WHITE DWARFS

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Abstract. The effects of core crystallization and of convection in the envelope on the cooling of white dwarfs are reviewed. The case of a 0.6 $M_\odot$ white dwarf composed of an oxygen core and a helium envelope is taken as example. Also the amount of hydrogen that a white dwarf can accrete before nuclear burning occurs is estimated and possible evolutionary relations between white dwarfs of types DA, DB and DC, as advanced by Baglin and Vauclair, are presented.

1. Introduction

I will review the classical theory of white dwarf cooling and its modifications due to core crystallization and convection in the envelope, the question of how much hydrogen a white dwarf can accrete before nuclear reactions cause a thermal runaway, and finally a possible evolutionary relation between white dwarfs of types DA, DB and DC.

2. Classical Theory

The classical theory of cooling for white dwarfs was proposed by Mestel (1952). It is also found in Schwarzschild (1958) and Weidemann (1968). It considers the star to be composed of an isothermal core and a radiative envelope. The opacity of the envelope is given by Kramer's law $K=K_0\varrho T^{-3.5}$, and matter obeys the perfect gas law $P=(k/\mu H)\,\varrho T$. Since the mass and the luminosity are essentially constants in the envelope the stellar structure equations are reduced to the two equations:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\varrho\frac{GM}{r^2} \quad \text{and} \quad \frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3}{4ac}\frac{K\varrho}{T^3}\frac{L}{4\pi r^2}.$$

These two equations are integrated with the boundary conditions $P=T=0$ and the opacity and pressure laws given. One obtains two relations:

$$P = AT^{4.25} \quad \text{and} \quad \varrho = BT^{3.25}, \tag{1}$$

where A and B depend on M, L, and other parameters.

This envelope joins a degenerate core at the point where the pressures of both are equal

$$\frac{k}{\mu_e H}\varrho T_t = K_1\left(\frac{\varrho}{\mu_e}\right)^{5/3}. \tag{2}$$

Substitution of (2) into (1) gives a condition for T_t of the form

$$L = aT_t^{3.5}. \tag{3}$$

If all the thermal energy is contributed by the ions and those are considered as a

Tayler (ed.), Late Stages of Stellar Evolution, 222–230.

perfect gas, the cooling law is

$$L = -\frac{d}{d\tau}\left(\frac{3}{2}\frac{kT}{\mu_i H} M\right), \tag{4}$$

where μ_i is the ion molecular weight, H the unit of atomic weights and τ is time. Substitution of (3) into (4) and integration with the initial condition $\tau = 0$ at $T_t = \infty$ gives a cooling time

$$\tau = b\left(\frac{M}{L}\right)^{5/7}. \tag{5}$$

Significant deviations from the classical theory are due to crystallization of the ions and convection in the envelope.

3. Ion Crystallization

The theory is found in Salpeter (1961), Van Horn (1968), and Ostriker (1971). The properties of the ion component depend on the parameter Γ defined by

$$\Gamma = \frac{1}{kT}\frac{(Ze)^2}{R} = 2.28\,\frac{Z^2}{A^{1/3}}\frac{\varrho_6^{1/3}}{T_7}, \tag{6}$$

where ϱ_6 is the density in units of 10^6 gm cm^{-3} and T_7 the temperature in units of 10^7 K.

The parameter Γ is the Coulomb energy between one ion and a sphere of uniformly distributed negative charge containing Z electrons divided by kT. If $\Gamma \gg 1$ the ions arrange themselves in a crystalline lattice. If $\Gamma \ll 1$ they form a perfect gas.

The specific heat of a crystalline solid is given by Debye's theory. At high temperatures it is twice the value corresponding to the gaseous case because free particles have three degrees of freedom and bound ones six. At low temperatures the high modes of oscillation of the lattice are not excited, and the specific heat drops in value. The specific heat per ion is given in all cases by

$$C_v = 3kD(\Theta/T) \tag{7}$$

where Θ is the Debye temperature given by

$$\Theta = 1.74 \times 10^3\left(\frac{2Z}{A}\right)\varrho^{1/2} \tag{8}$$

The function D tends to unity for $T \gg \Theta$ and varies as $(T/\Theta)^3$ for $T \ll \Theta$.

The decrease in C_v at $T \ll \Theta$ causes a marked drop in the heat capacity of white dwarfs of low luminosity.

At intermediate temperatures, $\Gamma \sim 1$, we have a partially ordered state that may

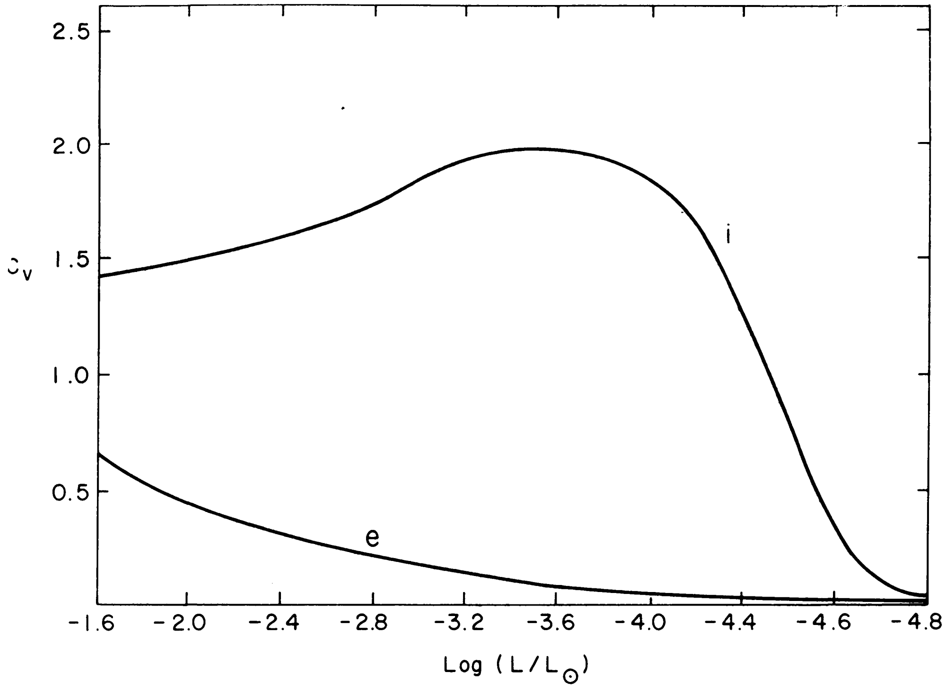

Fig. 1. Specific heat C_v of an ion (i) and Z electrons (e) for the half mass point of 0.6 $M_\odot$ white dwarf.

be compared to a liquid. Its properties have been studied by Brush *et al.* (1966). Their values for the specific heat show a gradual transition from the perfect gas values to the crystal values as Γ increases.

In Figure 1 I present C_v for one ion measured in units of $3k/2$. The values correspond to the half mass point of a 0.6 $M_\odot$ white dwarf as it evolves. This star, described later in the text, has an oxygen core and a helium envelope. It is to be noted that there is an increase to about twice the perfect gas value followed by a drop at low temperatures. The specific heat of Z electrons is also given.

4. Envelope Convection

The idea of convection in white dwarf envelopes was suggested by Schatzman (1958). Convection is caused by the decrease in the adiabatic gradient due to partial ionization and the high values of the radiative gradient produced by high opacities. Calculation of this envelope involves the evaluation of the ionization by Saha's equation including the treatment of pressure ionization and also accurate estimation of the opacities. Helium envelopes have been calculated for a 0.6 $M_\odot$ star by Böhm (1970) and Böhm and Casinelli (1971) and hydrogen and helium envelopes for stars of 0.32, 0.57, 0.89 and 1.17 $M_\odot$ by Koester (1972).

In Figure 2 the maximum temperatures in the helium envelopes for a 0.6 $M_\odot$ star by Böhm and a 0.57 $M_\odot$ star by Koester are plotted versus effective temperature. The agreement is remarkable. It turns out from these results that convection is important for these stars only at temperatures below 15000 K.

5. White Dwarf Evolution

Using the envelopes given by Böhm, I have calculated the evolution of a 0.6 $M_\odot$ white dwarf composed of an oxygen core and a helium envelope. The details of the model construction are published (Vila, 1971). This evolution includes both the effects of ion crystallization and core convection and shows the combined influence of both in shortening the cooling times in advanced stages of white dwarf evolution.

In Figure 2 the upper solid curve gives the central temperature as function of the effective temperature. The straight line represents the corresponding relation in the classical treatment. Using Kramer's law for opacity one obtains the proportionality $T_c \propto T_e^{8/7}$. As can be seen in the figure, this law applies at high temperatures but at low ones, T_c is lower than the classical value. Hence, for a given central temperature, convective envelopes require a higher effective temperature than radiative ones and a correspondingly higher luminosity. The rate of cooling is increased by convection.

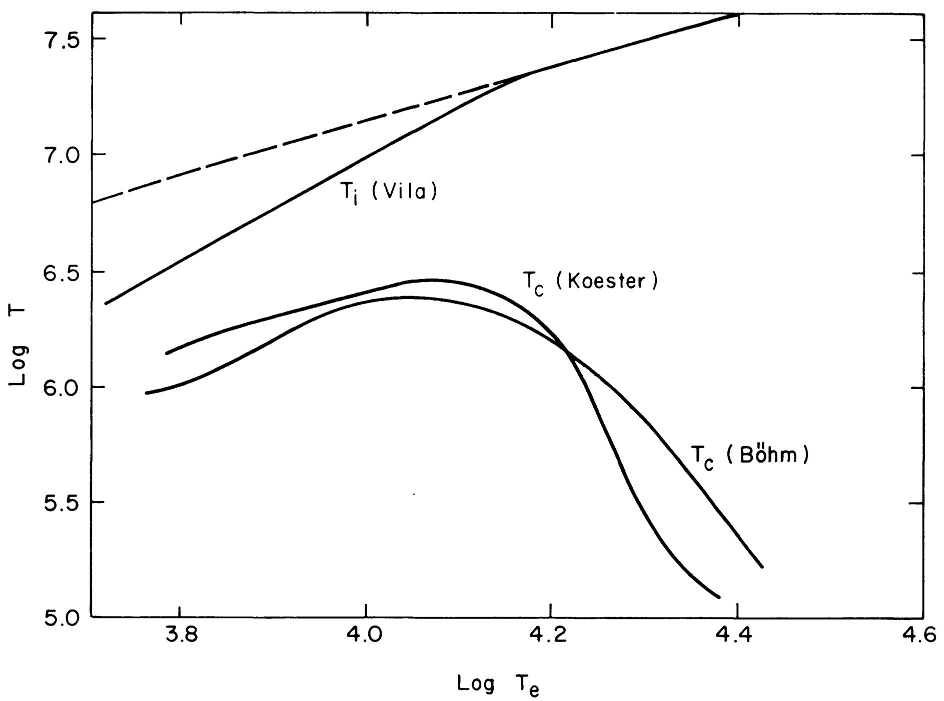

Fig. 2. Maximum temperatures of convective envelopes by Böhm and Koester. At top central temperatures of models by Vila including convection and of classical models without convection.

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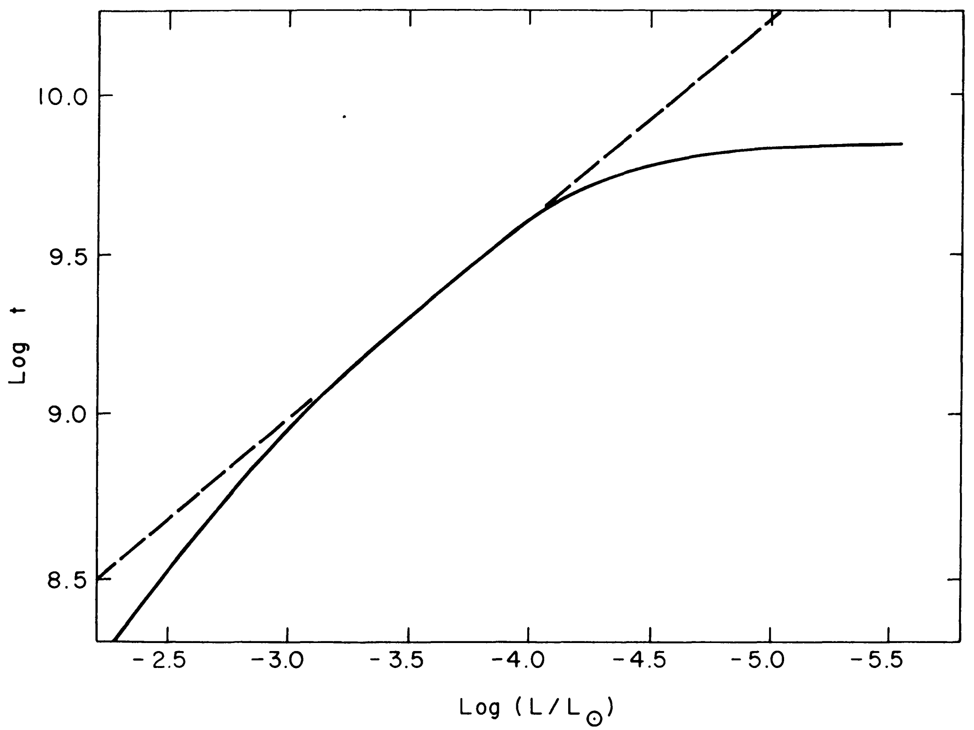

Fig. 3. Cooling times of a 0.6 $M_\odot$ white dwarf with crystallization and envelope convection com. pared to the ones in the classical case.

In Figure 3 I give the cooling time in years vs the luminosity in solar units. The straight line gives the corresponding quantity in the classical case with Kramer's opacity, that is $\tau \propto L^{-5/7}$. The discrepancy at small times is due to a different time origin for the two cases. The discrepancy at large times is significant and shows a rapid cooling in the real case as compared with the idealized one. This has also been found by Ostriker and Axel (1969). It is due both to the drop in the ion specific heat and to envelope convection. The mass of the star is the average mass of white dwarfs. The evolution time was always under 7×10^9 years. This is less than the current estimates of the age of the universe and supports the suggestion by Weidemann (1968) that cold degenerate dwarfs may contribute significantly to the local mass density in the Galaxy.

6. Accreted Hydrogen Envelopes

A problem arises in close binaries in which one star is a white dwarf receiving matter from its companion: how much matter can be transferred before the ignition of nuclear reactions causes a thermal runaway? To answer this question I extended the models by Koester for hydrogen envelopes of white dwarfs of 0.32, 0.57, 0.89 and 1.17 $M_\odot$ and several effective temperatures until nuclear reactions produce energy

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equal to the luminosity of the star at the given effective temperature. This is certainly an upper limit to the hydrogen mass.

In this integration the partial degenerate pressure of the electrons has been treated as in Chandrasekhar (1939) and numerically evaluated from the table by Grasberger (1961). The opacities used were from Cox and Stewart (1970) that were extended to higher densities by the tables for conductive opacities by Hubbard and Lampe (1969). The nuclear energy rates for hydrogen burning were taken from Cox and Guili (1968).

In Figure 4 I present the temperatures at the bottom of the hydrogen convective

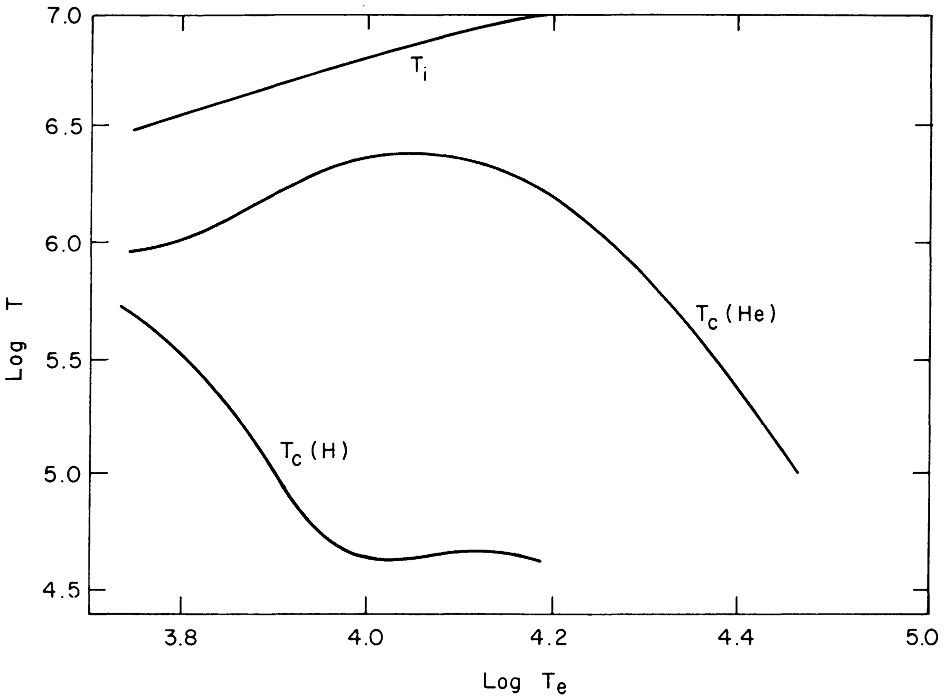

Fig. 4. Maximum temperature for hydrogen and helium convection zones and maximum envelope temperature for a 0.57 $M_\odot$ white dwarf.

zone and similar quantities for the case of a helium one. The upper curve gives the temperature of the inner boundary of the hydrogen envelope.

In Figure 5 the envelope mass is given as a function of the effective temperature for the 0.57 $M_\odot$ star. The results show that this quantity does not change very much with effective temperature. In Figure 6 the envelope mass is a function of the star's mass. The effective temperature is 10000 K. It shows that the envelope mass strongly increases with decreasing star mass.

What is the future evolution of a white dwarf that has ignited a hydrogen shell and is still accreting more hydrogen? This is an interesting question that will require detailed calculation. Two alternatives seem possible. Either part of the hydrogen will

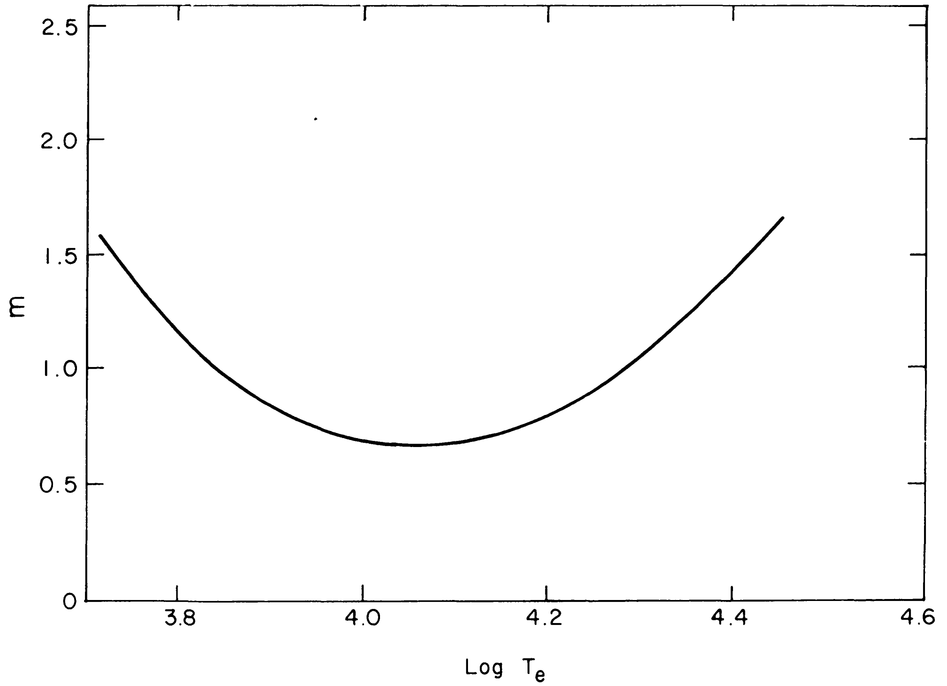

Fig. 5. Maximum mass of hydrogen envelope for a 0.57 $M_\odot$ white dwarf as function of effective temperature.

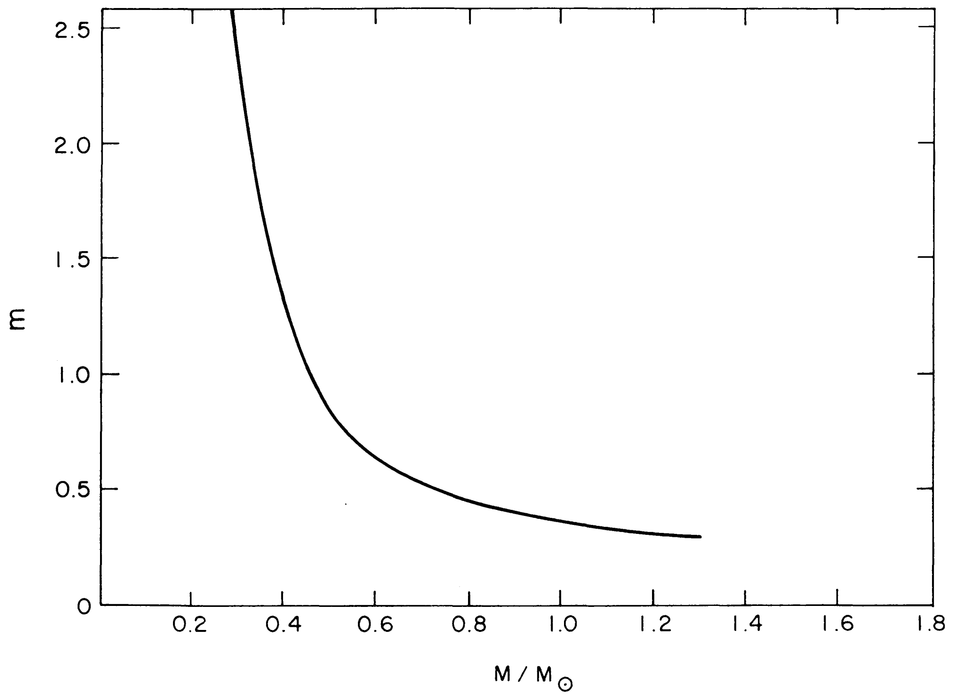

Fig. 6. Maximum mass of hydrogen envelope for a white dwarf of $T_e = 10000$ K as a function of its mass.

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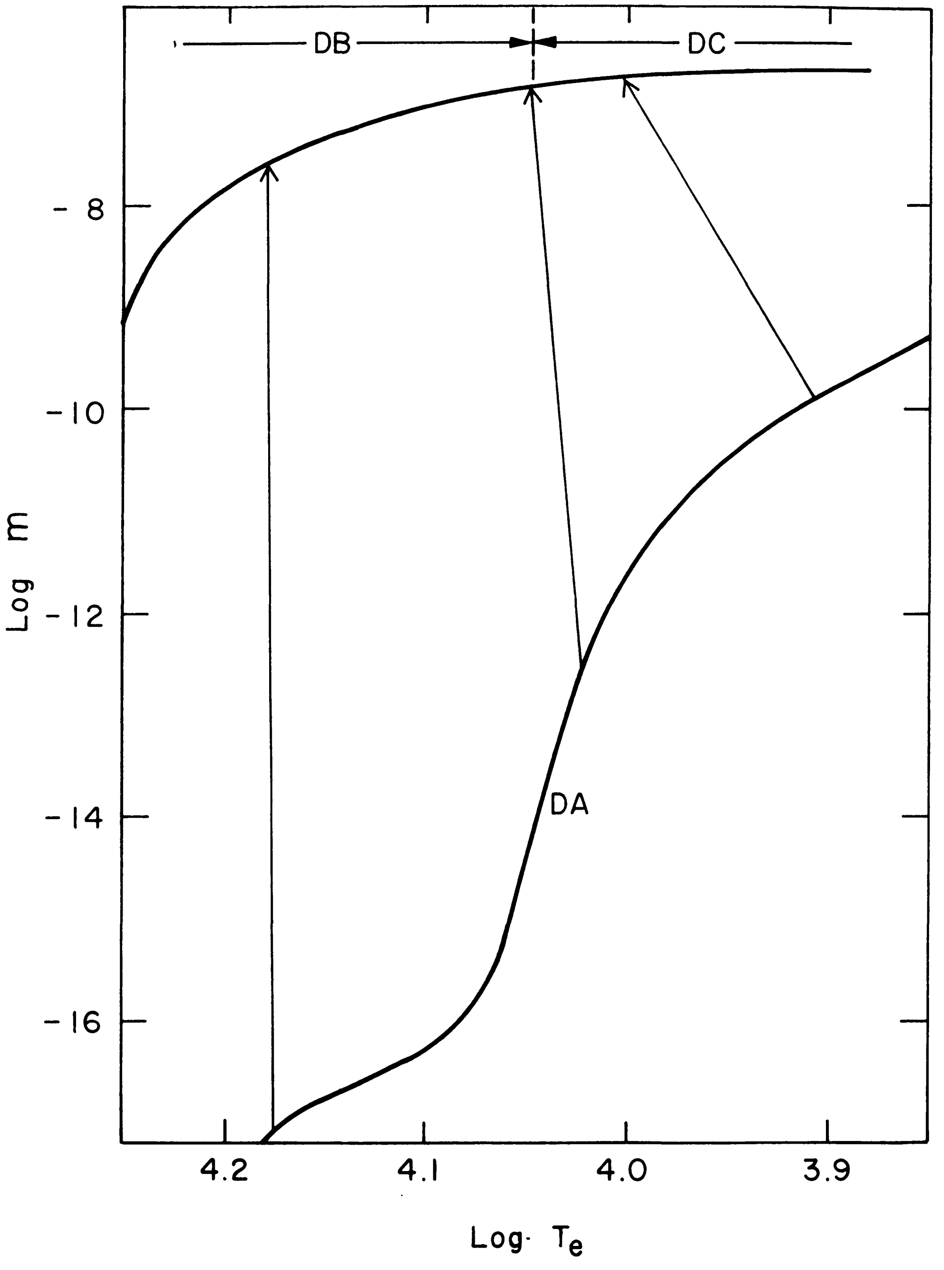

Fig. 7 Schematic evolution through the different spectral types by mixing according to Baglin and Vauclair. See text for explanation.

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be ejected in a shock wave and the process will repeat itself or the thermal runaway will take the star to the region of the red giants burning hydrogen in a shell.

7. White Dwarfs of DA, DB and DC Types

The possible interrelations between white dwarfs of types DA, DB, and DC have been considered by Strittmatter and Wickramasinghe (1971), Shipman (1973), Sion (1973), and possibly others. I will only give a brief review of a recent work by Baglin and Vauclair (1973) that was sent to me in preprint form as I was preparing this paper.

Baglin and Vauclair start from the consideration that there exist two classes of white dwarfs; the DA's, that show strong hydrogen features and the rest that show no hydrogen. It is then reasoned that if there is hydrogen left in the star it will show unless it is mixed by convection with the inner layers. They constructed envelopes of different effective temperatures and amounts of hydrogen above a helium envelope. The helium is convective. These envelopes are represented in Figure 7. For a given hydrogen mass a DA star cools to an effective temperature at which the whole mass of hydrogen becomes convective. This temperature is plotted in the lower curve. Mixing of the hydrogen with the helium produces an envelope of almost pure helium. The mass of these helium envelopes is given by the upper curve. The vertical lines show the transition by mixing from a DA to DB or DC according to the initial hydrogen content.

In this work accretion is neglected and it is considered that the gravitational energy liberated by accretion would produce enough energy to cause a coronal outflow preventing accretion from taking place.

References

Baglin, A. and Vauclair, G.: 1973, *Astron. Astrophys.* **27**, 307.
Böhm, K. H.: 1970, *Astrophys. J.* **162**, 919.
Böhm, K. H. and Cassinelli, J.: 1971, *Astron. Astrophys.* **12**, 21.
Brush, S. G., Sahlin, H. L., and Teller, E.: 1966, *J. Chem. Phys.* **45**, 2102.
Chandrasekhar, S.: 1939, *An Introduction to the Study of Stellar Structure*, University of Chicago Press, Chicago.
Cox, J. P. and Giuli, R. T.: 1968, *Principles of Stellar Structure*, Gordon and Breach, New York.
Cox, A. N. and Stewart, J. N.: 1970, *Astrophys. J. Suppl.* **19**, 261.
Grasberger, W. H.: 1961, *U.C.R.L. Report*, No. 6196.
Hubbard, W. B. and Lampe, M.: 1969, *Astrophys. J. Suppl.* **18**, 297.
Koester, D.: 1972, *Astron. Astrophys.* **16**, 459.
Mestel, L.: 1952, *Monthly Notices Roy. Astron. Soc.* **112**, 583.
Ostriker, J. P.: 1971, *Ann. Rev. Astron. Astrophys.* **9**, 353.
Ostriker, J. P. and Axel, L.: 1969, in S. Kumar (ed.), *Low Luminosity Stars*, Gordon and Breach, New York, p. 357.
Salpeter, E. E.: 1961, *Astrophys. J.* **134**, 669.
Schatzman, E.: 1958, *White Dwarfs*, North-Holland Publ. Co., Amsterdam.
Schwarzschild, M.: 1958, *Structure and Evolution of the Stars*, Princeton University Press, Princeton.
Shipman, H. L.: 1972, *Astrophys. J.* **177**, 723.
Sion, E. M.: 1973, *Astrophys. Letters* **14**, 219.
Strittmatter, P. A. and Wickramasinghe, D. T.: 1971, *Monthly Notices Roy. Astron. Soc.* **152**, 47.
Van Horn, H. M.: 1968, *Astrophys. J.* **151**, 227.
Vila, S. C.: 1971, *Astrophys. J.* **170**, 153.
Weidemann: 1968, *Ann. Rev. Astron. Astrophys.* **6**, 351.