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A SIMPLE ENERGY-BALANCE MODEL TO CALCULATE ICE ABLATION AT THE MARGIN OF THE GREENLAND ICE SHEET

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ABSTRACT. Data for daily ice ablation on two outlets from the Greenland ice sheet, Nordbogletscher (1979–83) and Qamanârssûp sermia (1980–86), are used to test a simple energy-balance model which calculates ablation from climate data. The mean errors of the model are only −1.1 and −1.3 mm water d^{-1} for Nordbogletscher (14 months) and Qamanârssûp sermia (21 months), respectively, with standard deviations of ±13.6 and ±18.9 mm water d^{-1} for calculating daily ablation. The larger error for Qamanârssûp sermia may be due to variations in ice albedo but the model also underestimates ablation during Föhn events.

According to the model, radiation accounts for about two-thirds of mean ablation for June–August at the two sites, while turbulent fluxes account for about one-third. The average ablation rate is higher at Qamanârssûp sermia than at Nordbogletscher because both sensible-heat flux and short-wave radiation are higher.

INTRODUCTION

The Geological Survey of Greenland (GGU) has made glacier-climate studies at several locations in Greenland for planning hydro-electrical power (Olesen and Braithwaite, 1989). The measurements included almost daily readings of ablation on two outlet glaciers from the Greenland ice sheet, Nordbogletscher and Qamanârssûp sermia (Fig. 1), in parallel with collection of simple climate data. Braithwaite and Olesen (1985, 1990) used these data to correlate ice ablation with air temperature, while we now describe the calculation of ablation by a simple energy-balance model based on Ambach (1986) and Ohmura (1981).

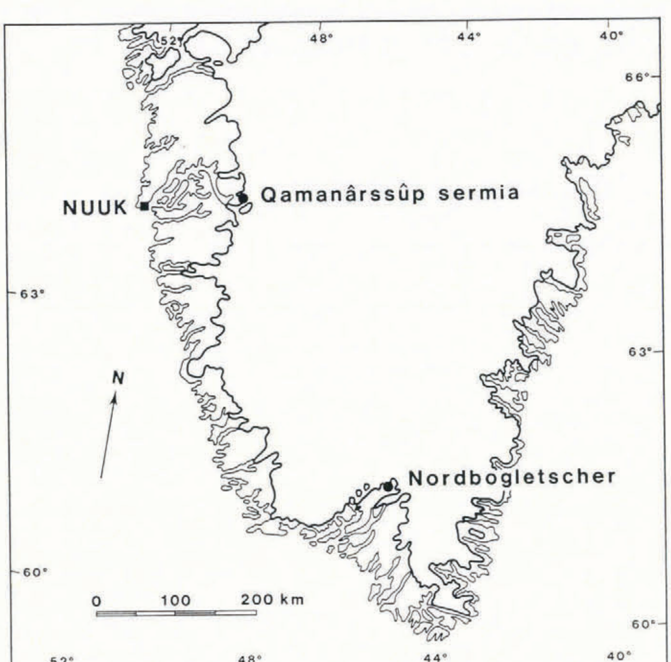

Fig. 1. Locations of glacier-climate stations of the Geological Survey of Greenland (GGU).

DATA

Data were collected over varying periods from May until September in each summer but, for convenience of making comparisons, the present analysis is based on data for June–August which represents the main ablation period at both stations. The availability of combined glacier-climate data for each summer is shown in Table I.

Ablation

Ablation readings were made at many stakes on both Nordbogletscher and Qamanârssûp sermia but the present paper refers only to measurements at stake 53 (at 880 m a.s.l.) on Nordbogletscher and at stake 751 (at 790 m a.s.l.) on Qamanârssûp sermia near the margins of the respective glaciers, i.e. about 200 m from the margin at Nordbogletscher and about 100 m at Qamanârssûp sermia. These were the so-called "daily stakes" which were measured almost every day in the late afternoon or early evening. The ablation "day" is not therefore identical to the reference period for daily climate data (see below).

The data mainly refer to ice ablation as both sites have little or no winter snow, although traces of new snow occur occasionally in cold periods during the summer

TABLE I. NUMBER OF DAYS WITH COMBINED ABLATION AND CLIMATE DATA FOR JUNE–AUGUST AT TWO GLACIERS

	Nordbogletscher	*Qamanârssûp sermia*
Stake	53	751
Elevation	880 m a.s.l.	790 m a.s.l.
Latitude	61°28′ N.	64°28′ N.
Summer		
1979	62	
1980	92	61
1981	91	85
1982	81	72
1983	89	76
1984		68
1985		66
1986		84
Total	415	512

(averages of 4 and 3 d/month at Nordbogletscher and Qamanârssûp sermia, respectively). Ice ablation is determined by measuring the lowering of the ice surface relative to the top of the stake and assuming a constant density for ice. The latter is not exactly correct as the density of the glacier-surface layer depends on weather conditions; a whitish "weathering crust" (Ambach, 1963, p. 185-86: Müller and Keeler, 1969) several centimetres deep often develops in sunny weather due to internal melting and disappears again under rainy or cloudy conditions. Although important on a short-term basis, these density variations have little effect on calculated ablation totals for longer periods.

The "daily stakes" at Qamanârssûp sermia, and at Nordbogletscher since 1981, are actually three separate stakes within a few metres of each other. Despite their closeness, the stakes seldom register the same ablation because of measurement errors and differences in micro-topography. The inter-stake difference has standard deviations of ±13 to ±19 mm water d^{-1} for daily ablation (Braithwaite, 1985, p. 21-22).

Climate

The meteorological measurements at the field stations are made by simple recording instruments, supplemented by hand observations in the mornings and evenings. All data are analysed with respect to the day 0–24 h Greenland summer time (UCT minus 3 h).

Air temperature and relative humidity are recorded continuously by Lambrecht thermohygrographs in standard instrument shelters 2 m above ground. The vapour pressure of the air is calculated from air temperature and relative humidity (Wilson, 1974, p. 8). The run-of-wind is read twice-daily from Lambrecht cup anemometers mounted 4 m above the ground, and average wind speeds are calculated for 12 and 24 h intervals. Global radiation, i.e. short-wave radiation from sun and sky, is recorded with Belfort actinographs, supplemented by daily sunshine duration from Campbell–Stokes recorders. The actinographs were installed in 1981 and values for earlier periods, i.e. for 1979–80 at Nordbogletscher and for 1980 at Qamanârssûp sermia, are calculated from observed sunshine duration by an empirical equation (Appendix).

Climate data are available for longer periods than indicated by Table I which refers to availability of combined ablation and climate data. For example, climate measurements were made at Nordbogletscher in 1978 and June 1979 before daily ablation measurements were started in July 1979.

THE ENERGY-BALANCE MODEL

The ablation stakes are located close to the ice margin in both cases and are presumed to have the same climate as the field stations aside from being about 0.2 deg colder in both cases because they are 30 m higher in elevation. Ablation is simulated by the model using daily means of air temperature, wind speed, vapour pressure, sunshine duration, and daily totals of global radiation measured at the field stations as described above.

Albation

The simulated ablation ABL* is obtained from

$$ABL^* = SHF + LHF + SWR + LWR \qquad (1)$$

where SHF and LHF are turbulent sensible- and latent-heat fluxes, and SWR and LWR are the short-wave and long-wave radiation fluxes. The observed ablation ABL is given by

$$ABL = ABL^* + ERR \qquad (2)$$

where ERR accounts for errors in both the data and in the model, or caused by neglected terms, e.g. heat flux into the ice. As defined in Equation (2), ERR can be regarded as a fifth energy-balance component. For convenience, all the ablation sources are expressed in equivalent ablation units, i.e. as mm water d^{-1} or kg m^{-2} d^{-1}.

Turbulent-heat fluxes

Turbulent-heat fluxes are often described by flux-gradient relations where SHF and LHF are proportional to the vertical gradients of air temperature and absolute humidity in the air immediately over the glacier surface. The correct formulations of these relations are difficult but Ambach (1986) has suggested simple approximations based upon energy-balance measurements on the Greenland ice sheet (Ambach, 1963, 1977). These approximations are valid for a melting glacier surface, i.e. temperature equal to 0 °C and vapour pressure equal to the saturation vapour pressure at 0 °C, and assuming an adiabatic stratification in a Prandtl-type boundary layer with different aerodynamic roughness parameters for ice and snow surfaces. The suggested relations are

$$SHF = K_S \cdot P \cdot T_2 \cdot V_2 \qquad (3)$$

and

$$LHF = K_L \cdot \Delta e_2 \cdot V_2 \qquad (4)$$

where K_S and K_L are coefficients, P is atmospheric pressure, T_2 is air temperature, V_2 is wind speed, and Δe_2 is the difference between vapour pressure of the air and saturation vapour pressure at the glacier surface. The subscript "2" indicates that temperature, wind speed, and vapour pressure are taken at 2 m above the glacier surface. A constant air pressure, depending only on elevation, is used for each station (91.3 and 92.4 kPa, respectively) as pressure variations due to different weather are small.

The numerical values of K_S and K_L are given in Table II in SI units for SHF and LHF in mm water d^{-1}, T_2 in °C, V_2 in m s^{-1}, and Δe_2 and P in Pa.

TABLE II. NUMERICAL VALUES OF SENSIBLE- AND LATENT-HEAT FLUX PARAMETERS (AMBACH, 1986)

Parameter	*Ice*	*Snow*
Sensible-heat flux, K_S	6.34×10^{-6}	4.42×10^{-6}
Latent-heat flux, K_L		
(Condensation)	9.83×10^{-3}	6.86×10^{-3}
(Evaporation)	11.14×10^{-3}	7.77×10^{-3}

For SI units, see Equations (3) and (4).

The assumptions that sensible- and latent-heat fluxes are proportional to air temperature and vapour pressure, respectively, are similar to those made by Kuhn (1979), Escher-Vetter (1985), and Hay and Fitzharris (1988). The heat-transfer coefficient of Kuhn, also used by Escher-Vetter, is approximately equal to $K_S P V_2$ in present terminology. The bulk-exchange coefficient K of Hay and Fitzharris (1988) is proportional to Ambach's K_S parameter (Braithwaite, 1988)

Short-wave radiation

The short-wave radiation flux in mm water d^{-1} is given by

$$SWR = (1 - \alpha)G/0.335 \qquad (5)$$

where α is the albedo, G is the global radiation in MJ m^{-2} d^{-1}, and 0.335 MJ kg^{-1} is the latent heat of fusion. Ambach (1986) assumed the albedo α is 0.3 for ice and 0.7 for snow.

Long-wave radiation

The long-wave radiation flux in mm water d^{-1} is given by

$$LWR = (L_{\downarrow n} - 27.35)/0.335 \qquad (6)$$

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where $L_{\downarrow n}$ is the incoming long-wave radiation and 27.35 is the outgoing long-wave radiation from the melting glacier surface (both in MJ m^{-2} d^{-1} units). The incoming long-wave radiation is given by

$$L_{\downarrow n} = \varepsilon^* \cdot \sigma T_a^4 \quad (7)$$

where ε^* is the effective emissivity of the sky, σ is the Stefan–Boltzmann constant, and T_a is the air temperature on the absolute scale. The effective emissivity ε^* is expressed in terms of cloud cover n, and the emissivity of the clear sky ε_0 by

$$\varepsilon^* = (1 + kn)\varepsilon_0 \quad (8)$$

where k is a constant depending on cloud type. Ohmura (1981, p. 243) listed k values for eight different cloud types but a constant value of 0.26 is assumed here (the average of Ohmura's k values for A_c, A_a, S_c, and S_t cloud types). According to Ohmura (1981, p. 229), the clear-sky emissivity is

$$\varepsilon_0 = 8.733 \times 10^{-3}.\ T_a^{0.788} \quad (9)$$

where the temperature-dependence accounts for the increase of absolute humidity with temperature.

With present assumptions, the effective emissivity according to Equation (8) varies from 0.73 to 0.96 at both Nordbogletscher and Qamanârssûp sermia.

Surface conditions

The model takes account of differences between ice and snow surfaces, e.g. according to Table II sensible and latent fluxes to a snow surface are 30% lower than those to an ice surface under the same climatic conditions. The short-wave radiation flux is also 57% less for a snow surface (assumed albedo α = 0.7) than for an ice surface (α = 0.3) with the same global radiation.

The model assumes a melting glacier surface but there are days when the combined ablation sources are not strong enough to maintain the glacier surface at the melting point. This usually occurs with air temperatures below zero but sometimes at positive temperatures as discussed by Kuhn (1987). The calculated ablation in these cases is re-set to zero in the model.

ACCURACY OF ABLATION SIMULATION

Daily and monthly ablation

On average, the simulations are surprisingly accurate considering the simplicity of the model. For example, the mean of the error ERR = ABL − ABL* is only −1.1 and −1.3 mm water d^{-1} for Nordbogletscher (14 months) and Qamanârssûp sermia (21 months), respectively. However, errors are much bigger on a day-to-day basis, e.g. the standard deviation of ERR is ±13.6 and ±18.9 mm water d^{-1} for the two cases, which means that errors account for 45 and 42%, respectively, of the day-to-day ablation variance.

Daily-averaged values of ablation and energy balance for different months are listed in Tables III and IV, respectively, while observed and simulated ablation rates are plotted against each other in Figure 2.

The error ERR for daily-averaged ablation is much lower than for raw daily data, i.e. with standard deviations of ±3.0 and ±7.0 mm water d^{-1} for Nordbogletscher and Qamanârssûp sermia, respectively. Apart from the greater amplitude of error at Qamanârssûp sermia, there appears to be a seasonal trend from negative errors in June to positive errors in August.

Errors

The errors in measuring ablation are an obvious source of the error ERR. For example, there is remarkable agreement between the standard deviations of ERR for daily data and the range of ±13 to ±19 mm water d^{-1} quoted by Braithwaite (1985, p. 21-22) for this error in measuring daily ablation but measurement errors cannot be the only source of error. For example, the daily ablation and climate data are based on different definitions of "day", although it is difficult to estimate the magnitude of error here.

Another cause of error is neglect of terms in the energy-balance model. The model does not include heat

TABLE III. ABLATION AND SIMULATED ENERGY BALANCE FOR 14 months AT NORDBOGLETSCHER. UNITS ARE mm water d^{-1}

Year	*Month*	*ABL*	*SHF*	*LHF*	*SWR*	*LWR*	*ERR*
1979	Jun						
	Jul	33.4	7.1	−1.3	40.3*	−11.6	−1.1
	Aug	33.2	10.6	−1.4	30.6*	−10.7	4.1
1980	Jun	26.0	7.0	−0.8	27.5*	−8.7	1.0
	Jul	35.2	8.4	0.2	36.1*	−9.2	−0.3
	Aug	23.5	8.7	−1.8	32.0*	−12.0	−3.4
1981	Jun	34.7	8.8	−0.2	37.6	−8.0	−3.5
	Jul	41.3	15.9	0.2	27.4	−4.6	2.4
	Aug	17.4	6.1	0.3	21.6	−7.6	−3.0
1982	Jun	27.7	5.3	1.5	37.9	−10.0	−7.0
	Jul	35.5	10.2	4.5	26.3	−5.6	0.1
	Aug	25.2	8.1	1.9	25.0	−9.1	−0.7
1983	Jun	17.3	5.9	0.2	24.0	−8.4	−4.4
	Jul	29.0	7.2	4.2	20.5	−4.9	2.0
	Aug	16.8	5.3	1.4	17.2	−6.0	−1.1
Means	Jun	26.4	6.8	0.2	31.8	−8.8	−3.5
	Jul	34.9	9.8	1.6	30.1	−7.2	0.6
	Aug	23.2	7.8	0.1	25.3	−9.1	−0.8
		28.3	8.2	0.6	28.9	−8.3	−1.1

*Global radiation estimated from sunshine duration.

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TABLE IV. ABLATION AND SIMULATED ENERGY BALANCE FOR 21 months AT QAMANÂRSSÛP SERMIA. UNITS ARE mm water d^{-1}

Year	*Month*	*ABL*	*SHF*	*LHF*	*SWR*	*LWR*	*ERR*
1980	Jun	30.3	13.4	−3.6	40.3*	−7.7	−12.1
	Jul	42.9	17.2	−2.3	40.2*	−6.5	−5.7
	Aug	32.6	19.6	−11.0	37.9*	−11.5	−2.4
1981	Jun	37.0	14.7	−1.9	42.7	−7.8	−10.7
	Jul	60.0	26.0	4.4	38.9	−4.0	−5.3
	Aug	25.8	9.4	0.1	23.3	−6.4	−0.6
1982	Jun	35.7	13.7	−7.4	49.5	−9.7	−10.4
	Jul	51.0	14.4	0.1	33.2	−3.3	6.6
	Aug	37.4	15.8	−9.0	31.0	−8.5	8.1
1983	Jun	29.7	12.2	−4.2	36.1	−8.2	−6.2
	Jul	47.1	14.0	3.6	33.0	−5.1	1.6
	Aug	18.4	5.1	−2.6	16.7	−5.8	5.0
1984	Jun	33.3	12.6	−6.2	45.7	−10.0	−8.8
	Jul	54.8	22.6	1.9	36.2	−4.0	−1.9
	Aug	35.7	13.4	1.9	17.6	−4.5	7.3
1985	Jun	48.7	20.6	1.2	33.7	−5.3	−1.5
	Jul	49.4	19.6	−1.3	38.9	−5.9	−1.9
	Aug	46.8	19.6	−1.6	30.6	−8.6	6.8
1986	Jun	19.0	7.7	−6.8	35.1	−9.7	−7.3
	Jul	54.8	21.6	2.3	37.9	−6.1	−0.9
	Aug	47.1	16.5	−0.2	22.1	−4.3	13.0
Means	Jun	33.4	13.6	−4.1	40.4	−8.3	−8.1
	Jul	51.4	19.3	1.2	36.8	−5.0	−1.1
	Aug	34.8	14.2	−3.2	25.6	−7.1	5.3
		39.9	15.7	−2.0	34.3	−6.8	−1.3

*Global radiation estimated from sunshine duration.

conducted into the ice as there are no data but this can be roughly assessed by analogy with other situations. For example, this heat flux amounted to −1.0 to −1.9 MJ m^{-2} d^{-1} for four series from Arctic Canada (Braithwaite, 1981), which is equivalent to only −3 to −6 mm water d^{-1} in ablation units. However, the active layer at both Nordbogletscher and Qamanârssûp sermia must be much warmer than in the Canadian cases, with lower englacial temperature gradients, so that heat conduction into the ice in the present cases is even smaller than in the Canadian cases. The heat provided by cooling of rainwater is also neglected in the model but a rough calculation shows that it is equivalent to less than 0.2 mm water d^{-1} of ablation, which can be neglected.

Radiation errors

The error ERR has a negative correlation with SWR, i.e. r = −0.28 and r = −0.51 for Nordbogletscher and Qamanârssûp sermia, respectively, suggesting that errors in radiation are partly responsible for ERR. For example, albedo seems higher in sunny weather and lower in cloudy weather (according to subjective observation) due to formation of "ablation crust". If true, this would give a negative correlation between ERR and SWR and, because SWR is highest in June, it would explain the apparent seasonal trend from negative to positive values of ERR. A trial re-calculation of the energy balance for Qamanârssûp sermia with an albedo of 0.4 for June (which is plausible) instead of 0.3 reduces the mean error for June ablation to only −2.4 compared with −8.1 mm water d^{-1}.

Another possibility is reduction in albedo from June to August due to increasing dirtiness of the glacier surface through the season (subjective observation). Routine measurements of albedo in future would help solve the problem.

Turbulent errors

Correlations between ERR and the turbulent fluxes SHF and LHF are not especially high but the model often underestimates ablation during Föhn events with high temperature and wind speed, and low humidity. This is curious as Ambach (1963, p. 121) suggested that non-adiabatic stratification should reduce the tubulent fluxes by up to 12% compared with those calculated for the adiabatic assumption implicit in the model, i.e. the model should overestimate ablation under Föhn conditions. The under-estimation found here may occur because we use daily means of climate data which might not accurately reflect the coincidence of high temperatures and wind speeds during Föhn. Although these events are fairly rare, they involve high ablation rates, i.e. 100–150 mm water d^{-1}, so it would be useful to improve the calculation of turbulent fluxes.

ABLATION CONDITIONS

In the previous section, we examined the model accuracy while in the present section we use the model results to discuss ablation conditions. Errors in the model may cause some misinterpretation but the results, spanning four and seven complete summers, respectively, should be quite representative in a statistical sense.

Sources of ablation energy

The importance of the various ablation sources varies from year to year and throughout the summer but the basic pattern is represented by the mean values at the bottom of Tables III and IV, respectively.

The largest source of energy is short-wave radiation followed by sensible-heat flux and long-wave radiation. The

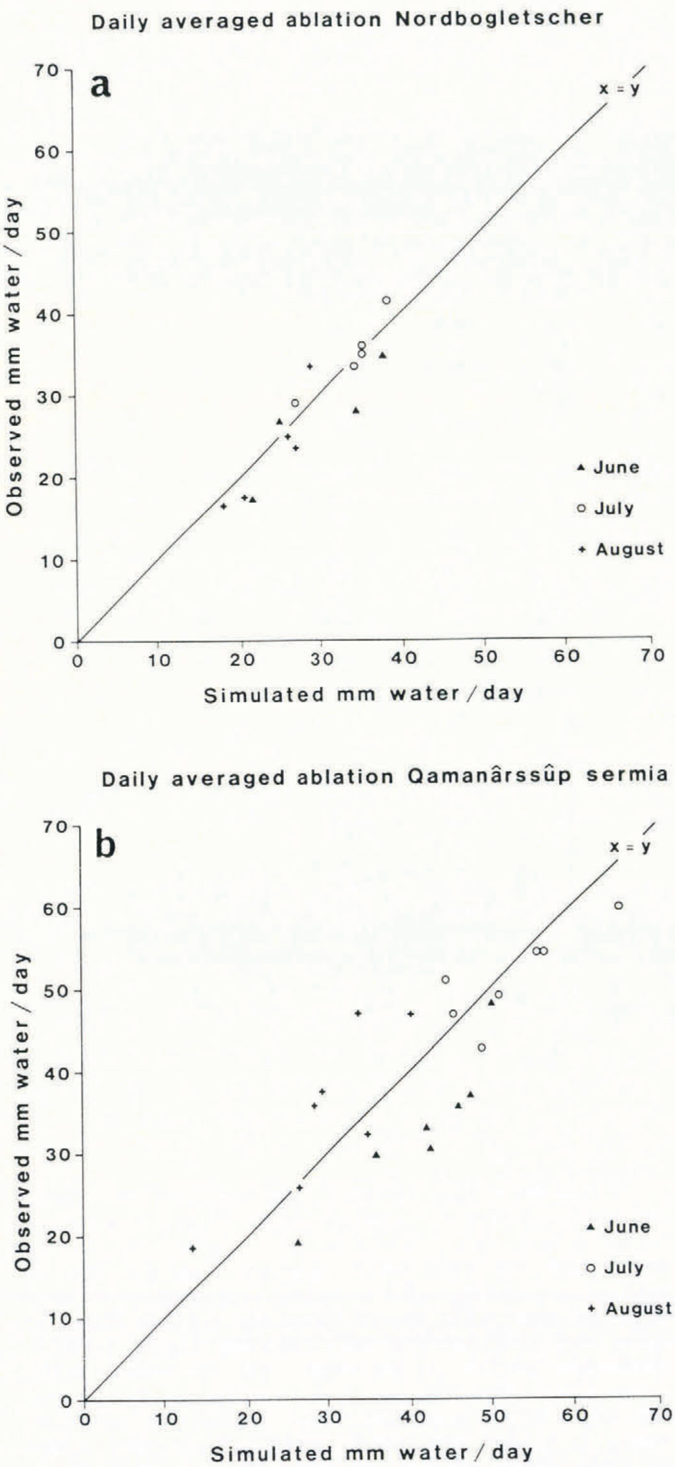

Fig. 2. Observed and simulated ablation rates. (a) 14 months at Nordbogletscher; (b) 21 months at Qamanârssûp sermia.

latent-heat flux is very small on average but this is the result of substantial fluctuations between negative and positive daily fluxes, i.e. evaporation and condensation, respectively, which nearly cancel out over longer periods.

In conventional terms, radiation (SWR and LWR) accounts for about two-thirds of mean ablation at the two sites (73 and 69% at Nordbogletscher and Qamanârssûp sermia, respectively) and turbulence (SHF and LHF) accounts for one-third (31 and 34%, respectively). Errors only account for respectively −4 and −3% of mean monthly ablation. These relative contributions by radiation and turbulence agree quite well with the estimates by Braithwaite and Olesen (1985) and with results of measurements by Knudsen and others (1987).

Differences between the two locations

The average ablation rate is higher at Qamanârssûp sermia than at Nordbogletscher because sensible-heat flux and short-wave radiation are both higher on average although slightly offset by lower latent-heat flux. This is because average temperature, wind speed, and global radiation are all generally higher at Qamanârssûp sermia (5.0 deg, 4.8 m s^{-1}, and 16.5 MJ m^{-2} d^{-1}) than at Nordbogletscher (3.7 deg, 3.3 m s^{-1}, and 14.6 MJ m^{-2} d^{-1}).

Ablation variations

Variations of ablation between different summers are illustrated by the deviations in Table V which refer to deviations of the summer averages from the means for four and seven summers, respectively.

Ablation at Nordbogletscher was low in summer 1983 mainly because of low short-wave radiation SWR (high cloudiness) but also due to low sensible-heat flux SHF (low temperature). Short-wave radiation was high in 1980 (low cloudiness) but this was nearly offset by low latent-heat flux (low humidity) and low long-wave radiation (low cloudiness), so the resulting average ablation was not exceptionally high in 1980.

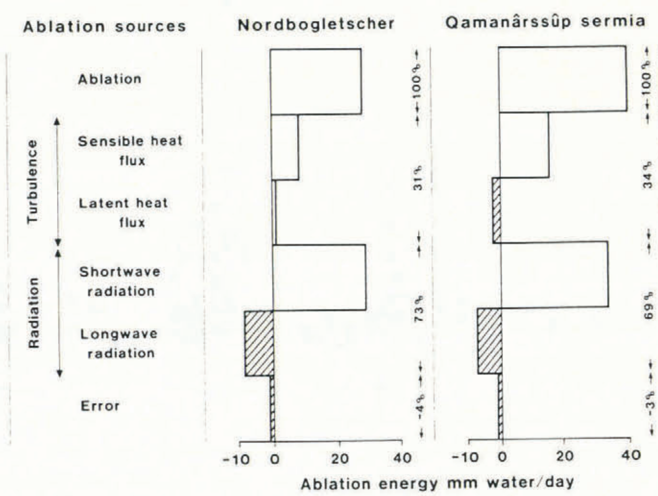

Fig. 3. Means of ablation and simulated energy balance for Nordbogletscher (14 months) and for Qamanârssûp sermia (21 months).

At Qamanârssûp sermia, the interpretation is more difficult because the amplitude of the error ERR is generally larger. For example, the error deviation in 1980 is larger than the ablation deviation, so the apparent low ablation in 1980 cannot be explained. However, there were clear cases of low ablation in 1983 and high ablation in 1985. The former was caused by low sensible-heat flux SHF (low temperature) and low short-wave radiation SWR (high cloudiness), while the high ablation in 1985 was due to high turbulent fluxes (high temperature and humidity) with short-wave radiation close to average.

DISCUSSION

Energy-balance measurements are difficult and expensive to make. This is why there are few measurements from Greenland and even first-class series like those of Ambach (1963, 1977) are limited in time coverage. By contrast, it is relatively easy to measure simple climate data over a few seasons and use them as input to the simple energy-balance model.

The energy-balance model can be used for research on ablation conditions. For example, we have used it to simulate ablation under a future greenhouse climate (Braithwaite and Olesen, 1990).

Another possible application of the model is real-time forecasting of run-off from glacier basins where hydro-electric power stations may be operated in the future.

TABLE V. SUMMER DEVIATIONS OF ABLATION AND ENERGY BALANCE. UNITS ARE mm water d^{-1}

Summer	*δABL*	*δSHF*	*δLHF*	*δSWR*	*δLWR*	*δERR*
Nordbogletscher						
1980	0.7	−0.1	−1.8	4.1	−2.2	0.6
1981	3.6	2.2	−0.9	1.1	1.1	0.1
1982	2.0	−0.2	1.6	1.9	−0.4	−1.0
1983	−6.5	−2.0	0.9	−7.2	1.4	0.3
Qamanârssûp sermia						
1980	−4.6	1.0	−3.6	5.2	−1.8	−5.4
1981	1.0	1.0	2.9	0.7	0.7	−4.2
1982	1.5	−1.1	−3.4	3.6	−0.4	2.7
1983	−8.2	−5.3	0.9	−5.7	0.4	1.4
1984	1.4	0.5	1.2	−1.1	0.6	0.2
1985	8.4	4.2	1.4	0.1	0.2	2.4
1986	0.4	−0.4	0.4	−2.6	0.1	2.9

Automatic weather stations could be used to measure all the necessary variables and the latest generation of "smart" data-loggers could even be programmed to make on-site model calculations. However, the accuracy of the model should be improved if possible.

CONCLUSIONS

The energy-balance model is surprisingly accurate considering its simplicity and deserves to be used more widely. Variations in ice albedo, neglected in the model, may be an important source of error and should be measured in future glacier-climate studies in Greenland. The calculation of turbulent fluxes, especially during Föhn events, should also be improved.

According to the model, radiation supplies about two-thirds of ablation energy at the two sites and turbulent fluxes supply about one-third. Ablation rate is higher at Qamanârssûp sermia than at Nordbogletscher because sensible-heat flux and short-wave radiation are both higher.

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APPENDIX

ESTIMATION OF MISSING DATA FOR GLOBAL RADIATION

Following Wilson (1974, p. 39), global radiation is estimated from sunshine duration for days when measured data are missing according to the formula

$$G/G_0 = a + b \cdot S/S_0 \qquad \text{(A1)}$$

TABLE VI. ERRORS IN CALCULATING GLOBAL RADIATION FROM SUNSHINE DURATION. UNITS ARE MJ m^{-2} d^{-1}

Month	Days	Global radiation		Error	
		Mean	S.D.	Mean	S.D.
Nordbogletscher					
Jun	81	17.0	±6.0	1.2	±3.2
Jul	88	12.8	±6.3	−0.8	±2.3
Aug	92	11.4	±4.9	−0.2	±1.9
	261	13.6	±6.2	0.0	±2.6
Qamanârssûp sermia					
Jun	167	20.8	±7.3	0.7	±2.9
Jul	174	17.3	±7.2	−0.1	±2.9
Aug	170	12.8	±5.5	−0.4	±2.4
	511	16.9	±7.4	0.1	±2.8

where G is the global radiation at the station, G_0 is the extra-terrestrial short-wave radiation, S is the observed sunshine duration, and S_0 is the potential sunshine duration. The variables G_0 and S_0 depend upon latitude and are calculated for each day by equations in Sellars (1965, p. 232).

The intercept a and the slope b in Equation (A1) are calculated by linear regression of observed G and S values for the days on which data are available for both. This gives $a = 0.22$ and $b = 0.47$ for Nordbogletscher (correlation coefficient $r = 0.90$ for 261 d), and $a = 0.27$ and $b = 0.52$ for Qamanârssûp sermia ($r = 0.91$ for 511 d). These a and b constants are not quite the same as given by Wilson (1974, p. 39) for middle latitudes.

Commenting on an early draft of this paper, Professor W. Ambach (personal communication) suggested that the a and b parameters might depend upon season. Ambach (1963, p. 75) also gave a non-linear relation between G/G_0 and cloud amount. We therefore re-examined the validity of Equation (A1) by re-calculating the a and b parameters for each month separately. Although different values were found for different months, differences were not statistically significant at the 5% level. As a further check, the error in estimating global radiation from sunshine duration with constant a and b parameters was calculated for each month (Table VI) and was found to be small compared with the error ERR in the energy-balance calculation.

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