BOOK REVIEWS

Advances in Mathematics, volume 1, fascicle 1, 1961, edited by H. Busemann, Academic Press, New York and London, 30s. 6d.

This is the first part of a new journal devoted to expository articles, and is to be warmly welcomed. There has been for some time a need for a Western periodical on the lines of the Russian *Uspehi*. The papers published will be of two main types, those addressed to all mathematicians and those addressed to a smaller number of mathematicians working in related fields. The first article published, entitled " Recent developments in the theory of connections and holonomy groups ", by Katsumi Nomizu, is of the second type, while the second article on " Banach algebras and analytic functions " by John Wermer, is of the first type or is, at any rate, addressed to a wider set of readers. To secure prompt publication of material, *Advances in Mathematics* will be released in paper-bound fascicles, which will be combined into volumes of between 350 and 400 pages.

R. A. RANKIN

STOLL, R. R., *Sets, Logic, and Axiomatic Theories* (W. H. Freeman and Co., San Francisco, 1961), x+206 pp., $2.25.

This book is one of the Golden Gate series of undergraduate mathematical texts. It is intended, however, not only for undergraduate use but also for school teachers who want to learn something of the spirit of modern mathematics ; probably its main appeal in this country will be to the latter class of readers, and possibly also to their pupils. Much of it would be accessible to an intelligent sixth-form schoolboy.

Chapter 1 provides an eminently readable introduction to the elementary parts of the theory of sets and relations, with special reference to equivalence relations, functions and ordering relations. Chapter 2, on Logic, covers the statement calculus and the predicate calculus. The elementary sentence connectives, truth tables and quantifiers are introduced and validity and valid consequences discussed. Axiomatic theories are introduced in Chapter 3, which includes a clear elementary description of the notions of consistency, completeness, decidability and categoricity. The concluding section, on metamathematics, gives an informal account of Hilbert's programme and the theorems of Gödel and Church. Chapter 4, on Boolean algebras, is " the icing on the cake "; it is intended to draw together the ideas of the previous chapters and to show them at work in a modern mathematical theory. Two formulations of the theory are given and it is developed far enough to give the Stone representation theorem.

The book is clearly and attractively written, with some lively turns of phrase—for example, " The principle of duality yields a free theorem for each theorem which has been proved ". It should be of interest and value to the readers for whom it is intended.

I. F. ADAMSON

PHILLIPS, E. G., *A Course of Analysis* (Cambridge Students' Editions, C.U.P., 1961), 361 pp., 15s.

This is a reprint at a reasonable price of the second edition of a book first published in 1930, which covers an undergraduate course on the definition of number, the theory of convergence and differential and integral calculus; multiple integrals and functions

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of several variables are included, but not Lebesgue integration or Fourier series. The treatment of the selected topics is careful and thorough, sometimes even laboured; the only omissions which the reviewer would question are those of Abel's and Dirichlet's tests for convergence and the classification of conditional stationary values.

In his preface to the second edition, the author remarks that experience has convinced him that the book is difficult enough for those using it as a first introduction to rigorous analysis. The reviewer feels that the book is unsuitable for a first introduction, since no attempt has been made to separate the easier topics from the harder ones or to use elementary arguments. It may prove a useful stimulus to an able student, especially as the text includes frequent summaries of the difficulties inherent at particular stages.

P. HEYWOOD

KOLMOGOROV, A. N., AND FOMIN, S. V., *Elements of the Theory of Functions and Functional Analysis*, vol. 2: *Measure, the Lebesgue Integral, Hilbert Space*, translated by H. KAMEL AND H. KOMM (Graylock Press, Rochester, N.Y., 1961), ix+128 pp., 34s.

Also published as *Measure, Lebesgue Integrals, and Hilbert Space*, translated by N. A. Brunswick and A. Jeffrey (Academic Press, New York and London, 1961), xii+147 pp., $4.

The Graylock Press set a high standard when they published the English translation by L. Boron of the first volume of this course. The standard is fully maintained in their version of the second volume. The other translation, published almost simultaneously by the Academic Press, is not a serious rival: it is in fact a remarkably inept piece of work, but fortunately there is no need to consider its faults in detail, most of them being absent from the Graylock version.

Although this second volume can be read independently of the first, the Graylock version follows the Russian original in having the chapters and sections numbered consecutively: Volume 2 consists of Chapters V-IX. Chapter V is an excellent introduction to measure theory. It starts with the construction of Lebesgue outer measure for bounded plane sets. The fundamental properties of measurable sets are then discussed in a way that illuminates the concept of measurablity very clearly. There follows a discussion, in an abstract setting, of the extension of measures from semi-rings to rings and to sigma-rings (a " measure " being defined here as a non-negative, finite-valued, additive function on a semi-ring of sets). The connexion between the full extension of a measure and the completion of an associated metric space is pointed out.

Measurable functions are discussed in Chapter VI in preparation for Chapter VII in which the Lebesgue integral is introduced. Integration is defined in the first place with respect to an arbitrary (finite-valued) measure on a sigma-algebra, and the usual theorems are proved. The Lebesgue integral over a finite interval is considered as a special case and is compared with the Riemann integral. There is then a discussion of product measures, leading to a general version of Fubini's theorem. Chapter VII ends with a brief reference to the Radon-Nikodym theorem.

Chapter VIII is concerned with the space L_2 of real-valued square-integrable functions with respect to a finite measure. The fundamental properties of this space are established, including completeness with respect to the norm, and separability when the measure satisfies a countability condition (equivalent to separability of the metric space associated with the measure). Orthogonal systems of functions are then considered, with the Riesz-Fischer theorem and the mean-square theory of Fourier series. (" Fischer " is spelt " Fisher " in the Graylock edition.) At the end of the chapter, the isomorphic isometry between l_2 and a separable L_2 is established, in preparation for Chapter IX.

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