RADIATIVE TRANSFER PROBLEMS IN PLANETARY NEBULAE

David G. Hummer†
Joint Institute for Laboratory Astrophysics, University of Colorado and National Bureau of Standards, Boulder, Colorado, 80309

INTRODUCTION

In view of the enormous importance of the UV observations of planetary nebulae made possible by the IUE, this review will concentrate primarily on the formation of resonance lines in nebulae; an important special case is that of He II Lyα and its role in the Bowen mechanism. Special attention is given to the effects of dust on the line and continuum formation.

RESONANCE LINES

With the IUE a considerable number of resonance lines formed in planetary nebulae have been observed. In interpreting the strengths and profiles of these lines, several factors have to be considered. First, the optical thickness of the nebular shell in these lines will be very large, with values of perhaps 10^3 for lines of metal ions to as much as 10^6 or more for He II Lyα. Under these circumstances the details of the redistribution in frequency are important. Moreover, the effect of absorption can become crucial; the extinction of UV line photons by dust grains is of primary interest. The expansion of the nebular shell with a velocity on the order of the mean thermal velocity is relevant, although probably not crucial in preliminary modeling. Finally, the inhomogeneous distribution of the nebular gas should be taken into account. Because line photons escape preferentially in the least opaque direction, models based on a uniform shell with either planar or spherical geometry could be quite misleading. Although the effects of severely nonuniform gas distributions are expected to be important in many astrophysical situations, no really practical way of attacking this problem has been developed.

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†Staff Member, Quantum Physics Division, National Bureau of Standards.

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The description of the frequency redistribution mechanism for resonance lines in planetary nebulae in terms of a combination of natural and Doppler broadening appears to be basically correct, as the effect of collisions on redistribution at the low nebular densities is negligible. Thus the use of the so-called R_{II} redistribution function is adequate. For lines of the less abundant species, in which the optical thickness is sufficiently small that transfer occurs within the Doppler core, complete redistribution appears to be a reasonable approximation for our present level of modeling. Techniques for the numerical solutions of these problems are too well known to warrant discussion here.

The effect of dust absorption on the intensities of UV resonance lines is important in interpreting the IUE observations of planetary nebulae. The absorption of line photons by dust or photoionization depends on the mean path length of those photons within the nebula. For a line with an extremely large optical depth this mean photon path length can be many times the geometrical size of the nebula. This problem has been examined on the basis of Monte Carlo calculations by Bonilha, Ferch, Salpeter and Slater (1979), and by Adams (1975) and Hummer and Kunasz (1980), who used an accurate numerical solution of the transfer equation for a wide range of parameters. Subsequently Frisch (1980) obtained asymptotic expressions for the path length and other relevant quantities. In all of these calculations the R_{II} redistribution function was used, as the mean path length is sensitive to the distribution of the free paths between scatterings, and thus to the distribution of frequencies of a photon as it undergoes scatterings. We consider the quantity

$$\rho \equiv \langle \ell \rangle / 2T$$

where $\langle \ell \rangle$ is the mean photon path length and T is the optical thickness of the shell, both measured on the mean optical depth scale. In Figure 1, ρ is plotted versus T for a number of values of the Voigt parameter a. Here, photons are created at the midplane of the shell, although results for a uniform creation model are similar. Although ρ decreases over some range of optical thickness T, for which transfer starts to occur in the wings, the mean path length itself increases monotonically. With complete redistribution, the results for large T are qualitatively different.

From this work it emerged that the fraction of line photons that ultimately escapes depends on the extinction cross section of the dust only through the product of the cross section and the mean path length for no absorption, i.e. on

$$\alpha = n_d \sigma_d \langle \ell \rangle_o / k_{line} \quad .$$

Moreover the numerical results show that for large T, the escaping fraction is a universal function of this parameter α; i.e. results for very different Voigt parameters, optical depths, absorption cross sections, etc., all lie on the same curve as a function of α. The escap-

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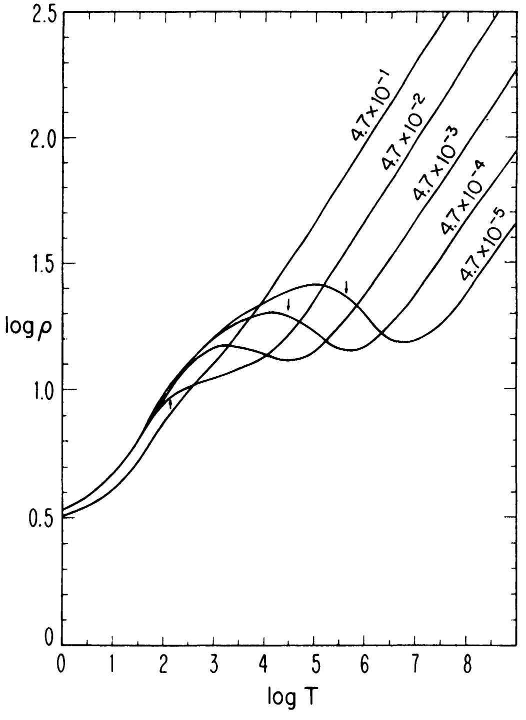

Fig. 1. The ratio ρ of mean path length to the optical half-thickness of the slab, versus the thickness, for indicated values of the Voigt parameter, for a midplane source. The arrows indicate the thickness at which transfer begins to occur in the line wings.

ing fraction decreases much more slowly than a negative exponential, which one might at first expect.

How does a velocity field modify the above results? Bonilha et al. (1979) have examined cases with both expansion and dust, and found that for typical expansion rates the escaping fraction was quite insensitive to the expansion rate. These authors note that dust destroys the photons that one might expect the expansion to help most in escaping, i.e. those in or near the line core.

All of the calculations to date have taken the resonance line as a single component, i.e. the multiplet structure was ignored. If the components do not overlap, have a common lower level and distinct upper levels with sufficiently small collisional or radiative coupling, then each component can be treated with the existing theory. "Sufficiently small" coupling means that the product of the largest transition probability and the mean number of scatterings is much less than unity. Expressions and tables for the mean number of scatterings are given by the above authors.

Even if the coupling is negligible, a very curious situation occurs that gives unexpected intensity ratios among the components, in that the stronger component of, say, a doublet may suffer less absorption than the weaker. This may be seen by recognizing that the parameter α can be written as

$$\alpha = \frac{1}{2} \rho_o T_d \quad ,$$

where T_d is the dust optical depth. In the region where ρ decreases with increasing T, ρ and therefore α for the stronger component can be smaller than for the weak component. But smaller α implies a larger escaping fraction, i.e. the stronger component experiences less absorption. This occurs because the distance between scatterings for a photon of given frequency is smaller for the strong component than for the weak component. Since the number of scatterings needed for a photon to escape is only very weakly dependent on the line strength, the path length and hence the probability of absorption of the weaker component is larger than the strong one. Harrington, Seaton, Adams and Lutz (1982) have used this theory to infer from the C IV resonance line in NGC 7662 a dust optical depth of approximately 0.1 for radiation at 1549 Å.

The effects of macroscopic expansion, or more generally, flow velocities may play a role. In nebulae the flow velocities are comparable to the thermal and turbulent velocities, so that the Sobolev approximation is not available, at least in its original form. One possiblity of simplifying transfer problems with complete redistribution in such media has recently appeared through the improvements to the escape-probability approximation. In escape probability methods the source function at a point is related to the probability that a photon will escape from that point. Two kinds of improvements allow such approximations to be used for planetary nebulae. Expressions have been obtained and evaluated by Grachev (1976,1977a,b,1978a,b) and by Hummer and Rybicki (1982a) for the escape probability for an arbitrary ratio of flow to random velocities; previously only the static and the very high velocity or Sobolev limit were used.

The second improvement is the derivation by Hummer and Rybicki (1982b) of a new expression for the source function in terms of the escape probability that is very much more accurate than the previous algebraic relation. Now a simple first-order differential equation and a few integrals must be evaluated for each line, but typical errors of 10% and maximum errors of 25% are found, whereas maximum errors of hundreds of per cent could be obtained previously. This method is limited to complete redistribution, but it should considerably simplify the modeling of most resonance lines, except those of H, He I and He II.

HELIUM II Lyα AND THE BOWEN MECHANISM

Of course, the resonance line par excellence in planetary nebulae is that of ionized helium, which among other things drives the Bowen fluorescent mechanism. Kallman and McCray (1980) have re-examined the Bowen mechanism in a static shell, considering not only the primary cycle, in which He II Lyα at 303.783 Å overlaps O III 303.799 Å and pumps the $2p^2\ ^3P_2$ - $2p3d\ ^3P_2^o$ transition, but also the secondary cycle, in which one of the O III lines at 374.436 Å overlaps N III 374.434, 374.441 Å to pump the 2p $^2P_{3/2}$ - 3d $^2D_{5/2}$ transition. These authors took as the mean escape probability for He II Lyα the inverse of the asymptotic mean number of scatterings obtained by Harrington (1973) for the R_{II} function, and reduced the resulting depth-independent equation to a simple algebraic problem using the Kneer(1975) approximation to the redistribution function. The O III transfer problem was treated in complete redistribution. The fluorescent efficiencies agreed well with those of Weymann and Williams (1969) and Harrington (1972) for similar values of the parameters. Because of the semi-analytic treatment, results could easily be obtained for a very wide range of parameters. In particular, the O III and N III fluorescent intensities are tabulated as a function of optical thickness. Although the means of the O III and N III fluorescent intensities for a large sample of nebulae agree well with the prototypical results of Kallman and McCray, the considerable variation among the nebulae may be worth analyzing for diagnostics of individual objects.

Although this work includes the ionization of neutral hydrogen and helium by the resonance lines, it does not account for dust absorption specifically. However, as the continuum absorption is parameterized by an equivalent density of hydrogen atoms, it can be interpreted as dust absorption. Kallman and McCray calculated the efficiencies of various processes as functions of the equivalent absorber density. In view of these results, it would be interesting to relate the Bowen efficiencies to the observed intensities of IR dust radiation, bearing in mind, however, that only a small fraction of this radiation comes from He II Lyα.

Another factor of possible interest not treated here is the expansion of the nebula. Because the primary O III line lies in the red wing of the He II line, while the primary N III line lies in the blue wing of the O III 374 Å line, the relative intensities of the O III and N III Bowen lines might give a useful velocity diagnostic.

The recalculation by Saraph and Seaton (1980) of the radiative parameters of O III removes much of the uncertainty in previous discussions of the Bowen mechanism. The primary product of this calculation is the probability $P(\lambda)$ that excitation of O III 2p3d $^3P_2^o$ is followed by the emission of a photon of wavelength λ. This provides the coupling strength of the N III cycle to the O III cycle. The calculated relative intensities of the observable O III multiplets agree well with observations and the inferred probability that excitation of

2p3d $^3P_2^0$ by Lyα gives Bowen lines agrees well with the calculations of Harrington (1972).

The fate of He II Lyα photons that escape the region in which they are created has been discussed in some detail by Flower and Perinotto (1980), in connection with the well-known [O II] and [N II] intensity anomaly. Although the authors avoided the transfer problem by parameterizing the fraction of photons escaping the He^{+2} zone, they showed that He II Lyα did not strongly influence the [O II] and [N II] intensities because the Lyα photons were absorbed by H and He before reaching the zone in which these lines are formed. The [O III] and [N III] intensities were increased, however.

CONTINUUM TRANSFER

In modeling planetary nebulae, the transfer problem for the diffuse radiation must be solved in conjunction with the ionization balance and the energy balance. Various procedures have been developed, most of which are refinements of the on-the-spot (OSA) approximation introduced by Zanstra (1951) and developed by Hummer and Seaton (1963). It is not generally recognized that the OSA is a form of the escape-probability approximation with the escape probability set to zero, and that considerably better results can be obtained by using the correct non-zero value. The OSA and the escape-probability method were compared with accurate numerical solutions by Van Blerkom and Hummer (1967), who referred to the escape-probability method as the normalized on-the-spot (NOSA) approximation. It is unfortunate that the utility of the NOSA is not more widely known. Now an even more accurate approximation is available in the second-order escape probability method of Hummer and Rybicki (1982b), which should be nearly exact for continuum problems.

In the early 1970's considerable progress was made in treating spherical-geometry transfer problems. The basic idea was to introduce the so-called Eddington factors, defined as certain ratios of the first three moments of the intensity. In the Eddington approximation these quantities are simply constants, whereas now they are functions of depth that are to be determined by a non-linear iteration procedure. The convergence is very rapid and leads to essentially exact results, although a rather large amount of computational work is required. Following the earliest applications of this method for spherical geometry by Hummer and Rybicki (1971) and by Kunasz and Hummer (1974), for continuum and line problems respectively, Leung (1975) applied this method under the name of quasi-diffusion method to pure dust clouds; a substantial literature now exists on this subject.

One particularly interesting application of the quasi-diffusion or variable-Eddington factor method has been made by Petrosian and Dana (1980) to nebulae containing dust with hydrogen and helium. This paper has two goals, a methodological one of comparing various approximations for the nebular continuum problem, and the physical one of examining in a systematic way the effect of dust, and especially of the dust albedo, on the ionization structure and the temperature of the dust and gas in the nebula. Although this is not the place for an extensive discussion of dust in nebulae, it does appear that Petrosian and Dana have explored the rather extensive parameter space of this problem and that the observable properties of the nebula do depend on the parameters of the dust grains. More to the point of this review, however, is the conclusion that the on-the-spot approximation generalized to include dust absorption is reasonably accurate for purely-absorbing dust, while for dust with a significant non-zero albedo the Eddington approximation seems to be the method of choice, with the modification for extended spherical geometry hardly worth the additional effort.

In conclusion, it appears that transfer theory can account for dust in planetary nebulae, thus providing a bridge between the UV spectrum of the stellar and nebular radiation fields on one hand, and the IR observations on the other.

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OSTERBROCK: Do you see any hope of radiative transfer calculations for situations more complicated than plane- or spherical-symmetry, for instance, for a medium containing strong density fluctuations with sizes smaller than the size of the nebula, but not infinitesimal?

HUMMER: A Monte Carlo calculation would probably provide the most immediate solution to this problem. Such a calculation might be used to "calibrate" plane- or spherical-geometry calculations, so that one would have some idea of the magnitude and sign of the error one makes in numerical solutions with a convenient but artificial geometry.

Another approach might be to compute the response of a spherical element of gas to radiation of a given frequency, and then to work out statistically the transfer for a randomly distributed ensemble of such spheres, which could be either stationary or moving, as a kind of "macro-molecule".