the Hubble constant, H_0, has been a topic of much debate in cosmology, for more than half a century. Basically, it relates the cosmological distances and the recessional velocities of galaxies, $v = H_0 d$. Its value can be estimated from gravitational time delays, a concept that was introduced by Refsdal (1964).

One advantage of using time delays to derive the Hubble constant is that they are not very sensitive to the adoption of a given cosmological model. However, there are some uncertainties associated with determining mass distributions based on image deflections and distortions using gravitational lensing. Another source of uncertainty in mass is, of course, the 'missing mass' problem. This problem exists in nearly all galactic systems, clusters of galaxies, large-scale structure, and the cosmic microwave background (CMB). To solve this puzzle, we can introduce dark matter into the system. On the other hand, one can also modify Newton's law of motion or the laws of gravity. Milgrom (1983) proposed Modified Newtonian Dynamics (MOND) to explain both the flat rotation curves observed in most spiral galaxies and the Tully–Fisher (1977) relation. MOND asserts that when the acceleration of an object that is under the influence of gravity only is smaller than approximately $\mathfrak{a}_0 = 1.21 \times 10^{-10}$ m s^{-2}, Newton's second law of motion no longer holds. The proposed modification is

$$\tilde{\mu}(|\mathbf{a}|/\mathfrak{a}_0)\mathbf{a} = -\nabla\Phi_{\rm N} = \mathbf{a}_{\rm N}, \tag{1.1}$$

where $\mathbf{a}$ is the acceleration of the object and $\Phi_{\rm N}$ is the Newtonian gravitational potential. The function $\tilde{\mu}(x)$ is called the interpolation function. With $x = |\mathbf{a}|/\mathfrak{a}_0$, $\tilde{\mu}(x) \approx 1$ for $x \gg 1$ (Newtonian regime), and $\tilde{\mu}(x) \approx x$ for $x \ll 1$ (deep MOND regime). For convenience, we call $\Phi_{\rm N}$ the Newtonian potential and Φ the MONDian potential.

MOND is very successful in explaining the dynamics of galactic systems (see, e.g., the review by Sanders & McGaugh 2002). Recently, McGaugh (2011a) showed that MOND can perfectly explain the Tully–Fisher relation in gas-rich spiral galaxies without the need to invoke uncertainty parameters such as the mass-to-light ratios of galaxies. Nevertheless, many scientists consider that MOND is not quite successful on scales of clusters of galaxies (see, e.g., Clowe *et al.* 2006; Angus & McGaugh 2008). Two decades after the original proposal by Milgrom (1983), Bekenstein (2004) proposed the Tensor–Vector–Scalar (Te Ve S) covariant relativistic gravity theory with MONDian dynamics as its non-relativistic limit. Adopting Te Ve S, Chiu *et al.* (2006) derived the corresponding strong-lens equation. More recently, Milgrom (2009) proposed another relativistic version of MOND, called BiMOND. It turns out that Te Ve S and BiMOND have identical gravitational-lensing equations. The lens equation has been applied to some galaxy lensing data, where the masses of the galaxies were calculated and compared with population synthesis (e.g., Zhao *et al.* 2006; Ferreras *et al.* 2008; Chiu *et al.* 2011). In this paper, we turn our attention to the Hubble constant.

2. Gravitational Lensing and Time Delays in Relativistic MOND

The modern view of light deflection is that it represents a relativistic gravitational effect, in which both the time-like and space-like parts of the metric contribute to the deflection angle. The angle of deflection by a spherical lens in the small-angle approximation can be written as (Chiu *et al.* 2006, 2011)

$$\Delta\varphi = 2\int a_{\perp}\frac{\mathrm{d}t}{c} \approx \frac{2\varrho_0}{c^2}\int_{-D_{\mathrm{LS}}}^{D'_{\mathrm{L}}} \frac{1}{\varrho}\frac{\partial\Phi(\varrho)}{\partial\varrho}\,\mathrm{d}\zeta, \tag{2.1}$$

where c is the speed of light, θ the image position, ϱ the distance from the center of the spherical lens, $\varrho_0 \approx D_{\mathrm{L}}\theta$ the closest approach of the light path from the center of the lens, $\zeta^2 = \varrho^2 - \varrho_0^2$, and $\Phi(\varrho)$ is the MONDian potential. D_{L}, D'_{L}, and D_{LS} are the angular distances of the lens from the observer, observer from the lens, and the source from the lens, respectively. The image appears in the direction of the closest approach (projected on the sky). For a spherical lens, there are two images, located on both sides of the source, and the governing equation (the 'lens equation') is given by

$$\beta = \theta_+ - \alpha(\theta_+) = \alpha(\theta_-) - \theta_-\,, \quad \alpha(\theta) = \Delta\varphi\frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}}, \tag{2.2}$$

where β is the source position and $\theta_\pm$ are the image positions. The top sign denotes an image on the same side as the source and the bottom sign corresponds to an image on the opposite side of the source. $\alpha(\theta)$ is commonly called the reduced deflection angle.

A time delay is defined as the difference in time traveled by light along the actual path with respect to that expected from travel along the undeflected path. It can be derived from Fermat's principle or from the geodesic equation in relativistic gravitation theory. As for the deflection angle, the form of the time delay is the same for general relativity (GR) and for MOND (with Newtonian and MONDian potentials for GR and a MOND, respectively),

$$t(\theta) = \frac{(1+z_{\mathrm{L}})}{c}\left[\frac{D_{\mathrm{L}}D_{\mathrm{S}}}{2D_{\mathrm{LS}}}\alpha(\theta)^2 - \int_{-D_{\mathrm{LS}}}^{D'_{\mathrm{L}}}\frac{2\Phi(\varrho)}{c^2}\,\mathrm{d}\zeta\right]. \tag{2.3}$$

The first and seconds term in Eq. (2.3) are referred to as the geometric and potential time delays. If the difference in time delay for the two images is available, the value of H_0 (and the mass of the lens) can be obtained by solving the time-delay difference equation (henceforth 'time-delay equation'), $\Delta t = t(\theta_-) - t(\theta_+)$, and the lens equation, Eq. (2.2).

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For a full derivation and formulation, we refer the reader to Tian *et al.* (2012). Here, we only show the interesting results in the deep MOND regime for a point-mass lens. The interpolation function becomes $\tilde{\mu}(a/\mathfrak{a}_0) \simeq a/\mathfrak{a}_0$. If the extent of the luminous matter is also much smaller than θ (i.e., if it can be modeled by a point mass), then the time-delay difference is solely determined by the potential time delay, because the deflection angle approaches a constant in the deep MOND regime (Chiu *et al.* 2006). In this case, the time-delay (difference) equation is independent of the choice of interpolation function,

$$\frac{\tilde{D}_{\rm LS}}{\tilde{D}_{\rm L}\tilde{D}_{\rm S}}\frac{{\rm H}_0\Delta t}{(1+z_{\rm L})} = \frac{1}{2}\left(\theta_+^2 - \theta_-^2\right). \tag{2.4}$$

Note that Eq. (2.4) is identical to that in GR for an isothermal lens model (see Witt *et al.* 2000). This is expected, since both potentials have the same form, i.e. logarithmic potentials. However, in MOND Eq. (2.4) is valid only in the deep MOND regime.

3. Data and Modeling

Although hundreds of quasar lenses have been found, gravitational time delays have been measured for only a few. As far as we know, only 18 strong lenses have had their time delays measured (Paraficz & Hjorth 2010). To test our theory, we select lensed elliptical galaxy systems with double images. Only five cases satisfy our criteria. They are HE 2149−2745, FBQ J0951+2635, SBS 0909+532, SDSS J1650+4251, and HE 1104−1805. The remainder are clusters, spiral galaxies, multiple images, or multiple lenses. However, the lensing galaxy in the SDSS J1650+4251 system is very dark. It does not have a reliable measurement of its effective radius. In addition, the uncertainty in the time-delay measurement in HE 1104−1805 is too large. This leaves us with a total of three candidates: HE 2149−2745, FBQ J0951+2635, and SBS 0909+532 (Paraficz & Hjorth 2010). A rough estimate indicates that the gravitational acceleration of the three systems ranges from 10^{-9} to 10^{-10} m s^{-2}. Thus, these gravitational lenses are not in the deep MOND regime. The surface brightness profile of the lensing elliptical galaxies satisfies a de Vaucouleurs' profile. Therefore, we adopt a Hernquist (1990) mass density profile.

MOND has been criticized, implying that it can not form large-scale structure. In essence, this criticism originated from arguments based on consideration of GR with baryons only. Skordis *et al.* (2006) showed that TeVeS needs 2 eV massive neutrinos (which are treated as non-relativistic particles) to comply with CMB observations, and $(\Omega_{\rm B}, \Omega_\nu, \Omega_\Lambda) = (0.05, 0.17, 0.78)$. This is often called νCDM cosmology. In any case, the discrepancy between the traditional ΛCDM model and the νCDM model is small, at least for our sample. The difference in $\tilde{D}_{LS}/(\tilde{D}_L\tilde{D}_S)$ is less than 0.4%.

4. Result and Conclusion

In this paper, we consider lensing and the time-delay equation for MOND in the 'Bekenstein' form, $\tilde{\mu}(x) = (-1+\sqrt{1+4x})/(1+\sqrt{1+4x})$ (Bekenstein 2004). The common flat rotation curves of spiral galaxies and the Tully–Fisher relation for gas-rich galaxies give a consistent value of the acceleration constant $\mathfrak{a}_0 = 1.21\times10^{-10}$ m s^{-2} (Sanders & McGaugh 2002; McGaugh 2011a). We evaluate H$_0$ and M for the three selected systems summarized in Table 4. In last two columns of Table 4, $x = a/\mathfrak{a}_0$ is the ratio of the acceleration at the closest approach to the MOND acceleration constant. From Table 4, we see that the deep MOND point-mass model did not give reasonable values of H$_0$. This is understandable, because these three cases are not in the deep MOND regime.

A source of uncertainty is the choice of interpolation function. In both the Newtonian ($x \gg 1$) and the deep MOND regimes ($x \ll 1$), different interpolation functions should give the same result. However, our sample lies in the intermediate MOND regime. Table 4

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Table 1. Evaluated lens masses and the resulting Hubble constants. The mass unit is 10^{10} $M_\odot$ and the unit of of the Hubble constant is km s^{-1} Mpc^{-1}. $x = a/\mathfrak{a}_0$ is a measure of the MONDian regime at the closest approach, ϱ_0, and $x_\pm$ corresponds to $\theta_\pm$. The numbers in smaller font are results taking into account the corresponding maximum and minimum uncertainties.

	Mass		H_0					
	GR	MOND	GR	GR	Deep MOND	MOND	x_-	x_+
Mass Model	Hernquist	Hernquist	Hernquist	Isothermal	Point Mass	Hernquist		
HE 2149-2745	$23.2^{25.9}_{20.5}$	$16.1^{17.5}_{14.6}$	$78.3^{70.2}_{88.7}$	$51.7^{46.3}_{58.5}$	$47.8^{42.8}_{54.1}$	$57.6^{51.0}_{66.0}$	11.3	2.3
FBQS J0951+2635	$2.9^{3.3}_{2.6}$	$2.30^{2.54}_{2.05}$	$101.5^{90.2}_{116.0}$	$62.5^{55.6}_{71.4}$	$57.4^{51.1}_{65.6}$	$79.3^{69.9}_{91.5}$	28.5	4.0
SBS 0909+523	$77.2^{78.9}_{58.3}$	$55.8^{56.8}_{44.3}$	$91.4^{89.4}_{120.9}$	$88.5^{86.6}_{117.1}$	$82.1^{80.3}_{108.7}$	$70.2^{68.6}_{95.1}$	8.7	5.9

shows the results for the Bekenstein form of the equations. Other interpolation functions will give somewhat different results. In any case, the major uncertainty comes from the observations, in particular from the time-delay measurements.

In summary, this work is a first attempt to use MOND to interpret data from gravitational time delays. The Hubble constant obtained from lensing and from time delays must, of course, be consistent with values from other measurements. Recently Riess (2009) calibrated 240 low-redshift Type Ia supernovae (SNe Ia) with Cepheids and obtained a Hubble constant of $H_0 = 74.2 \pm 3.6$ km s^{-1} Mpc^{-1}. The values of H_0 found from time delays in this paper (see Table 4) are consistent with the value(s) from SNe Ia data. When comparing with GR and adopting a Hernquist model (without dark matter), the lens masses in MOND are 28 to 44% smaller than those found from GR, and the Hubble constant is 18 to 25% smaller than resulting from GR. In this paper, we test this effect in TeVeS, because we have a consistent cosmological model (i.e., νCDM). However, the method is not restricted to TeVeS only. Gravitational lensing promises to provide a testing ground for modified gravity.

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