

## Numerical studies of galaxy formation using special purpose hardware

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**Abstract.** I review recent progress in numerically simulating the formation and evolution of galaxies in hierarchically clustering universes. Special emphasis is given to results based on high-resolution gas dynamical simulations using the N-body hardware integrator GRAPE. Applications address the origin of the spin of disk galaxies, the structure and kinematics of damped Ly- $\alpha$  systems, and the origin of galaxy morphology and of galaxy scaling laws.

### 1. Introduction

Motivated by the increasing body of evidence that most of the mass in the universe consists of invisible “dark” matter, and by the particle physicist’s inference that this dark matter is made of exotic non-baryonic particles, a new and on the long run more fruitful approach to study the formation of galaxies has been developed: rather than to model the formation and evolution of galaxies from properties of present day galaxies, it is attempted to prescribe a set of reasonable initial conditions. The evolution of galaxies is then modeled starting from these initial conditions. Physical processes are taken into account that are considered to be relevant such as gravity, gas dynamics, radiative cooling and star formation. The outcome at different epochs is then confronted against observational data.

One scenario that has been extensively tested in that way is the model of hierarchical clustering, currently the most successful paradigm of structure formation in the universe. In this scenario, structure grows as objects of progressively larger mass merge and collapse to form newly virialized systems. The probably best known representative of this class of models is the *Cold Dark Matter* (CDM) scenario. The initial conditions consist of the cosmological parameters ( $\Omega$ ,  $\Omega_{\text{baryon}}$ ,  $\Lambda$ ,  $H_0$ ) and of an initial fluctuation spectrum such as the CDM spectrum. The remaining free parameter, the amplitude of these initial fluctuations, is calibrated by observational data, e.g., the measured anisotropies of the microwave background. Over the past few years, limits on the values allowed for these parameters have been consistently refined by improved observational techniques and theoretical insight, and it is widely accepted that a new “standard” model has emerged as the clear front-runner amongst competing models of structure formation. This  $\Lambda$ CDM model envisions an eternally expanding universe with the following properties (Bahcall et al. 1999): (i) matter makes up at

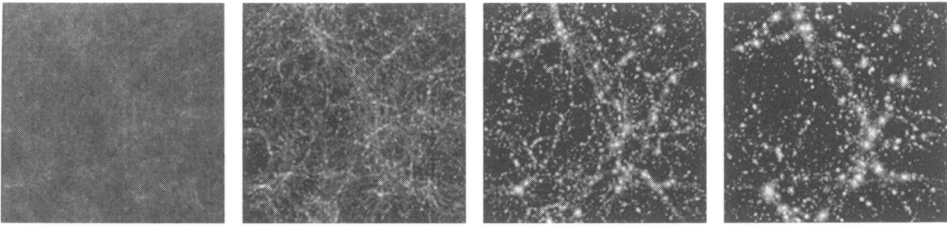


Figure 1. Time sequence of structure formation in a hierarchical clustering universe, here for the so-called  $\Lambda$ CDM model. The four snapshots correspond (from left to right) to redshifts of 9, 3.5, 1 and 0, respectively. The side length of the simulation box is  $32.5 h^{-1} \text{Mpc}$  comoving.

present less than about a third of the critical density for closure ( $\Omega_0 \approx 0.3$ ); (ii) a non-zero cosmological constant restores the flat geometry predicted by most inflationary models of the early universe ( $\Lambda_0 = 1 - \Omega_0 \approx 0.7$ ); (iii) the present rate of universal expansion is  $H_0 \approx 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$  ( $h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1} \approx 0.7$ ); (iv) baryons make up a very small fraction of the mass of the universe ( $\Omega_b \approx 0.019 h^{-2} \ll \Omega_0$ ); and (v) the present-day *rms* mass fluctuations on spheres of radius  $8 h^{-1} \text{ Mpc}$  is of order unity ( $\sigma_8 \approx 0.9$ ). The hierarchical structure formation process in this  $\Lambda$ CDM scenario is illustrated in Figure 1, which depicts the growth of structure within a  $32.5/h \text{ Mpc}$  box between redshifts nine and zero. The  $\Lambda$ CDM model is consistent with an impressive array of well-established fundamental observations such as the age of the universe as measured from the oldest stars, the extragalactic distance scale as measured by distant Cepheids, the primordial abundance of the light elements, the baryonic mass fraction of galaxy clusters, the amplitude of the Cosmic Microwave Background fluctuations measured by COBE, BOOMERANG, MAXIMA and DASI, the present-day abundance of massive galaxy clusters, the shape and amplitude of galaxy clustering patterns, the magnitude of large-scale coherent motions of galaxy systems, and the world geometry inferred from observations of distant type Ia supernovae, among others.

The hierarchical build-up is also thought to determine the morphology of a galaxy, most noticeably the difference between disk like systems such as spiral galaxies (some of them barred) and spheroidal systems such as elliptical galaxies and bulges. This picture envisions that whenever gas is accreted in a smooth fashion, it settles in rotationally supported disk-like structures in which gas is slowly transformed into stars. Mergers, however, convert disks into spheroids. The Hubble type of a galaxy is thus determined by a continuing sequence of destruction of disks by mergers, accompanied by the formation of spheroidal systems, followed by the reassembly of disks due to smooth accretion. This picture of a hierarchical origin of galaxy morphology has been schematically incorporated in so-called semi-analytical galaxy formation models used to study the evolution of the galaxy population, but its validity in a cosmological setting has just recently been directly demonstrated (Steinmetz & Navarro 2002).

Numerical simulations have been an integral part in the detailed analysis of the virtues of the CDM scenario. Only numerical techniques can account for the

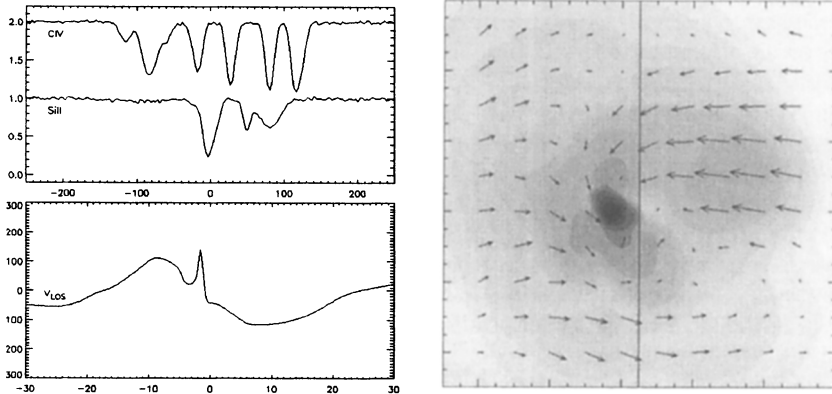


Figure 2. Right: Color map of the column density distribution in a 60 kpc box around a DLAS. Black correspond to HI densities  $\log n(\text{HI}) > 1.5$ , light gray to  $\log n(\text{HI}) \approx -3$ . Arrows indicate the velocity field. The solid line corresponds to the line-of-sight (LOS). The lower left plot shows, the velocity field along the LOS, the upper left plot the absorption line in CIV 1548 (top) and SiII 1808 (bottom). For readability, CIV has been displaced by 0.5 in flux.

highly irregular structure formation process and for at least some of the complicated interaction between gravity and other relevant physical processes such as gas dynamical shocks, star formation and feedback processes. Simulations also provide the required interface to compare simulations with observational data and are able to link together different epochs. While simulations of structure formation on the larger scales have mainly used large massively parallel supercomputers, studies how individual structures such as galaxies or clusters of galaxies form in the  $\Lambda$ CDM scenario have heavily used special purpose hardware like the GRAPE (=GRAVity PipeE) family of hardware N-body integrators (Sugimoto et al. 1990).

In this review I will concentrate on some examples, how high resolution gas dynamical simulation using special purpose hardware have illustrated some successes but also also some failures of  $\Lambda$ CDM scenario in reproducing structures as observed on the scales of galaxies.

## 2. Gas dynamical simulations – The kinematics of damped Ly- $\alpha$ systems

Although gas dynamical simulations were considerably successful in explaining some details of the galaxy formation process, the largest impact so far is related to the properties of QSO absorption systems. Numerical simulations can reproduce the basic properties of QSO absorbers covering many orders of magnitude in column density (Cen et al. 1994; Hernquist et al. 1996; Zhang, Anninos &

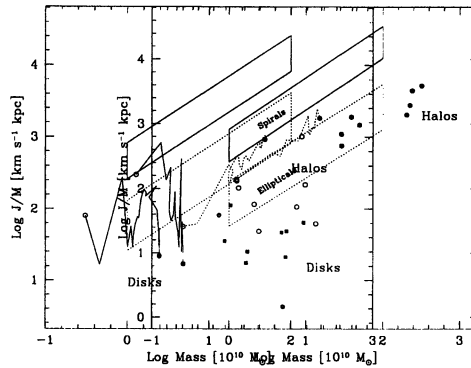


Figure 3. The specific angular momentum of dark halos and gaseous disks, as a function of mass. The boxes enclose the region occupied by spiral and elliptical galaxies, as given by Fall (1983). Open circles, solid squares and starred symbols correspond to the specific angular momenta of gaseous disks, solid circles for the hosting dark matter halos. Right: Evolution of the dark halo and central gaseous disk in the  $J/M$  versus  $M$  plane, from  $z = 5$  (open circles) to  $z = 0$  (solid circles). The mass of the system grows steadily by mergers, which are accompanied by an increase in the spin of the halo and a decrease in the spin of the central disk. The latter results from angular momentum being transferred from the gas to the halo during mergers.

Norman 1995). Indeed, gas dynamical simulations were even responsible for a paradigm shift, as QSO absorbers are no longer considered to be caused by individual gas clouds. Absorbers of different column density ( $\text{Ly-}\alpha$  forest, metal line systems, Lyman limit systems and damped  $\text{Ly-}\alpha$  absorption systems) are rather reflecting different aspects of the large-scale structure of the universe. While the lowest column density systems ( $\log N \approx 12 - 14$ ) arises from gas in voids and sheets of the “cosmic web”, systems of higher column density are produced by filaments ( $\log N \approx 14 - 17$ ) or even by gas that has cooled and collapsed in virialized halos ( $\log N > 17$ ).

The kinematics of damped  $\text{Ly-}\alpha$  absorption systems (DLAS) at high redshift serves as a very nice example to demonstrate how oversimplifying assumptions may lead to wrong implications on the validity of a cosmological model, while a full numerical treatment can avoid those artificial contradictions. DLASs have often been interpreted as large, high-redshift progenitors of present-day spirals that have evolved little apart from forming stars (Wolfe 1988). Kauffmann (1996), however, studied the evolution of DLASs in the CDM structure formation scenario in which disks form by continuous cooling and accretion of gas within a merging hierarchy of dark matter halos and found that the total cross section was dominated by disks with comparably low rotation velocities (typically 70 km/sec). Prochaska & Wolfe (1998) however, observed much larger

velocity spreads (up to 200 km/s) and came to the conclusion that only models in which the lines-of-sight (LOS) intersects rapidly rotating large galactic disks can explain both the large velocity spreads and the characteristic asymmetries of the observed low ionization species (e.g., SiII) absorption profiles, in strong contradiction to the prediction of the (semi-analytical) cold dark matter model.

So how may a numerical simulation solve this problem? The critical assumption that enters semi-analytical models is that DLASs are equilibrium disks. Numerical simulations (Haehnelt, Steinmetz & Rauch 1998) demonstrate that this is a poor assumption and that asymmetries and non-equilibrium effects play an important role. Figure 2 shows a typical configuration that gives rise to a high redshift DLAS with an asymmetric SiII absorption profile. The velocity width of about 120 km/s is also quite similar that of typical observed systems. No large disk has yet been developed and also the circular velocity of the collapsed object is only 70 km/s. The physical structures that underlie DLASs are turbulent gas flows and inhomogeneous density structures related to the merging of two or more clumps, rather than large rotating disks similar to the Milky Way. Rotational motions of the gas play only a minor role for these absorption profiles. A more detailed analysis also demonstrates that the numerical models easily pass the statistical tests proposed by Prochaska and Wolfe, i.e., hierarchical clustering, in particular the CDM model, is consistent with the kinematics of high- $z$  DLASs.

### 3. Gas dynamical simulations – The spin of disk galaxies

While the irregular structure of a galaxy in the process of formation was helpful to alleviate the problem concerning the kinematics of DLASs, one may wonder whether it hurts the assembly of large galactic disks at low redshifts as major mergers are usually associated with the transformation of spirals into ellipticals (for a review, see Barnes & Hernquist 1992).

Indeed, gas dynamical simulation are capable of producing disk like structures (see also section 4), but a closer inspection reveals that these model disks are too concentrated compared to observed spiral galaxies (Navarro & Benz 1991; Navarro, Frenk & White 1995; Navarro & Steinmetz 1997). However, this problem is not encountered in semi-analytical models, which relatively easily can reproduce the sizes of present day galaxies. One may again ask: What is the difference between the semi-analytical model (Mo, Mao & White 1997) and the numerical simulations, in particular since they are based on the same structure formation model? Again, the reason can be found in the assumptions that enter the semi-analytical models: Semi-analytical models assume that gas collapses under conservation of angular momentum (Fall & Efstathiou 1980), an assumption that, as will be shown, is only very poorly fulfilled.

Figure 3 (left) shows the specific angular momentum of dark matter halos and of their central gaseous disks at  $z = 0$ , as a function of mass. If, as suggested by Fall & Efstathiou (1980), the collapse of gas would proceed under conservation of angular momentum, the baryonic component would have the same specific angular momentum  $J/M$  as the dark matter, however, its corre-

sponding mass would be a factor of 20 smaller (for  $\Omega_{\text{bary}} = 0.05$ ,  $\Omega_0 = 1^1$ ). These disks would be located only slightly below the box for spiral galaxies. However, Figure 3 demonstrates clearly that the spins of gaseous disks are about an order of magnitude lower than that. This is a direct consequence of the formation process of the disks. Most of the disk mass is assembled through mergers between systems whose own gas component had previously collapsed to form centrally concentrated disks. During these mergers, and because of the spatial segregation between gas and dark matter, the gas component owing to dynamical friction transfers most of their orbital angular momentum to the surrounding halos. While the specific angular momentum of dark matter halos increases with decreasing redshift, that of gaseous disk decreases (Figure 3, right).

Up to now, it is unclear how this discrepancy can be overcome. One potential solution envisions the presence of strong feedback processes (Weil et al., 1998, Thacker & Couchman 2002) that prevent gas from clumping early on and that thus reduce the transfer of angular momentum due to dynamical friction. Alternatively, the low angular momentum gas may be ejected during early phases of galaxy formation or absorbed in the bulge component (Dominguez-Tenreiro et al. 1998, van den Bosch et al., 2002). If these solutions turn out to be unable to solve this problem, one may also have to consider modifying the CDM paradigm on the smallest scale, i.e., reducing number of mergers, e.g., by considering *warm dark matter* (WDM; see, e.g., Somer-Larson & Dolgov, 2001).

#### 4. Gas dynamical simulations including star formation – The origin of galaxy morphologies

A more realistic modeling of galaxy formation requires, of course, the inclusion of star formation and related feedback processes such as supernovae or stellar winds. Unfortunately, the physics of star formation and of the interaction of evolving stars with the interstellar medium is only ill understood. Explicit inclusion of these effects in direct numerical simulations can only be accomplished in the form of numerical “recipes” that contain a number of free parameters which can be calibrated e.g. such that Kennicutt’s (1998) relation between the HI surface density and the star formation rate per unit area (Schmidt’s law) is reproduced in isolated galaxy test cases.

The level of detail with which it is now possible to model the formation of individual galaxies is illustrated in Figure 4 and 5 which convincingly demonstrate that numerical simulations are now able to capture the main stages of galaxy formation, and to resolve their main morphological components. In particular, the following phases can be identified:

- **Formation of first disks.** At  $z \approx 4$  the first proto galactic clumps are numerically resolved (i.e. more than 500 particles per halo). Small disks (diameter  $\approx 3$  kpc) form near the centers of these clumps.

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<sup>1</sup>The angular momentum problem has first been demonstrated in simulations based on an  $\Omega_0 = 1$  CDM scenario. The problem is, however, generic to all hierarchical models and can also be seen in more recent simulations of galaxy formation in a  $\Lambda$ CDM cosmology

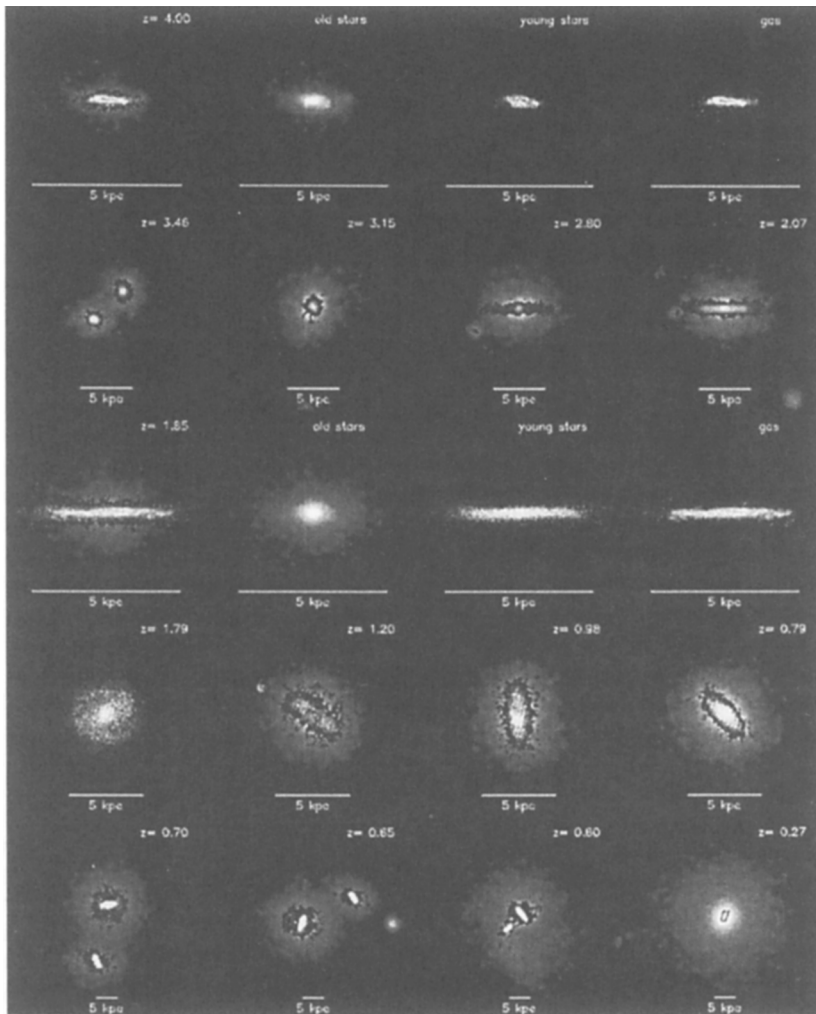


Figure 4. Surface density of the gaseous and stellar components of  $\Lambda$ CDM halo at various epochs. Horizontal bars in each panel are 5 (physical) kpc long and indicate the scale of each figure. Rows 2, 4 and 5 show time sequences near some key evolutionary stage. Rows 1 and 3 decompose a galaxy at a particular redshift (left) into its constituents: old stars, young stars and gas (from left to right). Top row: The most massive progenitor at  $z=4$  shown edge-on. Second row: The formation of a bulge and the rebirth of a disk. Third row: The appearance of the galaxy at  $z=1.8$ , seen edge-on. Fourth row: The tidal triggering of bar instability by a satellite resulting in the emergence of a rapidly rotating bar. Bottom row. A major merger and the formation of an elliptical galaxy.

- **Mergers and bulge formation.** The proto galactic clumps frequently merge ( $z \approx 3$ ) with objects of comparable size and mass. The small disks at the center are destroyed in this process and a central spheroidal component forms.
- **Regeneration of disks.** In the absence of major mergers, disk-like structures around the bulge regenerate from gas accreted both smoothly and through minor mergers ( $z \approx 2$ ). These objects feature all the major dynamical components of a bright spiral galaxy like the Milky Way: a rotationally supported disk of young stars, a centrally concentrated bulge, and a stellar halo of older stars.
- **Minor mergers and the formation of rapidly rotating bars.** Tidal torques during the close encounter with a minor satellite (mass ratio  $\approx 1 : 10$ ) can trigger the formation of a rapidly rotating bar in the disk. The bar pattern persists long after the disruption of the culpable satellite. The bar extends out to about 2.5 kpc and has a corotation radius of slightly less than 3 kpc, implying, in agreement with the few barred galaxies where this ratio has been measured (Debattista & Sellwood 1999), that the bar is “fast”. The bar pattern is clearly visible for more than 30 orbital periods, a consequence of repeated triggering by orbiting satellites aided by the fact that the baryonic component dominates the central potential (75% of the mass within 3 kpc is in the disk). Although the process described here is likely not the only bar formation mechanism acting in real galaxies, the simulation shows nevertheless a direct link between the presence of a bar pattern and the existence of satellites that may act as tidal triggers.
- **Major mergers and the formation of elliptical galaxies.** Mergers between close to equal mass galaxies effectively destroy disks and give rise to the formation of an elliptical galaxy. Residual gas in the disk progenitors is effectively channeled to the center where it is transformed to stars in a starburst like manner ( $z = 0.25$ ). The resulting object is mainly composed of “old” stars with a sprinkle of young stars at the center that formed during the last major merger.

Figure 4 illustrated the formation history for a galaxy that encountered a major merger at relatively low redshift ( $z \approx 0.7$ ) and experience little accretion of mass afterwards. The morphology at  $z = 0$  thus corresponds to that of an elliptical galaxy. Figure 5 shows a simulation of a different halo that did not experience any major merger after redshift 1.5, but more mass has been added by smooth accretion between  $z = 1.5$  and  $z = 0$ . Consequently, its final morphology is similar to that of a major disk galaxy.

The examples shown in Figures 4 and 5 demonstrate that high resolution simulations of the formation of individual galaxies are now capable to directly validate the long standing hypothesis, that depending on their merging history galaxies may repeatedly change their morphology and that morphology may be a transient phenomenon. These simulations also highlight some of the potential shortcomings and open issues of hierarchical galaxy formation scenarios. For example, is the large fraction of stars observed in disks today consistent with the ‘lumpy’ accretion histories expected in CDM-like scenarios? Can one account



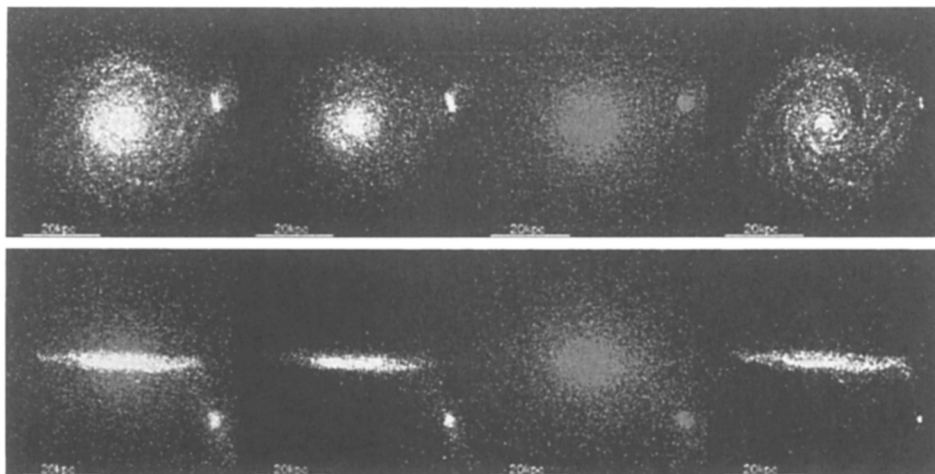


Figure 5. Face on (top) and edge-on (bottom) view of a simulated disk galaxy at redshift zero. Left: distribution of all stars and gas particles; Middle-left: distribution of stars younger than 0.6 Gyr; Middle-right: distribution of stars older than 5 Gyr; Right: distribution of gas particles

for the observed frequency, size, and dynamical properties of ‘pure disk’ (bulgeless) galaxies? Can variations in accretion history with environment account for the morphology–density relation? These questions are critical to the success of the hierarchical model and remain challenging puzzles to be elucidated within this so far highly successful paradigm. It will take a substantive computational effort to address them through direct simulation, but one that is within reach of today’s technological capabilities.

## 5. Gas dynamical simulations including star formation – The origin of disk galaxy scaling laws

The success of such a model can be further assessed by testing, to what extent such a model can reproduce scaling relations that link total luminosity, rotation speed, and angular momentum of disk galaxies such as the Tully–Fisher (TF) relation. Figure 6 shows the results of such an investigation, the simulated *I*-band TF relation at  $z = 0$  for the  $\Lambda$ CDM and for the  $\Lambda$ WDM scenario. The simulated TF relation is compared with the data of Giovanelli et al. (1997), Mathewson, Ford & Buchhorn (1992) and Han & Mould (1992). The slope and scatter of the simulated TF relation are in fairly good agreement with the observational data. This result also holds in other bandpasses: the model TF relation becomes shallower (and the scatter increases) towards the blue, just as in observational samples (see Steinmetz & Navarro 1999). The model TF relations are also very tight. In the *I*-band the *rms* scatter is only 0.25 mag, even smaller than the observed scatter of  $\sim 0.4$  mag. This must be so if the results are to agree with observations: scatter in the models reflects the intrinsic dispersion in

the TF relation, whereas the observed scatter includes contributions from both observational errors and intrinsic dispersion. If, as it is usually argued, both effects contribute about equally to the observed dispersion in the TF relation, then the intrinsic scatter in the I-band should be comparable to the  $\sim 0.25$  mag found in the models.

In addition, the zero-point is in rough agreement with observational data, at closer inspection the simulated galaxies appear to be  $\sim 0.3$  mag too dim. This disagreement can be accounted to a still sub optimal modeling of the star formation history in these models. Due to the particular parameterization of star formation, a large fraction of stars form already at high redshift resulting in a fairly high stellar mass-to-light ratio of 2-2.5 in I-band while observations indicate a value closer to 1-1.5 (Bottema 1997). It is also interesting that the zero-point of the TF relation seems to strongly depend on the assumed cosmological scenario. A similar simulation performed for a  $\Omega = 1$ ,  $\Lambda = 0$  CDM scenario results in a very serious disagreement of the zero point with the data, the simulated  $\Omega = 1$  TF relation being almost two magnitudes too faint at given rotation speed.

Another interesting clue on the origin of the Tully-Fisher relation is that the small scatter and the zero-point (for a given cosmological scenario) is only weakly dependent on the particular star formation and feedback parameterization: Very similar results can be obtained for quite different star formation prescriptions (Navarro & Steinmetz, 2000). This is a surprising result since the fraction of baryons that ends up in a disk strongly depends on the particular feedback model and can vary by factor 3-5 between different parameterizations.

The weak dependence of the scatter on the feedback parameterization is an immediate consequence of how the rotation velocity of the galactic disk responds to changes in the ratio between stellar and dark mass. As more and more baryon assemble in the central stellar disk, their luminosity increases but also the disk rotation velocity due to the gravity of the additional matter near the center. This effect is amplified as the additional gravity also pulls dark matter towards the center. Consequently a variation in the star to dark matter fraction results in a shift predominately parallel to the TF relation which thus does not cause substantial additional scatter (see Navarro & Steinmetz 2000 for a detailed discussion)

## 6. Summary and Conclusions

I presented some results of recent efforts to model the formation and evolution of galaxies in a hierarchical structure universe using high-resolution computer simulations. I demonstrated that only numerical simulation can take full account of the dynamics of the formation process and the complicated interplay between different physical processes such as, e.g., accretion and merging, star formation and feedback, photo heating and radiative cooling. Observational data can easily be misinterpreted if these effects are not properly included. For example, the apparent inconsistency of hierarchical structure formation models and the kinematics of high- $z$  damped Ly- $\alpha$  absorption systems could be easily solved by properly accounting for the complicated non-equilibrium dynamics of galaxies in the process of formation. Interestingly, the opposite is the case for the example

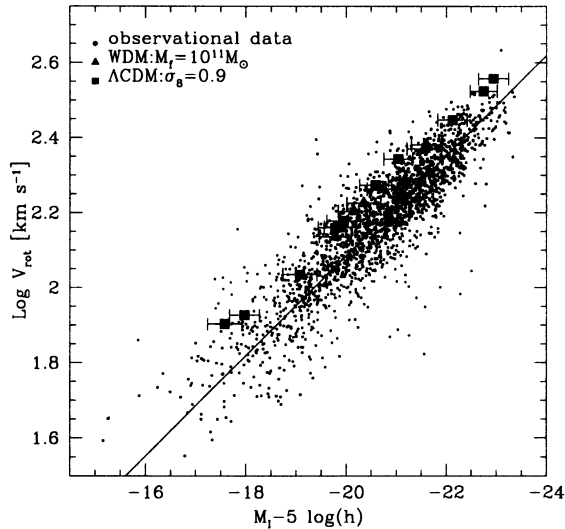


Figure 6. I-band TF relation at  $z = 0$  for a  $\Lambda$ CDM and a  $\Lambda$ WDM scenario. The error bars in the simulated data span the difference in magnitudes that results from adopting a Salpeter or a Scalo IMF.

of the galaxy spin. The simplifying assumption of collapse under conservation of angular momentum combined with success of this model in explaining the sizes of disk galaxies may lure someone to consider the origin of the sizes of disk galaxies as being solved. However, at closer inspection using numerical simulation, the physics behind the sizes of disk galaxies may be far more complicated. Maintaining the hierarchical build-up of galaxies and simultaneously avoiding substantial exchange of angular momentum from the gas to the dark matter due to mergers appears to be a major challenge to the scenario, and it cannot be excluded that the angular momentum crisis may finally lead to substantial revisions of the hierarchical structure formation scenario.

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