ORIGINAL PAPER

Eliciting utility curvature in time preference

Stephen L. Cheung[1,2]

Abstract
This paper examines the effects of alternative assumptions regarding the curvature of utility upon estimated discount rates in experimental data. To do so, it introduces a novel design to elicit time preference building upon a translation of the Holt and Laury method for risk. The results demonstrate that utility elicited directly from choice over time is significantly concave, but far closer to linear than utility elicited under risk. As a result, the effect of adjusting discount rates for this curvature is modest compared to assuming linear utility, and considerably less than when utility from a risk preference task is imposed.

1 Introduction

In both standard and behavioral theories of choice under risk and over time, the value of a risky or temporal prospect is typically modeled as a weighted sum of the utilities of its constituent elements. Thus, in the standard model of risk preference (von Neumann and Morgenstern 1944), the *expected utility* of a lottery is given by the probability-weighted sum of the utilities of its individual prizes, as evaluated by a Bernoulli utility function. Under expected utility, concavity of the Bernoulli function captures classical risk aversion, giving rise to a preference for more

✉ Stephen L. Cheung
Stephen.Cheung@sydney.edu.au

1   School of Economics, The University of Sydney, Social Sciences Building A02, Sydney, NSW 2006, Australia

2   IZA Institute of Labor Economics, Bonn, Germany

⦺ Springer

equally-distributed payoffs over states of nature. Analogously, in the standard model of time preference (Samuelson 1937), the *discounted utility* of a stream of payoffs is given by the (exponentially-) discounted sum of the utilities of its individual payoffs, as evaluated by an instantaneous utility function. Under discounted utility, concavity of instantaneous utility captures resistance to intertemporal substitution, giving rise to a preference to smooth payoffs over time. Leading behavioral alternatives, such as rank-dependent utility and cumulative prospect theory for risk (Quiggin 1982; Tversky and Kahneman 1992), and (quasi-) hyperbolic discounting for time (Laibson 1997; Loewenstein and Prelec 1992), retain this underlying additive structure while relaxing the assumptions of linear probability weighting and exponential discounting, respectively.

In principle, risk aversion and intertemporal substitution describe conceptually distinct preferences. Nonetheless, in settings where both risk and time are present it is common—and perhaps even natural—to assume that Bernoulli utility for risk is one and the same as instantaneous utility for time. In standard theory, this gives rise to the model of *discounted expected utility*, which has been a workhorse model of economics dating back at least to Phelps (1962). Alternatives to the standard model take divergent approaches to the question of whether interchangeability of Bernoulli and instantaneous utilities is maintained. On one hand, the class of recursive preference models developed by Kreps and Porteus (1978) and Epstein and Zin (1989) set out precisely to break the nexus—described by Weil (1990, p. 29) as a "purely mechanical restriction ... devoid of any economic rationale"—between risk aversion and intertemporal substitution. On the other hand, prospect-theoretic models of time-dependent probability weighting (Halevy 2008; Epper et al. 2011; Epper and Fehr-Duda 2012) posit a relationship between probability weighting and hyperbolic discounting, under the assumption that a single function characterizes utility for both risk and time.

The question of whether utility under risk is interchangeable with utility over time is also a core issue in the design of experiments to elicit time preference, *even though such experiments need not of necessity entail any interaction between risk and time*. The primary objective of such studies is usually to estimate the parameters of a discount function. However, since choices are a product of both the utility and discount functions, it is necessary to allow for the possibility of non-linear utility.

Unfortunately, until quite recently there were essentially no known methods to elicit the curvature of utility outside the domain of risk. This resulted in the prevalence of two main approaches. First, Coller and Williams (1999) estimate discount rates under the maintained assumption that utility is linear. These estimates are potentially biased if utility is in fact concave (Frederick et al. 2002, pp. 381–382).[1] Second, Andersen et al. (2008) measure utility by eliciting subjects' risk preferences, and combine risk and time preference data to jointly estimate a discount

---

[1] Consider a subject who is presented with a binary choice between a smaller-sooner payoff or a larger-later one, and suppose the former is chosen. There are two factors that combine to lead this subject to reject the larger-later alternative, namely time discounting and diminishing marginal utility. Therefore if the latter is assumed away, then the effect of the former will be overstated.

function adjusted for the curvature of utility. This assumes that utility under risk also represents utility over time; it is found that adjusting for this degree of curvature results in substantially lower discount rates than when utility is assumed to be linear.

The objectives of this paper are twofold. First, I introduce a novel experiment design that allows a clean comparison of the curvature of utility elicited under risk (in the absence of delay) and over time (in the absence of risk). This design builds upon and extends the well-known Holt and Laury (2002, hereinafter HL) procedure for risk preference, by transposing that design from state-payoff space into time-dated payoffs. The HL task is popular in its own right as means of eliciting the curvature of utility under risk, and also forms the basis for the curvature adjustment in the joint estimation approach. Second, I examine the effect upon estimated discount rates of alternative measurements of utility—namely whether utility is assumed to be linear, inferred from risk preferences, or revealed through choices over time.

Several related studies have likewise sought to measure the curvature of utility directly from choices over time.[2] In common with this paper, these studies share the key insight that to identify the curvature of instantaneous utility it is necessary to construct choices involving *bundles* of time-dated payoffs, as opposed to boundary choices between all-sooner versus all-later payoffs.[3] These studies find, again in common with this paper, that instantaneous utility is significantly concave yet close to linear. In the following paragraphs, I discuss these studies, and explain how this paper differs from each of them.

Abdellaoui et al. (2013) compare the curvature of utilities elicited under risk and over time, however they are not concerned with implications for the estimation of discount rates. For risk, they elicit the *certainty equivalent* (CE) of a risky prospect that pays x with probability p *or* otherwise y. For time, they elicit the *present equivalent* (PE) of a temporal prospect that pays x at time k *and* y today. Thus notice that these two procedures are not exactly comparable. For risk, the CE is an amount paid in *both states*. This implies, firstly, that the impact of diminishing marginal utility upon the marginal rate of substitution vanishes at the CE (see Eq. 2 in Sect. 2.1), and secondly that the CE lies between x and y. By contrast for time, the PE is an amount paid solely on a *single date*. The impact of diminishing marginal utility is thus maximized because the difference in payoffs between the two dates is also maximal (as the payoff on the second date, k, is implicitly zero), and the PE may be larger than both x and y. Thus Abdellaoui et al. measure the curvature of utility over different intervals of payoffs for risk and time, and in such a way that diminishing marginal

---

[2] For an expanded discussion of related literature, see Cheung (2016). Other relevant approaches, discussed in detail there, include those of Takeuchi (2011), Laury et al. (2012), and Attema et al. (2016).

[3] A distinct but complementary literature studies the preference for improving sequences. In a hypothetical survey framed as choice over wage profiles, Loewenstein and Sicherman (1991) document a preference for increasing over decreasing sequences. However, this was not replicated by Gigliotti and Sopher (1997) and Manzini et al. (2010) in incentivized experiments using money. Note that in this line of research, the undiscounted sum of payoffs is held constant, whereas experiments focused on discounting incorporate positive and varying interest rates. The literature on sequence preference also motivates psychological models, well outside the framework of discounted utility, such as those of Loewenstein and Prelec (1993) and Scholten et al. (2016).

utility has differing effects upon the trade-offs faced in the two domains. The design of the experiment in this paper seeks to avoid these confounds.

Andreoni and Sprenger (2012a) and Andreoni et al. (2015) compare estimates of utility curvature and discounting elicited using the Convex Time Budget (CTB) procedure to measures derived using the binary choice methodology of Andersen et al. (2008). The CTB design of Andreoni and Sprenger (2012a) identifies instantaneous utility by allowing subjects to choose any convex combination between an all-sooner and an all-later extreme, while the modified CTB of Andreoni et al. (2015) simplifies this to a multinomial choice. In this environment, the preference to smooth payoffs over time is expressed through the choice of an interior allocation. In fact, when payments on both dates are sent with certainty, choices occur predominantly at the corners of the budget set, indicating that utility is close to linear.[4] Andreoni and Sprenger (2012a) and Andreoni et al. (2015) compare this finding to that of a binary choice risk task of the type used by Andersen et al. (2008). They find that the latter indicates substantially greater utility curvature, and that the two curvature measures are uncorrelated at an individual level. Andreoni et al. (2015) further show that the risk-elicited curvature measure overstates the preference for interior allocations in the modified CTB.

Thus, Andreoni and Sprenger (2012a) and Andreoni et al. (2015) compare utilities for risk and time elicited using different experimental designs (binary choice for risk and CTB for time), with different associated estimation procedures. However, it is well-known that owing to violations of procedure invariance, risk and time preferences may not be stable across elicitation procedures (e.g., Tversky et al. 1990; Loomes and Pogrebna 2014; Freeman et al. 2016). Moreover, the bulk of previous research on time preference uses binary choices, and estimation techniques for such data are well established in both the risk and time preference literatures. Estimation methodology for continuous and multinomial CTB data is less settled (see discussions in Andreoni and Sprenger 2012a; Harrison et al. 2013; Andreoni et al. 2015). Thus, inferences from binary choice data for risk and CTB data for time may differ through any combination of: differences in experimental design, differences in estimation procedures,[5] or genuine differences in the curvatures of Bernoulli and instantaneous utility.

I seek to avoid these confounds by comparing the curvatures of utility for risk and time within a *unified design and estimation framework*, using binary choices for both. Moreover, my binary choice task for time is derived from a transposition of the standard HL task for risk: rather than varying probabilities (holding payoffs fixed), it is a payment date that varies instead. This ensures that when comparing these results to the risk preference task (or a joint estimation procedure as in Andersen

---

[4] Andreoni and Sprenger (2012b) study the case where payments on both dates are subject to risk, finding that interior allocations become more prevalent. One interpretation is that utility under risk may be more concave than under certainty. However, since the design involves an interaction of risk and time, the interpretation of this result is open to debate (Harrison et al. 2013; Cheung 2015; Epper and Fehr-Duda 2015; Schmidt 2014).

[5] To illustrate, Harrison et al. (2013) find convex utility in the data of Andreoni and Sprenger (2012a) using a different estimator.

et al. 2008), the estimation apparatus remains unchanged and it is only the source of information on the curvature of utility that differs.

The remainder of the paper proceeds as follows. Section 2 first interprets the HL design for risk in a state-preference framework before showing how it can be translated into time-dated payoffs and extended to identify both utility and discounting. The full experiment design consists of a series of choice lists that differ in whether the smaller-sooner option offers a more or less temporally-balanced combination of payoffs. If instantaneous utility is linear, a subject will have the same switch point in all lists, identifying the discount rate. However if utility is concave, this generates a preference for more temporally-balanced payoff bundles, resulting in systematic shifts in switching behavior across lists. Section 3 presents the results. The pattern implied by concave instantaneous utility is indeed observed, and is highly significant, but the magnitude is not large. The curvature of utility estimated from these choices is significantly concave, but less so than utility under risk, with the CRRA coefficient being an order of magnitude smaller. Adjusting for this degree of curvature has only a modest effect upon estimated discount rates compared to assuming linear utility, and a much smaller effect than when utility is inferred from risk preference using joint estimation. At an individual level, the curvatures of Bernoulli and instantaneous utility are uncorrelated. Joint estimates that constrain them to be the same predict time preference choices poorly because they overstate the preference for temporally-balanced payoff bundles. Section 4 concludes.

2 Design

2.1 State-preference representation of the HL design for risk

The HL experiment consists of a set of choices between two alternatives, labeled Options A and B, and is customarily presented as a choice list. Each alternative is a risky prospect that pays a low prize x_b in the "bad" state, with probability $1 - p_g$, or a high prize $x_g > x_b$ in the "good" state, with probability p_g. Options A and B represent two distinct payoff vectors, and in a given row of the choice list the probability p_g is the same for both alternatives. Moving down the rows of the list, the payoff vectors remain unchanged and it is only the probability p_g that varies.

Figure 1 illustrates using the payoffs used in this paper. Option A is a lottery that pays \$17 in the bad state (plotted on the horizontal axis), and \$20 in the good state (on the vertical).[6] Option B pays \$1 in the bad state, and \$38 in the good state.[7] Option A is safer in that the difference in payoffs $x_g - x_b$ is relatively small, whereas Option B is risky in comparison; in Fig. 1, this is represented by the fact that Option

---

[6] All payments are in Australian dollars. At the time of the experiments, one AUD was worth roughly USD 0.93 or EUR 0.68.

[7] These payoffs are thus approximately ten times the nominal stakes in the original HL experiment, however they have been modified slightly to generate more moderate interest rates when transposed into time-dated payoffs. The original HL payoffs were A = ($1.60, $2.00) and B = ($0.10, $3.85).

A lies closer to the diagonal, whereas Option B is close to the axis. In keeping with the original HL design, the probability p_g starts at 0.1 in the first row, and increases in increments of 0.1 up to a value of 1.0 in the final row.[8] The expected value of Option B thus increases more rapidly than that of Option A, and in the final row Option A is a dominated choice.

The rank-dependent utility of a risky prospect that pays x_b with probability $1 - p_g$ and $x_g > x_b$ otherwise is:

$$RDU(x_b, 1 - p_g; x_g, p_g) = [1 - w(p_g)] \cdot u(x_b) + w(p_g) \cdot u(x_g) \qquad (1)$$

where $w(p)$ is the probability weighting function, and $u(x)$ is the Bernoulli utility function. The (absolute) slope of an indifference curve is thus:

$$-\left. \frac{dx_g}{dx_b} \right|_{\overline{RDU}} = \frac{1 - w(p_g)}{w(p_g)} \cdot \frac{u'(x_b)}{u'(x_g)} \qquad (2)$$

This slope is a product of two terms: $[1 - w(p_g)]/w(p_g)$ is the probability-weighted odds of the bad state, while the ratio of marginal utilities $u'(x_b)/u'(x_g)$ captures the preference to smooth payoffs over the good and bad states of nature.

For the benchmark case of expected utility with a linear utility function, $w(p) = p$ and $u(x) = x$, the slope reduces to the objective odds $(1 - p_g)/p_g$ and the indifference curves are linear. In the early rows of the choice list p_g is small and the indifference curves steeper than the chord AB, such that a risk-neutral subject prefers Option A. Moving down the rows of the list, as p_g increases the indifference curves become flatter, and the subject eventually switches to Option B. In particular, a risk-neutral subject chooses Option A in the first four rows, and Option B thereafter.[9]

Relative to this benchmark, a risk-averse subject continues to choose Option A at higher probabilities of the good state p_g. This may occur as the subject over-weights the odds of the bad state,[10] such that $[1 - w(p_g)]/w(p_g) > (1 - p_g)/p_g$ and/or as Bernoulli utility is concave, such that $u'(x_b)/u'(x_g) > 1$. The impact of diminishing marginal utility vanishes when $x_g = x_b$, while it increases as the difference in payoffs grows. The indifference curves thus become steeper as they approach the vertical axis, such that the subject chooses Option A at larger values of p_g owing to a preference to avoid unequal payoffs across states.

---

[8] Full parameters of the risk preference experiment are enumerated in Appendix C.1.

[9] This is true both for the original HL parameters in footnote 7, as well as the modified parameters used here.

[10] Under rank-dependent utility, the indifference curves will be kinked at the 45-degree diagonal where the rank-ordering of prizes is reversed. Figure 1 thus depicts stylized indifference curves under expected utility.
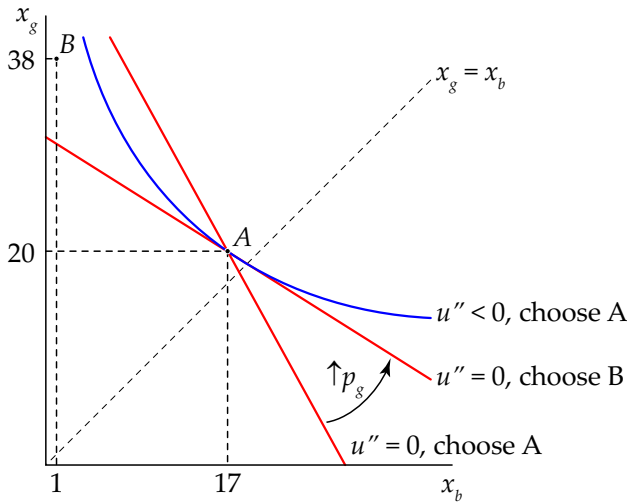
⚛ Springer

Fig. 1 State-preference representation of the HL design for risk

2.2 Time-dated translation of the HL design for time

To translate the logic of the HL procedure into the domain of time preference, Options A and B are recast as temporal prospects that pay an amount x_t on a "sooner" date t, *and* an additional amount x_{t+k} on a "later" date $t + k$. Letting the date of the experiment be 0, t is the "front-end delay" to the sooner payment, while k is the "back-end delay" between the sooner and later payments. Throughout this paper, t and k are expressed in weeks, while interest and discount rates are expressed in annualized terms. Consistent with the HL procedure for risk, each set of choices is presented as a choice list. Within a given list, Options A and B represent two distinct payoff vectors, and in a given row the payment dates are the same for both alternatives. Moving down the rows of the list, the payoff vectors remain unchanged and it is only the payment dates, and specifically only the back-end delay k, that varies.

Figure 2 presents the format of the choice list for the pair of payoff vectors corresponding to the risk preference task described in Sect. 2.1. In the first row, Option A offers $17 in 1 week *and* $20 in 28 weeks, while Option B offers $1 in 1 week *and* $38 in 28 weeks. Thus Option A is "smaller-sooner" in that it offers a smaller total payment in undiscounted terms, but more on the sooner date, while Option B is "larger-later". The front-end delay t is constant and equal to 1 week for all choices. The back-end delay k starts at 27 weeks in the first row and falls in decrements of 3 weeks down to 0 weeks in the final row. Thus in the final row all payments accrue after 1 week, such that Option A is a dominated choice.

By choosing Option B in a given row, a subject forgoes $17 − $1 = $16 from the sooner payment and in exchange receives an additional $38 − $20 = $18 in the later payment, a return of 12.5%. Since the subject must wait k weeks to attain this

<img>Springer</img>

return, the implied annual interest rate is $r = 1.125^{52/k} - 1$. As k falls, the subject waits a shorter length of time to realize the same return, and so the annual interest rate increases.[11]

Relative to a more conventional time preference choice list, this design differs in two key respects. First, all choices involve bundles of payments on two dates, as opposed to either the sooner or later date. Second, variation in the interest rate is generated by varying payment dates while holding the payoffs constant, rather than the other way around. As I explain next, this makes it possible to vary interest rates orthogonally to implications for intertemporal substitution, i.e. whether it is the sooner, later, or neither option that offers a more temporally-balanced bundle of payoffs.

2.3 Disentangling utility curvature and time discounting

The discounted utility of a temporal prospect that pays x_t on date t and x_{t+k} on date $t + k$ is:

$$DU(x_t, t; x_{t+k}, t + k) = D(t) \cdot v(x_t) + D(t + k) \cdot v(x_{t+k}) \tag{3}$$

where $D(t)$ is the discount function, and $v(x)$ is the instantaneous utility function.[12] The (absolute) slope of an indifference curve is thus:

$$-\left.\frac{dx_{t+k}}{dx_t}\right|_{\overline{DU}} = \frac{D(t)}{D(t+k)} \cdot \frac{v'(x_t)}{v'(x_{t+k})} \tag{4}$$

This slope is again a product of two terms: $D(t)/D(t + k)$ is the relative value of utility at date t compared to $t + k$, while $v'(x_t)/v'(x_{t+k})$ captures the preference to smooth payoffs over time.

For the benchmark case of exponential discounting with linear instantaneous utility, $D(t) = 1/(1 + \rho)^{t/52}$ (where ρ is the annual discount rate) and $v(x) = x$, the slope reduces to $(1 + \rho)^{k/52}$ and the indifference curves are linear. In early rows of the choice list, k is large and the indifference curves relatively steep, so a subject for whom ρ is sufficiently large initially prefers Option A. Moving down the list, as k decreases the indifference curves become flatter, and the subject eventually switches to Option B. In particular, the subject chooses Option A as $(1 + \rho)^{k/52} > 1.125$, i.e. as $\rho > r$, and Option B otherwise.

Thus in the benchmark case, this design functions exactly as a conventional time preference choice list, in that the "switch point" from smaller-sooner to larger-later identifies bounds on the discount rate. Therefore, in contrast to the benchmark case

---

[11] At ten times the original HL stakes for risk in footnote 7, we would have a return of 23.3% over k weeks, and the resulting annual interest rates would thus be considerably higher.

[12] The arguments in this section extend also to certain specifications in which intertemporal utility is not additively separable, such as the one studied by Andersen et al. (2018) and Cheung (2015). This is because, in contrast to those studies, the design here purposefully avoids interacting risk with time. See Appendix A.2 on this point.

DECISION TABLE 1

Make your choices by marking an "X" in the appropriate box in each row.

Weeks from today	Su	M	Tu	W	Th	F	Sa
						2014	
					May		
				1	2	3
0	4	5	6	7	8	9	10
1	11	12	13	14	15	16	17
2	18	19	20	21	22	23	24
3	25	26	27	28	29	30	31
				June			
4	1	2	3	4	5	6	7
5	8	9	10	11	12	13	14
6	15	16	17	18	19	20	21
7	22	23	24	25	26	27	28
8	29	30					
				July			
8			1	2	3	4	5
9	6	7	8	9	10	11	12
10	13	14	15	16	17	18	19
11	20	21	22	23	24	25	26
12	27	28	29	30	31		
				August			
12	31					1	2
13	3	4	5	6	7	8	9
14	10	11	12	13	14	15	16
15	17	18	19	20	21	22	23
16	24	25	26	27	28	29	30
				September			
17		1	2	3	4	5	6
18	7	8	9	10	11	12	13
19	14	15	16	17	18	19	20
20	21	22	23	24	25	26	27
21	28	29	30				
				October			
21				1	2	3	4
22	5	6	7	8	9	10	11
23	12	13	14	15	16	17	18
24	19	20	21	22	23	24	25
25	26	27	28	29	30	31	
				November			
25	30						1
26	2	3	4	5	6	7	8
27	9	10	11	12	13	14	15
28	16	17	18	19	20	21	22
29	23	24	25	26	27	28	29
	Su	M	Tu	W	Th	F	Sa

Decision	Option A	Your Choice	Option B
1	**$17** in **1 week** and **$20** in **28 weeks**	Option A ☐ Option B ☐	**$1** in **1 week** and **$38** in **28 weeks**
2	**$17** in **1 week** and **$20** in **25 weeks**	Option A ☐ Option B ☐	**$1** in **1 week** and **$38** in **25 weeks**
3	**$17** in **1 week** and **$20** in **22 weeks**	Option A ☐ Option B ☐	**$1** in **1 week** and **$38** in **22 weeks**
4	**$17** in **1 week** and **$20** in **19 weeks**	Option A ☐ Option B ☐	**$1** in **1 week** and **$38** in **19 weeks**
5	**$17** in **1 week** and **$20** in **16 weeks**	Option A ☐ Option B ☐	**$1** in **1 week** and **$38** in **16 weeks**
6	**$17** in **1 week** and **$20** in **13 weeks**	Option A ☐ Option B ☐	**$1** in **1 week** and **$38** in **13 weeks**
7	**$17** in **1 week** and **$20** in **10 weeks**	Option A ☐ Option B ☐	**$1** in **1 week** and **$38** in **10 weeks**
8	**$17** in **1 week** and **$20** in **7 weeks**	Option A ☐ Option B ☐	**$1** in **1 week** and **$38** in **7 weeks**
9	**$17** in **1 week** and **$20** in **4 weeks**	Option A ☐ Option B ☐	**$1** in **1 week** and **$38** in **4 weeks**
10	**$17** in **1 week** and **$20** in **1 week**	Option A ☐ Option B ☐	**$1** in **1 week** and **$38** in **1 week**

Fig. 2 Sample choice list instrument for time preference elicitation
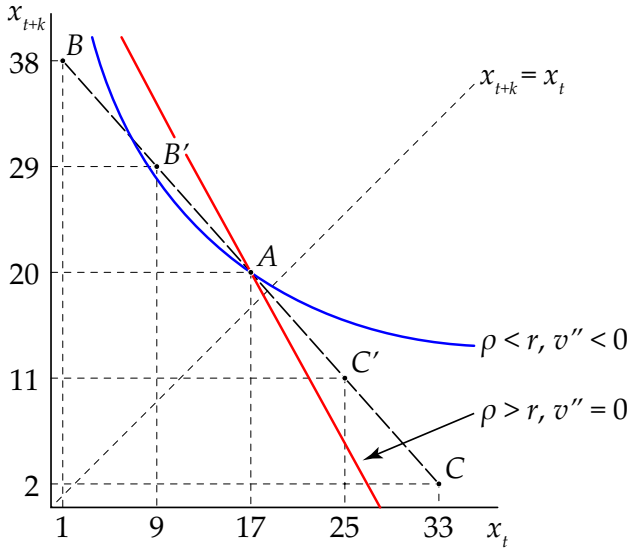
Springer

Fig. 3 Time-dated payoff representation of the HL design for time

for risk, *there is no point prediction* for the number of sooner choices. This simply reflects the fact that the discount rate ρ is an additional preference parameter that must be estimated, whereas in the case of risk the odds are objectively determined by the experimenter.

For the more general case of non-linear instantaneous utility, it follows that it will not be possible to also identify the curvature of utility from a single choice list. Figure 3 depicts indifference curves for two subjects who both prefer Option A in a given row of the list. The first, represented by the linear indifference curve, prefers Option A on account of impatience, i.e. $\rho > r$. The second, represented by the convex indifference curve, is relatively patient, i.e. $\rho < r$,[13] but has concave instantaneous utility (such that $v'(x_t)/v'(x_{t+k}) > 1$ for $x_{t+k} > x_t$) and prefers A because it offers a more temporally-balanced stream of payoffs. Clearly, it is not possible to distinguish these cases simply by observing the switch point in a single choice list.

Figure 3 also suggests two strategies by which it may be possible to distinguish between the cases. First, suppose subjects also face choices between A and C, where C is the payoff vector $(\$33, \$2)$. Relative to A, where B represents a deferral of payment, C represents expediting of payment at the same interest rate of 12.5% over k weeks. Then at the same row of an analogously-constructed CA choice list, the impatient subject with linear utility prefers C. On the other hand, the patient subject with concave utility continues to prefer A, both on account of the return for delay

---

[13] Where the indifference curve meets the diagonal $x_{t+k} = x_t$, the impact of diminishing marginal utility vanishes and the slope in Eq. 4 reflects the pure effect of discounting. At this point, the tangent of the indifference curve is flatter than the chord AB.

$\underline{\textcircled{2}}$ Springer

(discounting) and because A offers a more temporally-balanced stream of payoffs (utility). Second, consider choices between A and the payoff vector B' = $($9, 29)$, which is the midpoint of AB. Relative to A, B' again represents deferral of payment, however the amount being deferred is smaller such that B' is less unbalanced than B. At the same row of an AB' choice list, the impatient subject with linear utility continues to prefer A. However the patient subject with concave utility may choose B', but not B, if willing to save a smaller, but not a larger, amount.

2.4 Full design

The full design involves the five payoff vectors depicted in Fig. 3: C = $($33, 2)$, C' = $($25, 11)$, A = $($17, 20)$, B' = $($9, 29)$, and B = $($1, 38)$. Of these, C is "smallest-soonest", while B is "largest-latest". By construction, for any two vectors, the return for choosing the larger-later one is 12.5% over k weeks. Each subject completed six time preference choice lists, each in the format shown in Fig. 2, using the following pairs of payoff vectors: CA, C'A, AB', AB, CB, and C'B'.[14] In each list, the smaller-sooner option was shown on the left as Option A, while the larger-later one was shown on the right as Option B—thus the alternatives were not identified as C, B', etc. in materials presented to subjects. The front-end delay t was always one week, and the back-end delay k declined from 27 down to 0 weeks in each choice list, generating annual interest rates that increase from 25.46% up to infinity (in the final dominated choice).[15]

Because this design varies interest rates orthogonally to how near or far the payoff vectors are from the diagonal in Fig. 3, it is possible to identify both the discount rate and curvature of instantaneous utility directly from choices over time—without relying on a separate risk preference task or assuming that utility is the same for both risk and time.

Since by design the interest rate is the same at the corresponding row of each choice list, a subject with linear utility will have the same switch point in each. This is the analog to the point prediction that a risk-neutral subject makes four safe lottery choices in the risk task. On the other hand, a subject with concave instantaneous utility prefers to smooth payoffs over time. This subject will have a later switch point in the AB and AB' choice lists, in which the smaller-sooner option is more temporally-balanced, than in the CA and C'A lists, in which it is the larger-later option that is more balanced. Details of this prediction are set out in Appendix A.1. It should be

---

[14] Thus note that, had a subject faced a choice from the full menu of five bundles, this would amount to an instance of the modified CTB of Andreoni et al. (2015). Naturally, binary choices contain more information at the cost of requiring more responses. Suppose, for example, that B is chosen from the choice set {C,A,B} in Fig. 3. Then it can be inferred that B would also be chosen from {A,B}, but the choice from {C,A} cannot be determined. The experiment in this paper tests the prediction that a subject with concave utility switches from C to A before (at a lower interest rate than) switching from A to B.

[15] The annual rates are: 25.46% at $k = 27$ weeks; 29.07% at $k = 24$; 33.86% at $k = 21$; 40.53% at $k = 18$; 50.43% at $k = 15$; 66.59% at $k = 12$; 97.49% at $k = 9$; 177.54% at $k = 6$; 670.27% at $k = 3$; and infinity at $k = 0$. Full parameters of all time preference choice lists are enumerated in Appendix C.2.

emphasized that *this prediction holds regardless of the shape of the discount function*, and does not rely upon exponential discounting.

In addition to the six time preference choice lists, each subject also completed a single risk preference choice list, using the classic AB parameter set described in Sect. 2.1. This makes it possible to compare the curvature of utility elicited under risk and over time, in a within-subjects design.

Two limitations of the design may be acknowledged. First, the annual interest rates offered in the experiment are rather high,[16] as it was not possible to extend k beyond six months since the last payment date fell shortly before the start of summer vacation.[17] Second, as all choice lists have the same front-end delay of one week, it is not possible to identify parameters of a non-exponential discount function. Rather, it is only possible to estimate an exponential discount rate (which may also be interpreted as the exponential component of a quasi-hyperbolic model). This design choice was made because the focus of this paper is to examine implications of concave instantaneous utility that do not depend on the shape of the discount function.

2.5 Procedures

A total of 122 student subjects participated in the experiment at the research laboratory of the School of Economics at The University of Sydney between 6 and 13 May 2014. The mean age of subjects was 20.4 years, and 55.7% were males. Subjects were recruited using ORSEE (Greiner 2015). To ensure that subjects would still be at university when payments were sent, students already in their final semester of study were not eligible to participate. Each session ran for approximately 75 minutes including instruction and payment, and the average payment was $45.2 (approximately USD 42.0 or EUR 28.5), inclusive of a $10 show-up fee. A total of 12 sessions were conducted, and the order of presentation of time preference choice lists was varied between sessions.[18] Each choice list consisted of ten decisions, so each subject made 70 choices in total. The experiment was conducted by pen-and-paper.

At the end of the session, one decision was drawn randomly and independently for each subject, and they were paid according to the choice made in that decision. Following the procedure of Andreoni and Sprenger (2012a), the $10 show-up fee was split into two equal installments of $5 paid by check on the sooner and later payment dates of the decision selected to count for payment. The payments chosen by the subject were added to these checks. Since the subject would in any case have to bank two checks, this ensured that there was no convenience benefit from choosing a more unbalanced payoff vector in order to amass payment on a single date.

---

[16] The interest rates are comparable to those offered by Andreoni and Sprenger (2012a) (which vary from 20.5 to 1300.9%), but higher than those offered by Andersen et al. (2014) (which vary from 5 to 50%).

[17] Alternatively, it would be possible to generate lower interest rates by making the payoff vectors closer in undiscounted terms.

[18] Instructions for one of the orders are in Appendix D. There were four orders in total, enumerated in Appendix C.3. Within each, the first four choice lists were a different permutation of CA, C'A, AB' and AB, and the risk preference task was always last.

If one of the ten risk preference decisions was selected to count for payment, the realization of the chosen lottery was paid in cash at the end of the session, however the show-up fee was still paid in two checks of $5, sent one and sixteen weeks after the experiment. This ensured that any wealth effect attributable to the show-up fee would be the same for both risk and time preference decisions.[19]

The procedures also incorporated several measures introduced by Andreoni and Sprenger (2012a), as adapted by Cheung (2015), to enhance the credibility of payment and minimize the background risk of receiving payment in the future. First, all checks were drawn on the campus branch of the National Australia Bank and mailed by Australia Post guaranteed Express Post. Australia Post guarantees next-day delivery for articles mailed by Express Post, at a cost of $6 per envelope. Since every subject addressed their own envelopes prior to making their choices, they could observe that the experimenter was willing to pay $6 to mail a check to the value of as little as $5 by Express Post. This imparted a high level of credibility to the payments.[20] At the end of the session, each subject wrote their own payment amounts and dates on the inside of each envelope, and was given a copy of the receipt form showing these amounts and dates, as well as the business card of the experimenter to contact in the event of a payment not arriving as expected.

3 Results

Section 3.1 describes aggregate behavior in the risk and time preference tasks, before Sects. 3.2 and 3.3 report structural estimates of utility and discount functions for a representative agent. The key findings are that instantaneous utility is significantly concave, but less so than Bernoulli utility for risk, and the effect of correcting for the curvature of instantaneous utility upon the discount rate is modest. Section 3.4 considers joint estimation, which has a more pronounced effect, while Sect. 3.5 introduces an alternative to discounted utility that is compatible with more substantial utility curvature. Section 3.6 reports a number of robustness checks to the representative agent estimates. Section 3.7 turns to estimation and prediction at an individual level. It shows that the curvatures of Bernoulli and instantaneous utility are not significantly correlated, and individual estimates that infer the curvature of utility from choices over time predict subjects' time preference choices better than linear utility, while joint estimates do not.

3.1 Descriptive analysis

Figure 4 summarizes aggregate choice behavior in the experiment. The upper left panel reports the percentage of subjects who choose the safer Option A for each row of the risk preference task. The dashed line depicts the benchmark prediction under risk neutrality, the solid line depicts observed choices, and error bars represent

---

[19] Sixteen weeks represents the median of the (non-degenerate) later payment dates used in the time preference tasks.

[20] In the post-experiment questionnaire, all but two subjects reported trusting that they would be paid as stated in the instructions.

± one standard error of the mean for a binomial proportion. The lower left panel reports a histogram of the number of safe choices made by each subject. The median subject makes six such choices, and the number of safe choices differs significantly from the risk-neutral benchmark of four, with $p < 0.0001$ in both a sign test and a Wilcoxon signed-ranks test (all tests reported throughout the paper are two-sided).[21]

Turning to behavior in time preference tasks, the upper right panel reports the percentage of smaller-sooner choices as a function of the back-end delay, separately for the pooled AB/AB' and CA/C'A choice lists. Appendix B.1 reports separate figures for each list. The proportion of sooner choices declines smoothly as the back-end delay falls and the interest rate increases, suggesting that subjects understood the underlying trade-off entailed in waiting a longer or shorter time for a given-sized increase in undiscounted payoffs.

Under linear utility subjects are predicted to make the same choices in all lists, while departures from linearity are expressed as differences across lists. In particular, a subject with concave utility prefers to smooth payoffs over time, and thus makes more sooner choices in AB/AB' choice lists (in which the smaller-sooner option is more temporally balanced) than in CA/C'A lists (in which the larger-later option is more balanced). The upper right panel of Fig. 4 confirms a small, but clearly discernible shift in the direction predicted by concave utility. At every back-end delay except zero (where the sooner option is dominated), subjects make more sooner choices in AB/AB' than in CA/C'A. To illustrate the magnitude of these differences, the error bars represent ± one standard error of the mean for a binomial proportion.

The lower right panel of Fig. 4 reports a histogram of the difference in the number of sooner choices made by each subject between the AB/AB' and CA/C'A choice lists. The mode of this distribution is at zero, corresponding to linear utility, but there is greater mass to the right indicating a tendency toward concavity. The median subject makes a total of 13 sooner choices in the combined AB/AB' choice lists, compared to 11.5 in the CA/C'A lists. This difference is highly significant, with $p = 0.0013$ in a sign test, or $p = 0.0006$ in a Wilcoxon signed-ranks test.[22] This evidence of a systematic tendency to prefer the more balanced payoff vector A, consistent with a preference to smooth payoffs over time, does not rely on any assumptions on the functional form of utility.[23]

---

[21] The difference remains highly significant when four subjects who reswitch by choosing Option A after previously choosing Option B are excluded from the analysis. All four subjects reswitch exactly once, and one also makes a dominated choice by choosing Option A in the final row.

[22] The difference remains highly significant when 30 subjects who make one or more non-monotonic choices are excluded from the analysis. There are 26 subjects who reswitch by choosing smaller-sooner after previously choosing larger-later within a given list: 13 reswitch once, seven twice, and six more than twice over six choice lists. There are 17 who make dominated choices by choosing smaller-sooner in the final row of a list: six do so once, eight twice, and three more than twice. Note that dominated choices are less costly in the time preference tasks: in the AB choice list the cost is $2, compared to $18 in the risk preference task. The stochastic choice model in Eq. 9 allows that non-monotonic choices may occasionally occur.

[23] As discussed in Sect. 2.3, concave utility may also motivate differences in behavior between AB' and AB, or between CA and C'A, however no significant differences were found.
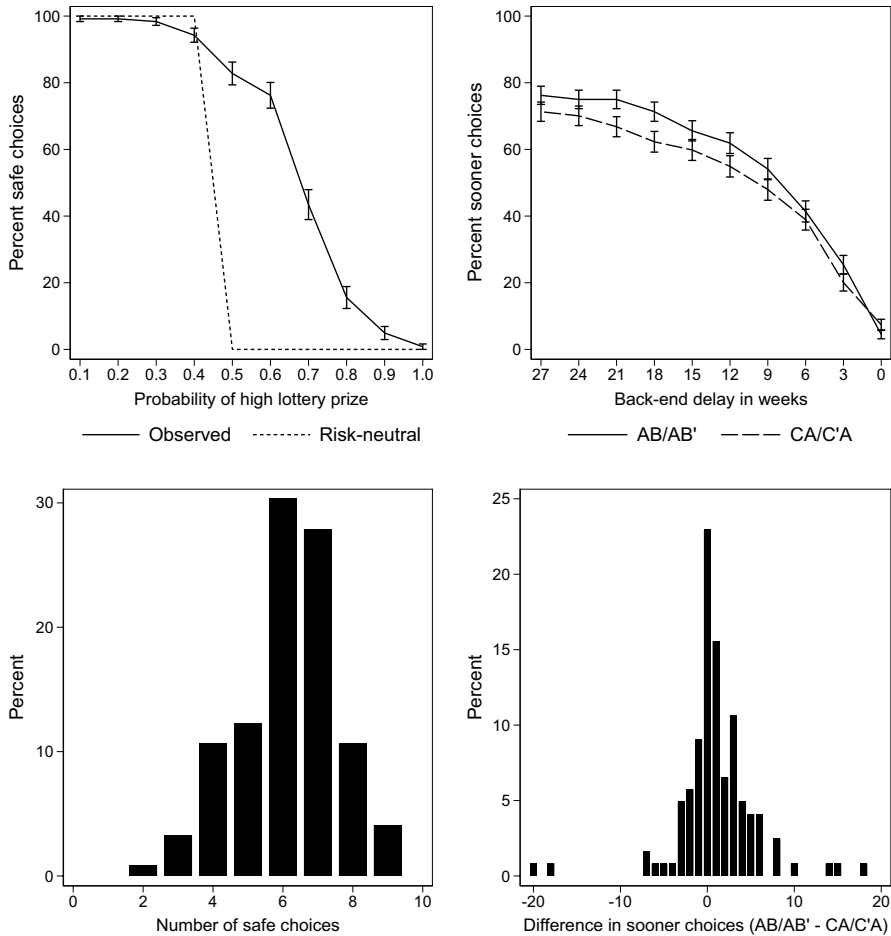
Note: Error bars represent +/- one SEM.

Fig. 4 Choice behavior in risk and time preference tasks

Figure 4 establishes that, in both risk and time preference, there is clear evidence of the choice patterns implied by concave utility. The finding for risk replicates other studies that use the HL design, while the finding for time is a novel result of transposing that design into the domain of time preference. Moreover, while both effects are highly significant, it is clear that the magnitude is smaller in choice over time. This suggests that while instantaneous utility for time is indeed concave, it may be less concave than Bernoulli utility for risk. To formalize this observation, I next estimate structural preference models for a representative agent, building upon well-established procedures documented by Harrison and Rutström (2008) for risk and Andersen et al. (2008, 2014) for time.

3.2 Utility curvature under risk

For risk preference, I assume a constant relative risk aversion (CRRA) functional form for Bernoulli utility:

$$u(x) = \frac{x^{1-\alpha}}{1-\alpha} \tag{5}$$

such that $\alpha = 0$ corresponds to linear utility, while $\alpha > 0$ implies concave utility. I begin with expected utility, but the exposition treats this as the special case of rank-dependent utility with $w(p) = p$. Given some candidate value of α (and probability weighting parameters), the rank-dependent utility of each lottery is evaluated. Then, adopting a "contextual" error specification (Wilcox 2011), the probability that Option B is chosen is modeled as:

$$\Pr(B) = \Lambda\big(\big((RDU_B - RDU_A)/\nu\big)/\mu\big) \tag{6}$$

where $\Lambda(\cdot)$ is the cumulative logistic distribution function, ν is the difference between the maximum and minimum utilities over all prizes in the choice set,[24] and μ is a structural "noise" parameter for the risk preference choices. As μ goes to zero, the lottery with the larger RDU is chosen deterministically, while as μ goes to infinity, the choice probability goes to one-half such that choices are essentially random. The data consists of 1220 observations, being ten binary choices in the risk preference task for each of 122 subjects. The parameters are estimated to maximize the likelihood of the observed choices using Stata 16, with robust standard errors clustered at the level of individual subjects.

Model (1) in Table 1 reports estimates under expected utility. The point estimate of α is 0.547, with a standard error of 0.036. This implies substantial concavity of Bernoulli utility, and sits comfortably within the range of previously-reported estimates using similar experimental designs and estimation procedures.[25]

As discussed in Sect. 2.1, risk aversion in HL-style tasks may be driven by curvature of the utility function, and/or by non-linear probability weighting. Therefore, just as assuming linear utility may cause estimates of the discount rate to be biased in choice over time, assuming linear probability weighting may cause estimates of the utility function to be biased in choice under risk. Indeed, Drichoutis and Lusk (2016) claim that risk aversion in HL tasks may be *solely* a product of probability weighting as opposed to utility curvature: in their estimates of a rank-dependent

---

[24] Division by ν ensures that the normalized utility difference lies in the unit interval. Since subjects only face a single choice set for risk, this amounts to re-scaling the "noise" parameter with no effect on estimates of the core preference parameters. However, this contextual error specification will be generalized to choices over time, where different decisions involve different payoff sets.

[25] For example, Harrison and Rutström (2008, Table 8) report CRRA estimates under expected utility for three data sets, using similar estimation techniques to those adopted here. For the data of Hey and Orme (1994), the CRRA estimate is 0.61 (standard error 0.03), while in their replication of that design it is 0.53 (standard error 0.05). For the data of Holt and Laury (2005), the estimate is 0.76 (standard error 0.04). More recently in a field setting, Andersen et al. (2014) report an estimate of 0.65 (standard error 0.04) in a model that employs the Wilcox (2011) contextual error specification.

🖄 Springer

Table 1 Representative agent estimates of utility curvature and discount rates

	(1) EU	(2) RDU	(3) Linear	(4) Linear-CB	(5) DU	(6) Joint-EU	(7) Joint-RDU	(8) DIU
CRRA utility curvature (α, θ, Θ)	0.547	0.206			0.018	0.456	0.421	0.226
	(0.036)	(0.095)			(0.006)	(0.012)	(0.020)	(0.060)
Probability weighting (η, γ)		0.788					0.789	
		(0.280)					(0.052)	
Annual discount rate (ρ)			0.639	0.622	0.626	0.065	0.107	0.504
			(0.077)	(0.075)	(0.074)	(0.017)	(0.025)	(0.063)
Decision "noise" for risk (μ)	0.082	0.088				0.083	0.073	
	(0.009)	(0.009)				(0.008)	(0.009)	
Decision "noise" for time (σ)			0.028	0.034	0.028	0.006	0.009	0.027
			(0.002)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)
AIC	689.875	685.734	8807.895	1412.456	8790.165	2082.889	2061.119	8785.710
BIC	700.089	701.054	8821.692	1422.670	8810.860	2106.088	2090.117	8806.405
LL	-342.938	-339.867	-4401.948	-704.228	-4392.083	-1037.445	-1025.559	-4389.855
Probability weighting function		Gul					Prelec-I	
Risk preference data	Yes	Yes				Yes	Yes	
Time preference data			Yes	CB only	Yes	CB only	CB only	Yes
N	1220	1220	7320	1220	7320	2440	2440	7320

Cluster robust standard errors in parentheses

⚫ Springer

model they find significant non-linear probability weighting, while the CRRA coefficient does not differ significantly from zero. While this finding is arguably at odds with other existing literature,[26] it highlights the importance of allowing for probability weighting when comparing the curvature of utility elicited under risk and over time.

To examine the robustness of concave Bernoulli utility to the possibility of non-linear probability weighting, I estimate rank-dependent models for each parametric form of the probability weighting function in Section 3.6 of the survey by Fehr-Duda and Epper (2012). The Bayesian information criterion (BIC) in fact selects the expected utility specification of model (1) in Table 1. The Akaike information criterion (AIC), which penalizes model complexity less severely than the BIC, selects a model with a single-parameter weighting function from the theory of disappointment aversion of Gul (1991): $w(p) = p/[1 + (1 - p)\eta]$. The resulting estimates are set out as model (2) in Table 1. This weighting function simplifies to linearity at $\eta = 0$, a restriction that is clearly rejected with $p = 0.005$. The implied weighting function is convex, and is depicted by the dashed line in Fig. 5. For the purpose of this paper, the effect on the estimate of utility curvature is of greatest interest. This estimate is now smaller (and less precisely estimated) than under expected utility. However, it will transpire that it is still considerably greater than the curvature of instantaneous utility estimated from choices over time.

3.3 Utility curvature and discounting over time

I next set out how the data from the six time preference choice lists can be used to estimate both the curvature of instantaneous utility and the discount rate, adopting similar procedures to Sect. 3.2 and Andersen et al. (2008, 2014). I assume a CRRA form for the instantaneous utility function:

$$v(x) = \frac{x^{1-\theta}}{1 - \theta} \tag{7}$$

and an exponential form for the discount function:

$$D(t) = \frac{1}{(1 + \rho)^{\frac{t}{52}}} \tag{8}$$

where θ captures the curvature of instantaneous utility and ρ is the annual discount rate. Given candidate values of θ and ρ, the discounted utility of each alternative is

---

[26] For example, in their re-analysis of the data of Holt and Laury (2005), Harrison and Rutström (2008, Table 8) estimate the same rank-dependent specification—with CRRA utility and a Tversky and Kahneman (1992) weighting function—as Drichoutis and Lusk (2016). In this specification, Harrison and Rutström do not find significant non-linear probability weighting, while their point estimate of the CRRA coefficient is actually (insignificantly) larger than in the corresponding expected utility specification. In a representative sample of adult Danes, Andersen et al. (2014) find evidence of non-linear probability weighting but conclude that the bulk of aversion to risk derives from concavity of the utility function.

evaluated and the probability that the alternative presented as Option B is chosen is modeled as:

$$\Pr(B) = \Lambda\big(\big(\big(DU_B - DU_A\big)/\lambda\big)/\sigma\big) \tag{9}$$

where λ is a contextual normalization term,[27] and σ is a noise parameter for the time preference choices.

It is worth emphasizing that this framework is essentially the same as that of Andersen et al. (2008, 2014), except that information on utility curvature is obtained directly from choices over time instead of a separate risk preference task, so it is not necessary to equate Bernoulli utility for risk with instantaneous utility for time. The data consists of 7320 observations, being ten binary choices in each of six time preference choice lists for each of 122 subjects. The parameters θ, ρ, and σ are estimated by maximum likelihood in Stata 16, with robust standard errors clustered at the level of individual subjects.

Before turning to the full results, model (3) in Table 1 reports a linear utility specification, with θ constrained to zero, giving an estimated annual discount rate of 63.9%. While this is higher than prevailing market interest rates, it is not extreme by the standards of the literature.[28] Model (4) reports a linear utility specification using only the data of the CB choice list in which the smaller-sooner option pays $\big(\$33, \$2\big)$ while larger-later pays $\big(\$1, \$38\big)$. This is included for comparability with more conventional designs that offer all-sooner or all-later payments, as well as the subsequent replication of joint estimation invoking utility from the risk preference task in Sect. 3.4. The resulting estimate of the discount rate is very close to that of model (3).

Model (5) reports the main discounted utility estimates, allowing non-linear instantaneous utility revealed through data on choices over time. In this model, the point estimate of θ is 0.018 with a standard error of 0.006, indicating significantly concave utility. This is consistent with the model-free analysis in Sect. 3.1. However, the estimate of θ is an order of magnitude smaller than estimates of α from risk preference data in models (1) and (2). Because the estimated curvature of instantaneous utility is modest, the effect of correcting for this concavity upon the discount rate is mild: the estimate of ρ falls from 63.9% in model (3) to 62.6% in model (5). Both the

---

[27] In Eq. 6 for risk, ν represented the difference between the best and worst lotteries in the choice context, being the utilities of the best or worst prizes *implicitly received in both states of nature*. In Eq. 9 for time, λ now represents the difference between the best and worst payoff streams, being the discounted utilities of the best or worst payoffs *received on both dates*. That is, $\lambda = (D(t) + D(t+k)) \cdot \big(v\big(x_{max}\big) - v\big(x_{min}\big)\big)$, where x_{max} and x_{min} are the best and worst payoffs in a given choice context. See Andersen et al. (2018, equation 12) for a related extension of contextual utility to choice over time.

[28] Among recent studies, Takeuchi (2011) imputes an annual discount rate of 726% in a design that theoretically controls for non-linear utility, while Benhabib et al. (2010) report annual discount rates on the order of 472%. However, neither of these studies employ a front-end delay. Laury et al. (2012) is an example of a modern study using student subjects and a front-end delay design. Their dollar discount rate task (Task D) is a standard time preference choice list in the manner of Coller and Williams (1999). From this task, they estimate an annual discount rate of 55.5% assuming linear utility, which is comparable to the estimate reported here.
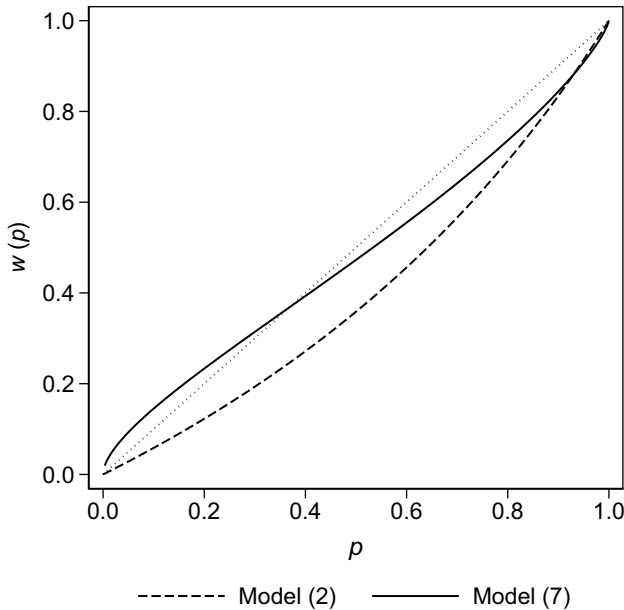
Fig. 5 Estimated probability weighting functions

AIC and BIC select the non-linear utility specification of model (5) over the more parsimonious linear utility specification in model (3).

3.4 Joint estimation

The modest effect of correcting for the curvature of instantaneous utility may be contrasted with that of a joint estimation procedure that combines risk and time preference data and imposes a single utility function upon both. To illustrate, I pool the data of the risk task with that of the CB choice list, which is comparable to conventional time preference data in that the payoffs are essentially at the all-sooner or all-later corners. The probability of making a risky lottery choice is modeled by Eq. 6 while that of making a larger-later choice is modeled by Eq. 9. The noise terms μ and σ are allowed to differ across risk and time preference tasks, but the utility function is constrained to be the same for both, such that $\alpha = \theta$. The data consists of 2440 observations, being ten risk and ten time preference choices for each of 122 subjects, and the parameters are estimated to maximize the joint likelihood of both sets of choices.

Model (6) in Table 1 reports joint estimates assuming expected utility for risk. The estimate of utility curvature is 0.456 with a standard error of 0.012, reflecting the influence of risk aversion in the lottery choices, while the estimated annual discount rate falls to 6.5%. This compares to an estimate of 62.2% in model (4), which uses the same CB time preference data but assumes linear utility.

🖄 Springer

To allow for non-linear probability weighting in the risk preference data, I again re-estimate the joint model assuming rank-dependent utility for each parametric weighting function in Fehr-Duda and Epper (2012). The effect upon utility and discounting is very similar across all specifications: the estimate of utility curvature falls to between 0.419 and 0.427 while the estimated discount rate increases slightly to between 10.0 and 10.9%. Both the AIC and BIC select a specification with a single-parameter weighting function from Prelec (1998): $w(p) = \exp(-(-\ln(p))^\gamma)$, and the resulting estimates are reported as model (7) in Table 1. This weighting function simplifies to linearity at $\gamma = 1$, a restriction that is rejected with $p < 0.0001$ in a Wald test. The implied weighting function is inverse-S shaped, and is depicted by the solid line in Fig. 5.

For the purpose of this paper, the key conclusions are twofold. First, even allowing for probability weighting, joint estimates of utility curvature which impose the restriction that $\alpha = \theta$ are considerably larger than the discounted utility estimate in model (5) which imposes no such restriction. Second, relative to the linear utility benchmark of model (4), the effect upon the discount rate of correcting for this amount of curvature is dramatic. The discount rates in models (6) and (7) are substantially smaller than the lowest interest rate offered in the experiment, suggesting that joint estimation may have yielded an over-correction.

3.5 Discounted incremental utility

In Sects. 3.3 and 3.4, and models (3) through (7) in Table 1, I explored the effect of alternative assumptions about the nature of instantaneous utility—whether it is taken to be linear as in models (3) and (4), revealed through responses to varying opportunities for intertemporal substitution as in model (5), or equated with Bernoulli utility for risk as in models (6) and (7). It was assumed throughout that the underlying framework to evaluate streams of payoffs over time is given by the discounted utility model in Eq. 3.

Blavatskyy (2016) has argued that discounted utility may give rise to violations of intertemporal monotonicity when utility is not linear. In particular, it is possible for discounted utility to increase when a payoff is split into two parts, one of which is slightly delayed. That is, if $v(x)$ is sufficiently concave while $D(t + k)$ is close to $D(t)$ it is possible to have:

$$D(t) \cdot v(x) < D(t) \cdot v(x/2) + D(t + k) \cdot v(x/2)$$

This implies that there may be a benefit to delay without any compensating increase in the magnitude of the payoff. The issue occurs when the impact of diminishing marginal utility as a payoff is divided in two outweighs the impact of discounting as a portion of it is delayed. This may occur in any model that has the discounted utility structure of Eq. 3, and is not specific to any functional form of the discount function $D(t)$ such as exponential discounting. Assuming discounted utility, it can be avoided only if utility is linear. It thus represents a theoretical argument for why discounted utility may be incompatible with substantial non-linearity of instantaneous utility, as found empirically in model (5) of Table 1.

⌂ Springer

The argument of Blavatskyy (2016) is analogous to how simple probability weighting may violate first-order stochastic dominance in the original prospect theory of Kahneman and Tversky (1979). In that setting, the solution proposed by Quiggin (1982) is to construct decision weights from a transformation of the cumulative probabilities instead of directly transforming the probabilities themselves. In the context of time, the solution proposed by Blavatskyy (2016) is to apply a utility transformation to the cumulative payoffs instead of directly to the payoffs themselves. This gives rise to the model of discounted incremental utility (Blavatskyy 2016, equation 3) which replaces discounted utility by:

$$DIU\big(x_t, t; x_{t+k}, t+k\big) = D(t) \cdot V\big(x_t\big) + D(t+k) \cdot \big[V\big(x_t + x_{t+k}\big) - V\big(x_t\big)\big] \quad (10)$$

This states that future payoffs are evaluated by the discounted value of their incremental contribution to the utility of the cumulated payoffs, $V(\cdot)$.

Estimation of the discounted incremental model requires data on choices over non-degenerate streams, such as that reported in this paper,[29] as opposed to conventional choices over all-sooner versus all-later payoffs which never cumulate. If the cumulative utility function is linear then Eq. 10, like Eq. 3, simplifies to discounted linear utility, in which case a subject is again predicted to have the same switch point in all six choice lists. However if the cumulative utility function is concave, I show in Appendix A.3 that discounted incremental utility again predicts a later switch point in AB/AB' than in CA/C'A choice lists.

To estimate the model I again assume a CRRA form, this time for the utility of cumulated payoffs, X:

$$V(X) = \frac{X^{1-\Theta}}{1-\Theta} \quad (11)$$

and the exponential discount function of Eq. 8, and define choice probabilities analogously to Eq. 9 except replacing DU by DIU. The model is estimated using the same set of 7320 observations from six time preference choice lists as used in models (3) and (5) of Table 1.

Estimates of the discounted incremental specification are reported as model (8) in Table 1. The curvature of cumulative utility is significantly concave, with a CRRA coefficient of 0.226 and standard error of 0.060. This is substantially larger than the estimated curvature of instantaneous utility in the discounted utility specification of model (5). Allowing for this amount of curvature reduces the estimate of the annual discount rate from 63.9% under linear utility in model (3) to 50.4% in model (8). The discounted incremental specification has the same number of parameters as the discounted utility model, and a superior log-likelihood. As a result, both the AIC and BIC select this model over both the discounted utility and linear models.

---

[29] See also Blavatskyy and Maafi (2018) who estimate points on the utility function non-parametrically, and are concerned with issues of model fit as opposed to the curvature of utility.

3.6 Robustness checks

In this section, I report several robustness checks of the key representative agent estimates to alternative structural assumptions. Table 2 reports expected utility, discounted utility, joint, and discounted incremental estimates assuming a constant absolute risk aversion form for utility: $u(x) = (1 - \exp(-ax))/a$. Table 3 reports corresponding estimates assuming expo-power utility (Saha 1993; Holt and Laury 2002): $u(x) = (1 - \exp(-ax^{1-r}))/a$. These analyses replicate all key implications of Table 1: instantaneous utility in model (5) is significantly concave[30] though substantially less so than Bernoulli utility in model (1); the discount rate in model (5) is only marginally smaller than that assuming linear utility (c.f. Table 1, model (3)), whereas the one obtained from joint estimation in model (6) is substantially lower; and the effects of discounted incremental utility in model (8) are similar to those in Table 1.

Table 4 reports linear, discounted utility, joint, and discounted incremental estimates for CRRA utility using an alternative contextual normalization that replaces the expression in footnote 27 with $\lambda = v(x_{max}) - v(x_{min})$ (see Blavatskyy and Maafi 2018, p. 278). That is, instead of normalizing by the difference in discounted utilities between the best and worst payoff *streams*, these estimates normalize by the difference in instantaneous utilities between the best and worst *payoffs*, which does not depend upon the discount function.[31] Again, all key implications of Table 1 are maintained.

3.7 Individual estimation and prediction

In this section, I extend the analysis to estimation at an individual level, and examine the performance of individual estimates in predicting subjects' choices in the risk and time preference decisions respectively.

In the full sample of 122 subjects there are fifteen who choose larger-later in all 60 time preference decisions, and four who choose smaller-sooner in all 54 decisions where it is not dominated. I thus focus on the remaining 103 subjects in individual estimation. Since there is only a single risk preference choice list, it is not possible to estimate probability weighting parameters at an individual level. I thus report individual estimates for five models: the expected utility specification in model (1) of Table 1, the linear utility specification in model (3), the discounted utility specification in model (5), the joint estimation specification in model (6), and the discounted incremental specification in model (8). Individual estimation was performed in Matlab R2018b, after first replicating the corresponding representative agent estimates from Table 1.

---

[30] For expo-power utility, the two utility parameters are jointly significant with $p = 0.004$ in model (5).

[31] There is no effect upon the expected utility estimates, as this is simply an alternative generalization of Wilcox (2011) contextual utility to choices over time and moreover, as noted in footnote 24, there was in any case only a single choice context for risk.

 © Springer

Table 2 Representative agent estimates under CARA utility

	(1) EU	(5) DU	(6) Joint-EU	(8) DIU
CARA utility curvature	0.041	0.001	0.040	0.018
	(0.003)	(0.000)	(0.002)	(0.005)
Annual discount rate (ρ)		0.624	0.068	0.424
		(0.073)	(0.016)	(0.065)
Decision "noise" for risk (μ)	0.081		0.081	
	(0.009)		(0.009)	
Decision "noise" for time (σ)		0.028	0.006	0.027
		(0.002)	(0.001)	(0.002)
AIC	689.875	8790.505	2063.699	8780.517
BIC	700.089	8811.200	2086.898	8801.212
LL	− 342.938	− 4392.252	− 1027.849	− 4387.259
Risk preference data	Yes		Yes	
Time preference data		Yes	CB only	Yes
N	1220	7320	2440	7320

Clustered standard errors in parentheses. Column numbers indicate corresponding models in Table 1

Table 3 Representative agent estimates under expo-power utility

	(1) EU	(5) DU	(6) Joint-EU	(8) DIU
Utility curvature (expo)	0.041	− 0.001	0.039	0.017
	(0.012)	(0.005)	(0.008)	(0.004)
Utility curvature (power)	0.002	0.036	− 0.015	− 0.123
	(0.185)	(0.059)	(0.132)	(0.120)
Annual discount rate (ρ)		0.627	0.068	0.390
		(0.074)	(0.015)	(0.066)
Decision "noise" for risk (μ)	0.081		0.081	
	(0.009)		(0.009)	
Decision "noise" for time (σ)		0.028	0.006	0.027
		(0.002)	(0.001)	(0.002)
H_0: Linear utility (p-value)	0.000	0.004	0.000	0.000
AIC	691.875	8792.053	2065.676	8781.630
BIC	707.195	8819.646	2094.675	8809.223
LL	− 342.938	− 4392.026	− 1027.838	− 4386.815
Risk preference data	Yes		Yes	
Time preference data		Yes	CB only	Yes
N	1220	7320	2440	7320

Clustered standard errors in parentheses. Column numbers indicate corresponding models in Table 1

Springer

Table 4 Representative agent estimates under alternative contextual normalization

	(3) Linear	(5) DU	(6) Joint-EU	(8) DIU
CRRA utility curvature		0.018	0.455	0.180
		(0.006)	(0.012)	(0.066)
Annual discount rate (ρ)	0.662	0.648	0.066	0.543
	(0.083)	(0.080)	(0.017)	(0.076)
Decision "noise" for risk (μ)			0.083	
			(0.008)	
Decision "noise" for time (σ)	0.053	0.052	0.012	0.046
	(0.004)	(0.004)	(0.003)	(0.004)
AIC	8771.565	8754.908	2082.580	8759.751
BIC	8785.362	8775.603	2105.779	8780.446
LL	− 4383.783	− 4374.454	− 1037.290	− 4376.875
Risk preference data			Yes	
Time preference data	Yes	Yes	CB only	Yes
N	7320	7320	2440	7320

Clustered standard errors in parentheses. Column numbers indicate corresponding models in Table 1

Table 5 Summary statistics of individual utility curvature and discount rate estimates

	Mean	SD	P10	P25	P50	P75	P90
Utility curvature							
Expected utility	0.557	0.461	− 0.064	0.336	0.504	0.789	1.127
Discounted utility	0.001	0.259	− 0.049	− 0.008	0.010	0.053	0.123
Joint estimation	0.418	0.320	− 0.065	0.230	0.476	0.504	0.787
Discounted incremental	0.050	1.394	− 1.394	− 0.295	0.064	0.577	1.056
Annual discount rate							
Linear utility	2.350	2.788	0.250	0.489	1.144	2.947	6.904
Discounted utility	2.219	2.699	0.240	0.464	1.035	2.535	6.707
Joint estimation	1.387	2.910	− 0.105	0.000	0.101	1.297	6.417
Discounted incremental utility	2.058	2.936	0.101	0.306	0.783	2.616	6.703

Table 5 reports summary statistics for four individual measures of CRRA utility curvature (not estimated under linear utility) and four individual estimates of the annual discount rate (not estimated under expected utility). Histograms for each set of estimates are reported in Appendix B.2. For utility curvature, the mean and median estimates under expected utility, discounted utility and joint estimation are very similar to the corresponding representative agent estimates. However for discounted incremental utility, the mean and median curvature are closer to linearity than the aggregate estimate, and the variance is considerable. For discounting, the mean and median estimates are consistently larger than the

<img> Springer

corresponding estimates in Table 1, in part reflecting the fact that the individual estimation sample excludes the fifteen most patient subjects.[32]

Table 6 reports the Spearman rank correlation matrix for the four individual measures of utility curvature. This strongly supports the conjecture that risk and time preferences reflect two distinct notions of utility. On one hand, the expected utility and joint estimates, which infer curvature from choices under risk, are highly significantly correlated. On the other hand, the discounted utility and discounted incremental estimates, which infer curvature from choices over time, are also highly significantly correlated. However the remaining four coefficients, which compare one measure elicited under risk to another measure elicited over time, are consistently small (Spearman *rho* less than 0.11) and far from statistically significant.

Figure 6 depicts a scatter plot of the individual estimates of α under expected utility and θ under discounted utility, these being the standard normative benchmarks for risk and time preference respectively.[33] According to the model of discounted expected utility, these measures are interchangeable and so all points should lie on a 45-degree diagonal. Instead, not only do the two sets of estimates differ considerably in magnitude, they are also essentially uncorrelated.[34]

To further examine the implications of alternative assumptions regarding the nature of utility, I use each set of estimates to generate predicted choices for each subject in each of the ten risk and 60 time preference decisions, and compare these predictions to their actual choices.[35]

For risk preference, simply assuming linear utility (i.e., that every subject switches from safe to risky after four rows) suffices to correctly predict 76.7% of the data. Relative to this benchmark, individual curvature estimates from the discounted utility model correctly predict 76.8% of risk preference choices, while those of the discounted incremental model correctly predict 74.6%. Treating individual subjects as independent observations, the proportion of their choices correctly predicted by either model does not differ significantly from the proportion correctly predicted by linear utility, in either a sign test or a signed-ranks test. By contrast, individual expected utility estimates correctly predict 98.5% of choices, and joint estimates 95.0%, both improving significantly upon linear utility with $p < 0.0001$ in both a sign test and a signed-ranks test.

For time preference, individual discount rates assuming linear utility correctly predict 86.6% of the data. This increases to 89.4% allowing non-linear utility in a discounted utility model, or 87.3% in a discounted incremental model. Again treating individual subjects as independent observations, both sets of estimates improve

---

[32] When the representative agent model (5) for discounted utility is re-estimated using only the 103 subjects used in individual estimation, the estimated annual discount rate increases to 80.6%, which is still less than the median individual estimate.

[33] For clarity, the scatter plot omits two subjects with large negative estimates of θ. However these subjects are included in the calculation of the linear fit depicted by the dashed line.

[34] Scatter plots for individual estimates of the discounted incremental utility parameter Θ may be found in Appendix B.3.

[35] The expected utility model for risk does not estimate a discount rate, and so cannot be used to predict choices over time.

Ⓓ Springer

Table 6 Spearman rank correlation matrix of individual utility curvature estimates

	EU	DU	Joint	DIU
EU	1.000			
DU	0.101	1.000		
	(0.309)			
Joint	0.931	0.106	1.000	
	(0.000)	(0.286)		
DIU	0.099	0.749	0.069	1.000
	(0.320)	(0.000)	(0.489)	

p-values in parentheses

significantly upon linear utility.[36] By contrast, individual joint estimates correctly predict only 68.4% of the data. Recall that joint estimation uses only the CB choice list for time. Individual linear utility estimates using only CB data (corresponding to model (4) in Table 1) correctly predict 84.1% of the data. The individual joint estimates predict significantly worse than this linear-CB benchmark, with $p < 0.0001$ in both a sign test and a signed-ranks test.

Figure 7 examines prediction performance of individual estimates in greater detail. The top panel relates to the risk preference task, while the lower two panels pool the AB/AB' and CA/C'A choice lists for time, respectively. In each panel, the solid line depicts observed safe or sooner choices at each row.[37] The shorter-dashed line depicts predictions of the joint estimation model (which infers utility curvature from choice under risk), while the longer-dashed line depicts predictions of the discounted utility model (which infers curvature from choice over time). The comparison between these predictions thus illustrates the effect of assuming equivalence between Bernoulli and instantaneous utility versus allowing for them to be separated. The figure illustrates how *both sets of estimates make poor out-of-sample predictions, because they reflect quantitatively different degrees of concavity*. Since instantaneous utility for time is near-linear, it wrongly predicts near risk-neutral behavior in choice under risk. But conversely, since Bernoulli utility for risk is more concave, it greatly exaggerates the preference for the temporally-balanced payoff vector A, both when it is smaller-sooner as in AB/AB', and when it is larger-later as in CA/C'A.

---

[36] For discounted utility, $p < 0.0001$ in both the sign test and signed-ranks test. For discounted incremental utility, $p = 0.0031$ in a sign test and $p = 0.0015$ in a signed-ranks test. Moreover, the discounted utility model improves significantly upon discounted incremental utility, with $p = 0.0054$ in a sign test and $p = 0.0020$ in a signed-ranks test.

[37] This differs from Fig. 4 as it only includes the individual estimation sample of 103 subjects.
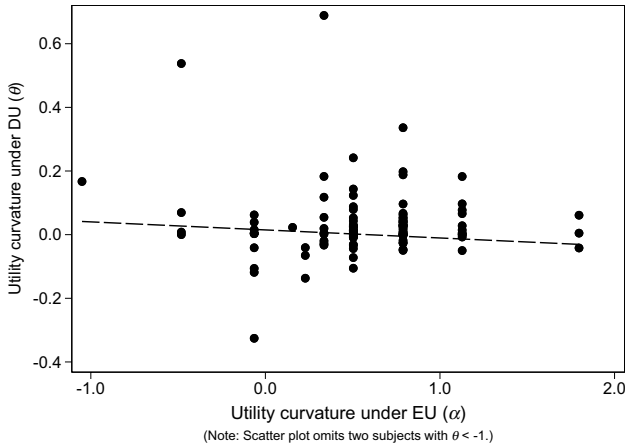
(Note: Scatter plot omits two subjects with θ < -1.)

Fig. 6 Scatter plot of individual estimates of α and θ

4 Conclusion

In this paper, I introduce a novel method to elicit the curvature of instantaneous utility, together with the discount rate, directly from binary choices over bundles of time-dated payoffs. Owing to a lack of suitable measurement techniques, until recently little was known about the shape of utility outside the domain of risk. This made it difficult to evaluate the assumption—frequently invoked in both theoretical and empirical literatures—that a single utility function characterizes preferences both under risk and over time.

My approach builds upon design and estimation principles well-established in the literature. Abdellaoui et al. (2013) and Andreoni and Sprenger (2012a) use outcome sequences to identify instantaneous utility, although the former are not concerned with implications for discounting while the latter compare the results of different elicitation procedures for risk and time. The Holt and Laury (2002) design is a standard instrument often used as the risk preference measure in studies which, following Andersen et al. (2008), assume equivalence of instantaneous and Bernoulli utility in jointly estimating risk and time preferences. By translating the HL design from state payoffs into time-dated payoffs, I retain the estimation apparatus developed by Andersen et al. (2008, 2014), with the important distinction that the curvature of utility is inferred directly from choices over time. This makes it possible to compare the estimated curvature of utility under risk and over time within a unified design and estimation framework. Thus, while many of my results replicate findings in Andreoni and Sprenger (2012a) and Andreoni et al. (2015), I establish that these results are not artefacts of differences between the elicitation procedures or estimation techniques they use for risk and time. Stated differently, I show that—as regards the curvature of instantaneous utility—the results of a binary choice methodology are in alignment with those of a continuous or multinomial choice paradigm.

The results demonstrate that instantaneous utility is significantly concave. This affirms the underlying theoretical concern of Frederick et al. (2002) over
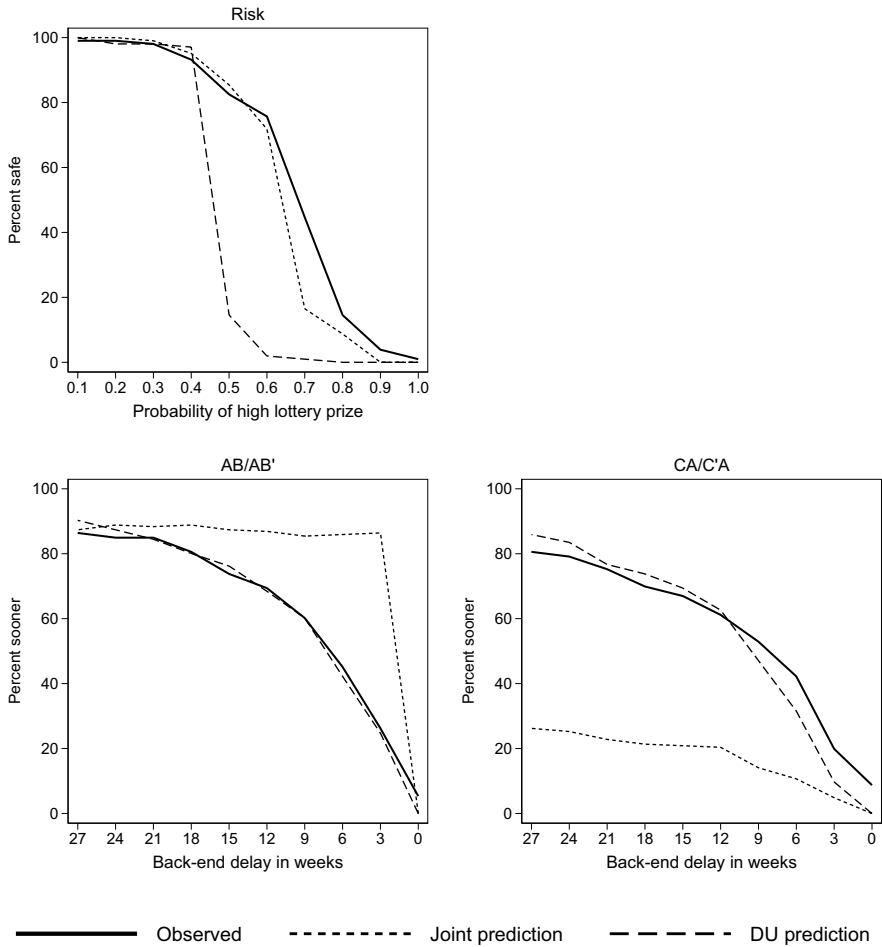
🌐 Springer

Fig. 7 Prediction performance of individual estimates

the confounding influence of utility curvature upon estimates of the discount rate, which has motivated the development of the joint estimation approach as well as alternative strategies such as those of Takeuchi (2011) and Laury et al. (2012). However, whereas each of those approaches assume a single utility function for both risk and time, I find the curvature of instantaneous utility elicited from choices over time to be substantially less than that of Bernoulli utility elicited from choices under risk, and moreover the two measures are uncorrelated at an individual level. It follows that just as assuming linear utility may cause estimates of discount rates to be biased, so too may assuming the equivalence of utility for risk and time. Indeed, I find the effect of correcting for time-elicited utility curvature, relative to assuming linear utility, to be no more than a few percentage points.

<img> Springer

The experiment in this paper involved choices over money, and in the structural estimation (though not the model-free analysis of Sect. 3.1) it was assumed that payoffs are consumed upon receipt. This presumes, generally, that subjects do not integrate experimental payments with money or consumption plans outside the experiment and, specifically, that they do not engage in arbitrage. Indeed, the finding of near-linear utility in the CTB experiment of Andreoni and Sprenger (2012a) has subsequently been interpreted by Andreoni and Sprenger (2015) and Sprenger (2015) as evidence suggestive of arbitrage. When subjects engage in arbitrage, their choices simply reveal their market interest rate as opposed to a true discount rate, yet the estimates in excess of 60% in Table 1 are unreasonably high by that standard. Moreover, the interest rates offered in the experiment were sufficiently high as to realistically permit only one direction of arbitrage, namely to "borrow low" outside the experiment and "save high" within it (Meier and Sprenger 2010). In that case, arbitrage would predict highly patient choices in the experiment, again inconsistent with the behavior of the majority of subjects. Andreoni et al. (2018) report a CTB experiment designed specifically to test implications of arbitrage, but find little support for it.

The more general issue, that subjects may integrate experimental rewards with money or consumption outside the laboratory,[38] has also been raised in the domain of risk. In that context, Rabin (2000) argues that an expected utility maximizer who integrates small-stakes lotteries with background wealth must exhibit near-linear utility toward such lotteries. This prediction is robustly rejected by a large body of experimental research, including results in this paper, strongly suggesting that—in choice under risk—the subjects in this experiment framed the payoffs narrowly and in isolation from external wealth. That being the case, it is not obvious why—in making choices over time—those same subjects would frame broadly and integrate.

If the results of this experiment reflect subjects' genuine preferences, and not some artefact of monetary payoffs, how are these preferences to be understood? A subtle but important distinction between the domains of risk and time, perhaps obscured by the formal analogy between Eqs. 1 and 3, concerns the realization of payoffs. In choice under risk, while the payoffs in a lottery are evaluated as a bundle *ex ante*, only one is ultimately realized *ex post*. By contrast in choice over time, all of the payoffs that make up a stream are to be realized, and it is only a question of when. Viewed in this light, it seems natural that subjects' choices would reflect a stronger motive to smooth payoffs over states of nature than over dates in time.

It has been argued by Blavatskyy (2016) that the discounted utility model is incompatible with substantial non-linearity of instantaneous utility as it results in violations of intertemporal monotonicity. In his alternative model of discounted incremental utility, I indeed find more substantial concavity when utility is defined over cumulated payoffs, at least in representative agent estimates. This points to the exciting opportunities for future empirical research to focus not only on the forms of

---

[38] This concern is not specific to choices over money. In a time preference experiment involving real consumption or effort, subjects may likewise adjust their outside consumption or leisure plans in conjunction with choices made within the experiment, confounding structural interpretation of the data.

the utility and discount functions, but also on alternatives to the discounted utility model itself.

References

Abdellaoui, M., Bleichrodt, H., l'Haridon, O., & Paraschiv, C. (2013). Is there one unifying concept of utility? An experimental comparison of utility under risk and utility over time. *Management Science*, *59*(9), 2153–2169.

Andersen, S., Harrison, G. W., Lau, M. I., & Rutström, E. E. (2008). Eliciting risk and time preferences. *Econometrica*, *76*(3), 583–618.

Andersen, S., Harrison, G. W., Lau, M. I., & Rutström, E. E. (2014). Discounting behavior: A reconsideration. *European Economic Review*, *71*, 15–33.

Andersen, S., Harrison, G. W., Lau, M. I., & Rutström, E. E. (2018). Multiattribute utility theory, intertemporal utility, and correlation aversion. *International Economic Review*, *59*(2), 537–555.

Andreoni, J., Gravert, C., Kuhn, M. A., Saccardo, S., & Yang, Y. (2018). *Arbitrage or narrow bracketing? On using money to measure intertemporal preferences*. Working Paper 25232, National Bureau of Economic Research.

Andreoni, J., Kuhn, M. A., & Sprenger, C. (2015). Measuring time preferences: A comparison of experimental methods. *Journal of Economic Behavior and Organization*, *116*, 451–464.

Andreoni, J., & Sprenger, C. (2012a). Estimating time preferences from convex budgets. *American Economic Review*, *102*(7), 3333–3356.

Andreoni, J., & Sprenger, C. (2012b). Risk preferences are not time preferences. *American Economic Review*, *102*(7), 3357–3376.

Andreoni, J., & Sprenger, C. (2015). Risk preferences are not time preferences: Reply. *American Economic Review*, *105*(7), 2287–93.

Attema, A. E., Bleichrodt, H., Gao, Y., Huang, Z., & Wakker, P. P. (2016). Measuring discounting without measuring utility. *American Economic Review*, *106*(6), 1476–94.

Benhabib, J., Bisin, A., & Schotter, A. (2010). Present-bias, quasi-hyperbolic discounting, and fixed costs. *Games and Economic Behavior*, *69*(2), 205–223.

Blavatskyy, P. R. (2016). A monotone model of intertemporal choice. *Economic Theory*, *62*(4), 785–812.

Blavatskyy, P. R., & Maafi, H. (2018). Estimating representations of time preferences and models of probabilistic intertemporal choice on experimental data. *Journal of Risk and Uncertainty*, *56*(3), 259–287.

Cheung, S. L. (2015). Comment on "Risk preferences are not time preferences": On the elicitation of time preference under conditions of risk. *American Economic Review*, *105*(7), 2242–2260.

Cheung, S. L. (2016). Recent developments in the experimental elicitation of time preference. *Journal of Behavioral and Experimental Finance*, *11*, 1–8.

Coller, M., & Williams, M. B. (1999). Eliciting individual discount rates. *Experimental Economics*, *2*(2), 107–127.

Drichoutis, A. C., & Lusk, J. L. (2016). What can multiple price lists really tell us about risk preferences? *Journal of Risk and Uncertainty*, *53*(2), 89–106.

Epper, T., & Fehr-Duda, H. (2012). *The missing link: Unifying risk taking and time discounting*. Working paper 096, Department of Economics, University of Zurich.

Epper, T., & Fehr-Duda, H. (2015). Comment on "Risk preferences are not time preferences": Balancing on a budget line. *American Economic Review*, *105*(7), 2261–2271.

Epper, T., Fehr-Duda, H., & Bruhin, A. (2011). Viewing the future through a warped lens: Why uncertainty generates hyperbolic discounting. *Journal of Risk and Uncertainty*, *43*(3), 169–203.

Epstein, L. G., & Zin, S. E. (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica*, *57*(4), 937–969.

Fehr-Duda, H., & Epper, T. (2012). Probability and risk: Foundations and economic implications of probability-dependent risk preferences. *Annual Review of Economics*, *4*(1), 567–593.

Frederick, S., Loewenstein, G., & O'Donoghue, T. (2002). Time discounting and time preference: A critical review. *Journal of Economic Literature*, *40*(2), 351–401.

Freeman, D., Manzini, P., Mariotti, M., & Mittone, L. (2016). Procedures for eliciting time preferences. *Journal of Economic Behavior and Organization*, *126*, 235–242.

Gigliotti, G., & Sopher, B. (1997). Violations of present-value maximization in income choice. *Theory and Decision*, *43*(1), 45–69.

Greiner, B. (2015). Subject pool recruitment procedures: Organizing experiments with ORSEE. *Journal of the Economic Science Association*, *1*(1), 114–125.

Gul, F. (1991). A theory of disappointment aversion. *Econometrica*, *59*(3), 667–686.

Halevy, Y. (2008). Strotz meets Allais: Diminishing impatience and the certainty effect. *American Economic Review*, *98*(3), 1145–62.

Harrison, G. W., Lau, M. I., & Rutström, E. E. (2013). *Identifying time preferences with experiments: Comment*. Working paper 2013-09, Center for the Economic Analysis of Risk, Georgia State University.

Harrison, G. W., & Rutström, E. E. (2008). Risk aversion in the laboratory. In J. C. Cox & G. W. Harrison (Eds.), *Risk aversion in experiments* (Vol. 12, pp. 41–196)., Research in experimental economics Bingley: Emerald.

Hey, J. D., & Orme, C. (1994). Investigating generalizations of expected utility theory using experimental data. *Econometrica*, *62*(6), 1291–1326.

Holt, C. A., & Laury, S. K. (2002). Risk aversion and incentive effects. *American Economic Review*, *92*(5), 1644–1655.

Holt, C. A., & Laury, S. K. (2005). Risk aversion and incentive effects: New data without order effects. *American Economic Review*, *95*(3), 902–904.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, *47*(2), 263–291.

Kreps, D. M., & Porteus, E. L. (1978). Temporal resolution of uncertainty and dynamic choice theory. *Econometrica*, *46*(1), 185–200.

Laibson, D. (1997). Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics*, *112*(2), 443–478.

Laury, S. K., McInnes, M. M., & Swarthout, J. T. (2012). Avoiding the curves: Direct elicitation of time preferences. *Journal of Risk and Uncertainty*, *44*(3), 181–217.

Loewenstein, G., & Prelec, D. (1992). Anomalies in intertemporal choice: Evidence and an interpretation. *Quarterly Journal of Economics*, *107*(2), 573–597.

Loewenstein, G., & Prelec, D. (1993). Preferences for sequences of outcomes. *Psychological Review*, *100*(1), 91–108.

Loewenstein, G., & Sicherman, N. (1991). Do workers prefer increasing wage profiles? *Journal of Labor Economics*, *9*(1), 67–84.

Loomes, G., & Pogrebna, G. (2014). Measuring individual risk attitudes when preferences are imprecise. *Economic Journal*, *124*(576), 569–593.

Manzini, P., Mariotti, M., & Mittone, L. (2010). Choosing monetary sequences: Theory and experimental evidence. *Theory and Decision*, *69*(3), 327–354.

Meier, S., & Sprenger, C. (2010). Present-biased preferences and credit card borrowing. *American Economic Journal: Applied Economics*, *2*(1), 193–210.

Phelps, E. S. (1962). The accumulation of risky capital: A sequential utility analysis. *Econometrica*, *30*(4), 729–743.

Prelec, D. (1998). The probability weighting function. *Econometrica*, *66*(3), 497–527.

Quiggin, J. (1982). A theory of anticipated utility. *Journal of Economic Behavior and Organization*, *3*(4), 323–343.

Rabin, M. (2000). Risk aversion and expected-utility theory: A calibration theorem. *Econometrica*, *68*(5), 1281–1292.

Saha, A. (1993). Expo-power utility: A 'flexible' form for absolute and relative risk aversion. *American Journal of Agricultural Economics*, *75*(4), 905–913.

Samuelson, P. A. (1937). A note on the measurement of utility. *Review of Economic Studies*, *4*, 155–161.

Schmidt, U. (2014). *Risk preferences may be time preferences: A comment on Andreoni and Sprenger (2012)*. Working paper 1942, Kiel Institute for the World Economy.

Scholten, M., Read, D., & Sanborn, A. (2016). Cumulative weighing of time in intertemporal tradeoffs. *Journal of Experimental Psychology: General*, *145*(9), 1177–1205.

Sprenger, C. (2015). Judging experimental evidence on dynamic inconsistency. *American Economic Review: Papers and Proceedings*, *105*(5), 280–85.

Takeuchi, K. (2011). Non-parametric test of time consistency: Present bias and future bias. *Games and Economic Behavior*, *71*(2), 456–478.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, *5*(4), 297–323.

Tversky, A., Slovic, P., & Kahneman, D. (1990). The causes of preference reversal. *American Economic Review*, *80*(1), 204–217.

von Neumann, J., & Morgenstern, O. (1944). *Theory of Games and Economic Behavior*. Princeton: Princeton University Press.

Weil, P. (1990). Nonexpected utility in macroeconomics. *Quarterly Journal of Economics*, *105*(1), 29–42.

Wilcox, N. T. (2011). 'Stochastically more risk averse:' A contextual theory of stochastic discrete choice under risk. *Journal of Econometrics*, *162*(1), 89–104.

✷ Springer