SIR GEORGE GREENHILL, F.R.S.

Born Nov. 29, 1847—Died Feb. 10, 1927.

In an attempt to give a permanent impression of a unique personality in the ranks of mathematics,—a personality whose like will certainly not be seen again,—for he lived, essentially, in the days which produced some of our most revered pioneers born outside their own appropriate age,—Augustus de Morgan was born too early, perhaps, and Greenhill too late,—we may give, in his own words, a succinct summary of his own impression of his career. It is modest to a degree.

George Alfred Greenhill was educated at Christ's Hospital and St. John's College, Cambridge. He became a Fellow of his College, and spent the greater part of his life, before retirement, as Professor of Mathematics in the Artillery College at Woolwich. He was knighted on retirement in 1908, and subsequently lived in famous rooms at 1 Staple Inn, W.C., pursuing mathematical researches. His publications include the following :

    1885.   Differential and Integral Calculus, with Applications.
    1892.   Applications of the Elliptic Function.
    1894.   Hydrostatics.
    1908.   Notes on Dynamics.
    1910 (and again 1916).   Report 19, on Theory of a Streamline, with Applications to an Aeroplane.
    1912.   The Dynamics of Mechanical Flight.
    1914.   (Report 146.)   Gyroscopic Theory.

The above account is not exactly his statement as made in books of reference which besieged him, but it is nearly so. When he had given this, he was tired, for publicity of any kind was obnoxious to him. This fact, however, renders it difficult for anyone in the present generation to obtain more precise information. As it would be absurd to make any attempt at concealment of the identity of the writer of this notice, he will at once say that he was Greenhill's neighbour for many years at 1 Staple Inn, and a lifelong friend after he took his own degree. Any account of Greenhill must necessarily be of an anecdotal form, and from multitudinous letters which the writer has seen, at one time or another, but will not quote with names and context, he feels that he has almost the whole Royal Artillery, of a certain generation, behind all he says. For Greenhill was loved by his old pupils to a degree which few Professors can have enjoyed. We shall, however, now refer back to his account of his career, for the omissions are serious, if characteristic. The value of his work, in a mathematical sense, is not at the moment in question.

His baptismal name was George Alfred Greenhill, but he subsequently reversed the order of his Christian names. Well-authenticated rumour has it that the reason was his dislike of a nickname the others led to in his youth. The best authentication is perhaps provided by the fact,—practically unknown even to his closest friends, and for which I am indebted to our mutual friend, Professor Ernest Wilson,—that he was once a Whitworth Scholar. This must be a considerable surprise to most mathematicians, but the entry in the " Whitworth book " is as follows :

    " Greenhill, Sir George, Kt., M.A. ; Wh. Sc. 1869 ; b. 1847. Formerly Professor of Mathematics in the Artillery College, Woolwich. Author of the following works : "   (These need no repetition.)

But there is a further entry, and under the heading " Scholars appointed in 1869 ", we read :

    " Greenhill, George A., (21) Student, Cambridge University.
    " Hopkinson, John, B.Sc., (19) Student, Cambridge University."

It is of interest that these two pioneers, one ultimately becoming a mathematician with a strong practical bent, and the other a great engineer with a

strong mathematical bent, should have met on this basis.   May we conjecture, as all must have done who have studied such works of Greenhill as the Treatise on Elliptic Functions, that at this time he was at the cross-roads where the practical engineer and the mathematician take their leave of each other ?—but neither able to cast off the old love entirely, or to resist the colour it gave to all their subsequent work.   In this, at least, I think we have the explanation of the very distinctive style which marks off all work done by 'Greenhill from work by others,—more professedly mathematicians,—in the same spheres.

After an intensely intricate and perhaps laborious calculation in regard to elliptic functions, no mathematician is unaware of the difficulty of some of these, and the uncanny facility which Greenhill had in their performance, he suddenly became equally intensely practical, and regarded his result as of no real value until its correspondence with phenomena shown by some inorganic " corpus vile " had been investigated.   This inclination is perhaps specially pronounced as regards his work on elliptic functions, but it went through all he did, and, to him, " practical", or " applicable", or even " useful", was determined by some activity of man against the inorganic corpus.   Perhaps two historical instances may suffice.   Once the writer and others were candidates at an examination.   He wished to set what was, in effect, a perfectly reasonable example on the catenary.   He set it as a problem (two-dimensional) of " flexible inextensible sailcloth ",—terrifying nearly all the candidates. Later, again in reference to the catenary, he gave an address, on a well-remembered evening, to the members of the Cambridge Mathematical and Physical Society on " Catenaries ".   It was probably as brilliant an address, from any mathematical standpoint, as they ever had, but he combined it with a kind of conjuring entertainment.   At intervals, from all kinds of remote pockets, he produced objects to illustrate his remarks, especially in regard to the unstated assumptions in all current books of the time,—and, incidentally, of this time, perhaps,—and in regard also to his distrust of the " idealised " problems of applied mathematics forced upon the student.   He produced, for example, from nowhere apparently, a long piece of thick and stiff rope, and held it up, with the supports at the same level.   Its shape is left to the imagination, but he said, " This is the common catenary ",—and proceeded to the next section of his address.

We now turn again to the omissions in his autobiography.   A brief summary, again, will serve, and is not meant to be complete.   He was elected to the Royal Society in 1888, served on its Council from 1896 to 1899, and was a Royal Medallist of the Society.   Many foreign distinctions came to him.   We mention, in particular, that he was Officier d'Académie de Paris, Corresponding Member of the Academy of Sciences of Paris, and Foreign Member of the Reale Accademia des Lincei.   He was almost an active member of some, for they had frequent visits from him, and he had a multitude of close friends among Continental mathematicians.   He had played his part, as a great figure, in other Societies.   He had been President of the London Mathematical Society, and much of his most valuable work is to be found in the publications of the American Mathematical Society.

A picture of such a figure as Greenhill, however, would only be blurred by any catalogue of his distinctions, or any attempt to appraise precisely his mathematical work in relation to that of others of his time.   It is sufficiently well known, and his influence has been so profound on all subsequent workers, that none interested in the subject, whether from the tabular standpoint or any other, can be unaware of it.   We shall only recall the fundamental originality of his treatment of problems in his reports published by the Stationery Office in his later years, for they all immediately became classical, and its needs no old Army pupil of his to understand the value they held at a very critical time in our history.

No fellow mathematician, and nobody else who had come much into contact
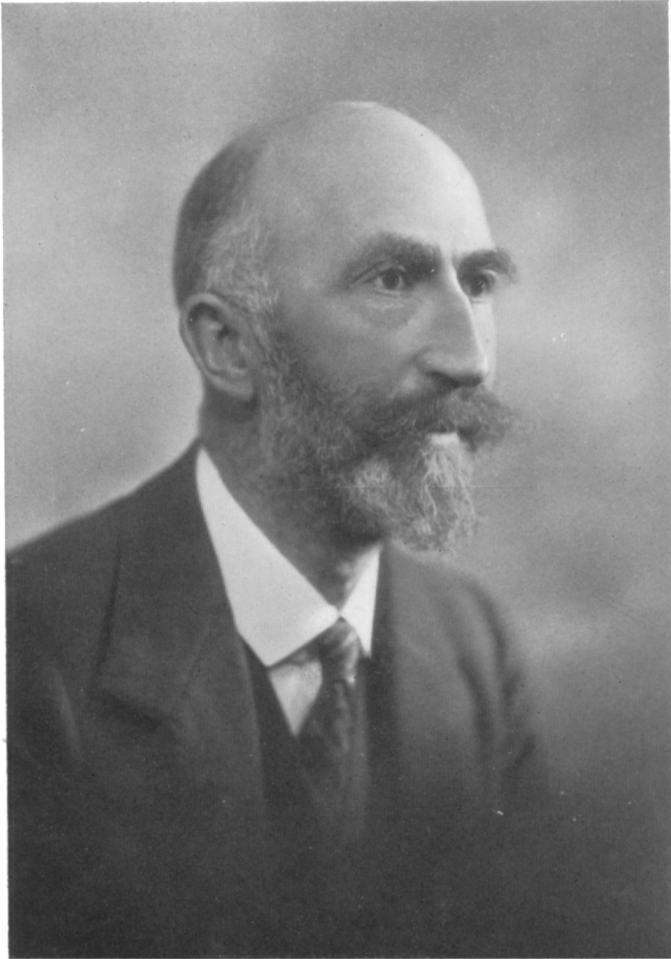
F. W. Hill.

with him, whatever his own private views, could fail to admire the inflexible purpose which he pursued when his mind was made up on any subject. The view of him, drilling, at his age, under a junior officer, in the Temple during the war, was one pathetic to some observers, depressing to others, and very stimulating to yet more, but surely an object of admiration to all. As our leading expert on any topic dealing with projectiles,—well are remembered his calculations while Big Bertha shelled Paris,—equal admiration must go to his efforts at this time to become a practical expert in musketry. It is an admiration of character, with claims on those of all views.

To some extent, the practical interests which inspired the lines of his main work in pure mathematics can be grouped under a small number of headings, which, however, give a most inadequate view of his profound contributions, either in outlook, mode of treatment, or results achieved, to subjects such as the theory of Elliptic Functions. They relate to, roughly, gyrostatic problems, problems of stability of ships and aeroplanes, apart from his more professional work on projectiles. They inspired, but did not bound, his achievement. He was a fine classical scholar, capable of sustaining a historical point with any opponent, if any scientific machine or a classical reference to one was in question. At the same time he could hold his own on a philological point, for classics constituted perhaps his main hobby.

He showed his friends, with delight, photographs and drawings of all historical attempts to evolve flying machines, navigable boats, and so forth, with all references ever made in literature. It is a pity that he never gave us, as he could have done, the last word on Leonardo da Vinci as an engineer. It would, in fact, have been a word full of profound admiration.

The editor must have found difficulty in obtaining any photograph of Greenhill,—or one he ever possessed of any of his friends. The reason is not well known, but may now be given. He considered that no photograph was of any value, as a criterion of the personality of its subject, unless it was of life-size. His greatest friend at one time, and the recipient at this, and all later times of his unbridled admiration, was James Clerk Maxwell. The photograph of Maxwell's head and shoulders, magnified to life-size, crowned his mantelpiece in his quaint room in Staple Inn. It had few companions, but all had undergone the same treatment. The results, to many, certainly bore out his contention,—so much so that he never, if it could be prevented, allowed himself to be photographed, and actually destroyed other photographs of his friends if they would not accept this condition of their appearance in his gallery.

At one time he possessed all Maxwell's notebooks, in which Maxwell jotted down, in pencil, rough drafts of his scientific ideas, amidst a welter of domestic details, reminders of appointments, and so forth. He lent two once to a mutual friend, and, during a reconstruction of this friend's study, they were lost. Greenhill never forgave this, and in spite of the unfortunate man's fervent protestation that they contained only Maxwell's laundry bills, and no mention of electrical calculations, he was belaboured with Greenhill's umbrella in the Strand. The matter was ultimately adjusted. The fate of the five volumes Greenhill still had is unknown, but would be of great interest.

Many of Greenhill's private papers are now before me. To attempt, even in an anecdotal form, to give extracts, would lead at once to a whole volume. Perhaps no mathematician of this or any other generation had so many stories associated with him,—but all appreciative of his impulsive and lovable nature. Many show light on his old friends, especially Klein,—and his domestic troubles when he wished to work,—perhaps later somebody, after a sufficient lapse of years, may undertake the task of producing such a volume.

Greenhill loved conferences of any international kind ; they did, in fact, constitute his holidays, and he followed them up for weeks later by visits to his friends. He officially disappeared at the end, and his whereabouts was

never known except to the friend he was then calling upon.   At the British Association meeting in Canada in 1925, he created a scare even in our newspapers, for he was presumed lost before the meeting started.   But he had only hurried on in front, and was found comfortably installed in the Students' Union.   He did not attend scientific societies to any great extent, but was a stimulating presence when he did.   He was President of the London Mathematical Society in 1890, and received from it the De Morgan Medal in 1902. This distinction he seemed to value more than any other he ever received. Was it because, as hinted already, he had more in common with Augustus de Morgan, either as mathematician or man, than anyone before or since ?—the main difference was, perhaps, that De Morgan married, and Greenhill did not.

But in later years, he felt somewhat out of touch with the newer trend of work, while still carrying on his own with extraordinary vigour at such an advanced age.   For in a letter, now before me, regarding the London Mathematical Society, he says regretfully that the younger generation appears to be interested only in convergence, function-theory, inequalities, and so forth, and implies that he is at least too old to give up the mathematical views he has always held.   But he is not hostile, and never could be, to any new development of value.

Readers of the *Gazette* will not need to be told that he was President of the Mathematical Association in 1913, and will equally need no reminder of his address on that occasion.   Our science has lost a unique figure, which will soon become legendary.   J. W. N.

Math. Note. 929. [V. 7].

RENATI FRANCISCI SLUSII

MESOLABUM

SEV

DVÆ MEDIÆ PROPORTIONALES

INTER EXTREMAS DATAS

PER CIRCVLVM ET ELLIPSIM

VEL HYPERBOLAM

INFINITIS MODIS EHIBITÆ

Leodij Eburonum

CIƆ IƆC LIX

(*Extract from Preface.*)

Problematis non novi nec incelebris effectionem tibi damus, Amice Lector, sed nobilis adeo & antiqui, ut consecrare audeat origines suas, & ad oraculum referre.

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Itaque forsitan actum, quod aiunt, agere videbor, dum post tot Clarissimorum Virorum conatus, ejusdem Problematis contemplationem rursus aggredior.   Sed nihilominus aliquid superesse credidi, in quo non inutiliter exercerer, cùm primùm illius naturam pressius examinavi.   Non quod ex eorum numero sim, qui rectâ & circulo illud construere inani labore contendunt : sed quod viderem, illos etiam qui vel organicæ rationis, vel sectionum Conicarum necessitatem agnovere, tam paucas nobis ejusdem demonstrationes hactenus ostendisse.

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Slusius rebukes the waste of time in attempting the Delian problem by straight line and circle.   For *demonstrationes* in the last line the edition of nine years later has *effectiones*, and alters *ehibitae* in the title to *exhibitae*.

Prestwich Lodge, Cheltenham.   A. A. BOURNE.