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A shell model for turbulent dynamos

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Abstract. A self-consistent nonlinear dynamo model is presented. The nonlinear behavior of the plasma at small scale is described by using a MHD shell model for fields fluctuations; this allow us to study the dynamo problem in a large parameter regime which characterizes the dynamo phenomenon in many natural systems and which is beyond the power of supercomputers at today. The model is able to reproduce dynamical situations in which the system can undergo transactions to different dynamo regimes. In one of these the large-scale magnetic field jumps between two states reproducing the magnetic polarity reversals. From the analysis of long time series of reversals we infer results about the statistics of persistence times, revealing the presence of hidden long-time correlations in the chaotic dynamo process.

Keywords. Dynamo models; magnetic reversals; MHD turbulence; shell models

1. Introduction

The understanding of dynamo problem is fundamental to explain the origin and the self-sustained of large scale magnetic field observed in natural systems like planets, stars, galaxies, black holes etc.

Many researchers (e.g., Kagemayama *et al.*, 2008) dealt with this problem using direct numerical simulation (DNS), even if realistic parameter regimes are beyond the power of actual supercomputers. Difficulties arise from the realistic description of both large-scales and small-scale (high Reynolds numbers) turbulence. Actual DNS are able to simulate only some few polarity reversals of the magnetic field. In order to overcome these difficulties we have built a model which takes into account very large Reynolds numbers and is able to reproduce very long time series of reversals which can be statistically analyzed giving the possibility to make a comparison with paleomagnetic data.

2. Turbulent Shell Dynamo Model

The starting point of our model is the decomposition of the fields in an average part, varying only on the large scale L, and a turbulent fluctuating part, varying at small-scales $\sim \ell$, with the assumption $\ell \ll L$ (Parker, 1955). Performing this scale separation we obtain, in the induction equation at large scale, a term which describes the action of small scales on the large one consisting in a turbulent e.m.f. that can be written in terms of the Fourier modes of velocity $(\mathbf{u}(\mathbf{k},t))$ and magnetic field $(\mathbf{b}(\mathbf{k},t))$ small scale fluctuations as follow:

$$\epsilon = -\sum_{\mathbf{k}} \mathbf{u}(\mathbf{k},t) \times \mathbf{b}^*(\mathbf{k},t) \ . \qquad (2.1)$$

Introducing a basis in the spectral space: $\widehat{e_1}(\mathbf{k})$, $\widehat{e_2}(\mathbf{k}) = \widehat{e_3}(\mathbf{k}) \times \widehat{e_1}(\mathbf{k})$, $\widehat{e_3}(\mathbf{k}) = i\mathbf{k}/|\mathbf{k}|$; and writing expression (2.1) in a form symmetric with respect to the change of $\mathbf{k}$ in $-\mathbf{k}$

we finally find:

$$\epsilon = - \sum_{\mathbf{k}(\mathbf{k_z}>\mathbf{0})} \widehat{e_3} \; [(u_1^* \; b_2 - u_2 \; b_1^*) + (u_2^* \; b_1 - u_1 \; b_2^*)] \tag{2.2}$$

where u_1 and u_2 (b_1 and b_2) are the components of $\mathbf{u}(\mathbf{k},t)$ ($\mathbf{b}(\mathbf{k},t)$), along $\widehat{e_1}$ and $\widehat{e_2}$.

We describe the dynamics of the system at large scale by integrating the induction equation in an axisymmetric situation and local approximation: we approximate the toroidal ($\widehat{e_\varphi}$) and the poloidal ($\widehat{e_p}$) unit vectors with the cartesian unit vectors, $\widehat{e_x}$ and $\widehat{e_z}$. Hence the field at large scale are: $\mathbf{u}_0 = V(y,z)\;\widehat{e_x}$; $\mathbf{b}_0 = B_\phi(y,t)\widehat{e_x} + B_p(y,t)\widehat{e_z}$.

The dynamics at small scales is described by a shell model. At variance with the original MHD shell model (Frick and Sokoloff, 1998; Giuliani & Carbone, 1998), here the nonlinear interactions avoid unphysical correlations of phases for each interacting triad. We can write the set of self-consistent equations for our dynamo model coupling the Eqs. for the small scales and the Eqs. for the field at large scale in which the e.m.f. is in a form consistent with the shell model and the spatial derivative associated with large scale is estimated dividing by the typical large scale L:

$$\frac{dB_\phi}{dt} = \frac{i}{L}\sum_n (u_n^* b_n - u_n b_n^*) + B_p \frac{V}{L} - \eta \frac{B_\phi}{L^2} \;, \tag{2.3a}$$

$$\frac{dB_p}{dt} = \frac{i}{L}\sum_n (u_n^* b_n - u_n b_n^*) - \eta \frac{B_p}{L^2} \;, \tag{2.3b}$$

$$\frac{du_n}{dt} = k_n(B_\phi + B_p)b_n + \mathrm{i}k_n\Big[(u_{n+1}^* u_{n+2} - b_{n+1}^* b_{n+2}) + \\ -\frac{1}{4}(u_{n-1}^* u_{n+1} - b_{n-1}^* b_{n+1}) + \frac{1}{8}(u_{n-2}u_{n-1} - b_{n-2}b_{n-1})\Big] - \nu k_n^2 u_n + f_n, \tag{2.3c}$$

$$\frac{db_n}{dt} = ik_n(B_\phi + B_p)u_n + \frac{ik_n}{6}\Big[(u_{n+1}^* b_{n+2} - b_{n+1}^* u_{n+2}) \\ +(u_{n-1}^* b_{n+1} - b_{n-1}^* u_{n+1}) - (u_{n-2}b_{n-1} - b_{n-2}u_{n-1})\Big] - \eta k_n^2 b_n; \tag{2.3d}$$

where ν is the viscosity and η is the diffusivity of the MHD flow; n is the shell number ($n = 0, ...N$); f_n is an external forcing term applied only on the first shell $k_0 \sim 2\pi/\ell$ ($n = 0$). This is an exponentially correlated Gaussian noise, characterized by a second moment $< f_1^2 >= \sigma^2/\ln 10$ and a correlation time $\tau_c = 1$, which has the property to preserve the energy flux to small scales (Giuliani & Carbone, 1998).

We solve the model Eqs. assuming $V = 0$, that is equivalent to solve the dynamo problem for Rossby number $\mathrm{Ro} = \frac{\delta u}{V}\frac{\delta b}{B_p} \gg 1$. Therefore $B_p(t) = B_\phi(t) = \mathrm{B}(t)$ and the model describes α^2 dynamo problem.

3. Numerical results

The model Eqs. are numerically solved by a fourth order Runge-Kutta scheme. The results are in dimensionless units: the field fluctuations are measured in Alfvén velocity unit c_A, the time in eddy-turn-over time $(1/(k_0 u_0))$, the lengths are normalized to $1/k_0$, and finally the dissipative coefficients are normalized to c_A/k_0.

At the beginning of each simulation, we let the system become turbulent at small scales. After that, we introduce a magnetic field seed of amplitude 10^{-10} at large scale and we check whether the dynamo effect starts to develop.

The numerical results reveal a strong sensitivity of the system with respect to the magnetic Reynolds number $\mathrm{Rm} \simeq \delta u/k_0\eta$ and a dependence on the hydrodynamic Reynolds number $\mathrm{Re} \simeq \delta u/k_0\nu$, where δu is the r.m.s. of the turbulent velocity fluctuations.

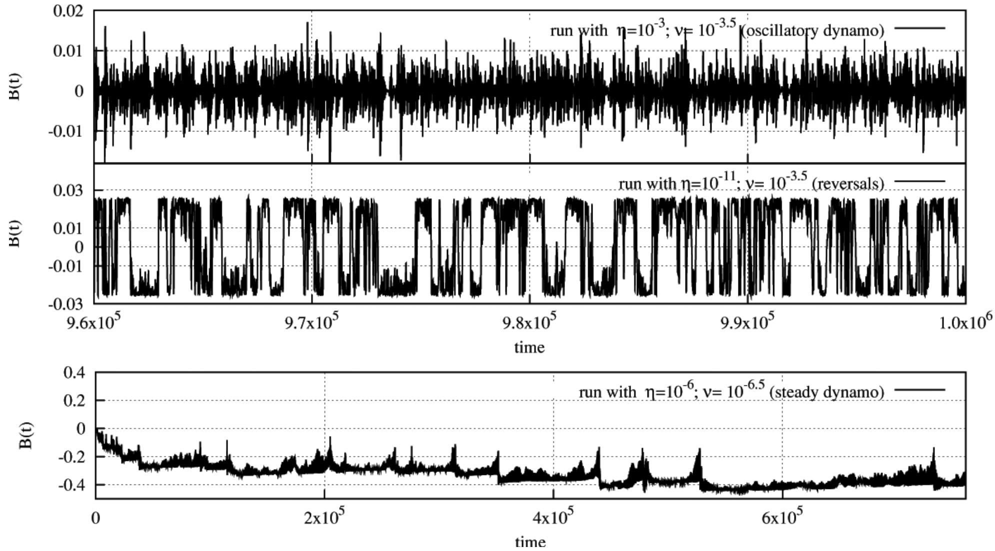

Figure 1. Time evolution of the large scale magnetic field in dimensionless units in different simulations.

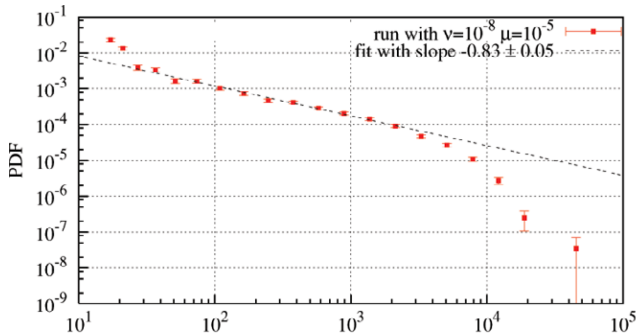

Figure 2. PDF of persistence times Δt in logarithmic scale.

Depending on these parameters, the system evolves towards different scenarios: i) no dynamo; ii) oscillatory dynamo; iii) magnetic reversals; iv) steady dynamo (see Fig. 1).

The model give us the capability to reproduce a long series of magnetic polarity reversals which can be statistically analyzed: the PDFs display a power law behavior (see Fig. 2), revealing the presence of hidden long-time correlations in the chaotic dynamo process. This is an argument in favor of some degree of memory in the chaotic dynamo as observed from analysis on the CK95 dataset of paleomagnetic inversions (Jonkers, 2003; Carbone *et al.*, 2006; Sorriso *et al.*, 2007; Nigro & Carbone, 2010).

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