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EFFECTS OF VELOCITY-SLIP AND VISCOSITY VARIATION ON JOURNAL BEARINGS

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Abstract

A generalised form of the Reynolds equation for two symmetrical surfaces is derived by considering slip at the bearing surfaces. This equation is then used to study the effects of velocity-slip for the lubrication of journal bearings using half-Sommerfeld boundary conditions. Expressions for pressure and load capacity and the coefficient of friction are obtained and numerically analysed for various parameters. It is found that the load capacity decreases with slip. This is unfavourable for lubrication. The coefficient of friction decreases with a high viscous layer and increases with slip.

1. Introduction

A journal bearing is the most common hydrodynamic bearing in use. It is a circular shaft or journal rotating inside a circular bush, that is, the inner constituent is a solid right circular cylinder called a journal and the outer body is in the form of a hollow right circular cylinder called a bearing. The inner diameter of the bush is between one and two parts per thousand bigger than the shaft. The gap between the cylinders is taken to be very small in comparison to the radii of both the cylinders and so there is a thin fluid layer between these two cylinders, which may be considered as a lubricant. Much work has been done in journal bearings with various lubricants [1, 2, 7, 13, 15–22, 27, 29, 30, 32, 34, 35].

In general, additives are added to the lubricant to improve the bearing characteristics. Various theories have been proposed for this. These additives are generally long-chain organic compounds and they may form a high viscous layer near the surface. It may be proposed that slip-velocity may occur near the surface [2].

Very little attention has been paid to the study of the effects of slip at the surface, although it may be of importance in the flow behaviour of gases and liquids particularly

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when the film is thin [10, 11, 14], the surface is smooth [8] and at the porous boundary [3–5, 24, 25, 33].

The slip phenomenon also plays an important role in bearings with porous facings. Beavers and coworkers [3–5] discussed this effect for an incompressible fluid and demonstrated the existence of slip velocity at the porous surface. This has been further supported by Saffman [25], Taylor [33] and Richardson [24]. The slip velocity at the porous surface can be written as

$$U_{\text{slip}} = \frac{\phi^{1/2}}{\xi}\left(\frac{\partial u}{\partial z}\right)_{\text{wall}},$$

where ξ is the slip coefficient at the wall and ϕ is the permeability of the porous facing.

In this paper, the effects of velocity-slip and viscosity variation in journal bearings are discussed using half-Sommerfeld boundary conditions.

2. Mathematical analysis

Consider the laminar flow of a fluid between two symmetric surfaces and the variation of fluid properties across as well as along the film thickness. The basic equations of motion and equation of continuity in the general form for a Newtonian fluid can be written as

$$\begin{aligned}
\rho\frac{Du}{Dt} &= \rho X - \frac{\partial P}{\partial x} + \frac{2}{3}\frac{\partial}{\partial x}\left\{\eta\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)\right\} + \frac{2}{3}\frac{\partial}{\partial x}\left\{\eta\left(\frac{\partial u}{\partial x} - \frac{\partial w}{\partial z}\right)\right\}\\
&\quad + \frac{\partial}{\partial y}\left\{\eta\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right\} + \frac{\partial}{\partial z}\left\{\eta\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right\},\\
\rho\frac{Dv}{Dt} &= \rho Y - \frac{\partial P}{\partial y} + \frac{2}{3}\frac{\partial}{\partial y}\left\{\eta\left(\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x}\right)\right\} + \frac{2}{3}\frac{\partial}{\partial y}\left\{\eta\left(\frac{\partial v}{\partial y} - \frac{\partial w}{\partial z}\right)\right\}\\
&\quad + \frac{\partial}{\partial z}\left\{\eta\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\right\} + \frac{\partial}{\partial x}\left\{\eta\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right\},\\
\rho\frac{Dw}{Dt} &= \rho Z - \frac{\partial P}{\partial z} + \frac{2}{3}\frac{\partial}{\partial z}\left\{\eta\left(\frac{\partial w}{\partial z} - \frac{\partial u}{\partial x}\right)\right\} + \frac{2}{3}\frac{\partial}{\partial z}\left\{\eta\left(\frac{\partial w}{\partial z} - \frac{\partial v}{\partial y}\right)\right\}\\
&\quad + \frac{\partial}{\partial x}\left\{\eta\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right\} + \frac{\partial}{\partial y}\left\{\eta\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\right\},
\end{aligned} \tag{2.1}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0, \tag{2.2}$$

with the following usual assumptions of lubrication theory:

(1) Inertia and body force terms are negligible compared with the pressure and viscous terms.

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(2) There is no variation of pressure across the fluid film, which means $\partial P/\partial Z = 0$.

(3) There is no slip in the fluid-solid boundaries.

(4) No external forces act on the film.

(5) The flow is viscous and laminar.

(6) Due to the geometry of the fluid film the derivatives of u and v with respect to z are much larger than the other derivatives of the velocity components.

(7) The height of the film h is very small compared to the bearing length, l. A typical value of h/l is about 10^{-3}.

The Navier-Stokes equations (2.1) can be simplified to

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial z}\left[\eta \frac{\partial u}{\partial z}\right], \quad \frac{\partial P}{\partial y} = \frac{\partial}{\partial z}\left[\eta \frac{\partial v}{\partial z}\right], \tag{2.3}$$

where $P = P(x, y)$ is the pressure in the film and η is the viscosity.

The boundary conditions considering slip at the surfaces [28] are

$$\begin{aligned} u &= (u)_1 = (\lambda)_1 \left[\frac{\partial u}{\partial z}\right]_1 + U_1, & v &= (v)_1 = (\delta)_1 \left[\frac{\partial v}{\partial z}\right]_1 + V_1, & &\text{at } z = H_1, \\ u &= (u)_2 = -(\lambda)_2 \left[\frac{\partial u}{\partial z}\right]_2 + U_2, & v &= (v)_2 = -(\delta)_2 \left[\frac{\partial v}{\partial z}\right]_2 + V_2, & &\text{at } z = H_2, \end{aligned} \tag{2.4}$$

where $(\,)_1$, $(\,)_2$ denote the value at $z = H_1$ and $z = H_2$. Here λ and δ are the molecular mean free path for gas lubrication and depend upon lubricant temperature, pressure and viscosity. In liquid lubrication λ and δ depend on viscosity and the coefficient of sliding friction. However, with porous bearings λ and δ are functions of the slip coefficient at the wall and the permeability parameter of the porous facing.

Integrating (2.3) and using boundary conditions (2.4), expressions for the fluid film velocities are obtained:

$$\begin{aligned} u &= U_1 + \left[\alpha_1 H_1 + \int_{H_1}^{z} \frac{z\,dz}{\eta}\right] \frac{\partial P}{\partial x} + \left[\frac{U_2 - U_1}{F_0} - \frac{F_1}{F_0}\frac{\partial P}{\partial x}\right]\left[\alpha_1 + \int_{H_1}^{z} \frac{dz}{\eta}\right], \\ v &= V_1 + \left[\beta_1 H_1 + \int_{H_1}^{z} \frac{z\,dz}{\eta}\right] \frac{\partial P}{\partial y} + \left[\frac{V_2 - V_1}{F_0^1} - \frac{F_1^1}{F_0^1}\frac{\partial P}{\partial y}\right]\left[\beta_1 + \int_{H_1}^{z} \frac{dz}{\eta}\right], \end{aligned}$$

where $F_0 = \alpha_1 + \alpha_2 + \int_{H_1}^{z} dz/\eta$, $F_0^1 = \beta_1 + \beta_2 + \int_{H_1}^{z} dz/\eta$, $F_1 = \alpha_1 H_1 + \alpha_2 H_2 + \int_{H_1}^{H_2} (z/\eta)\,dz$, $F_1^1 = \beta_1 H_1 + \beta_2 H_2 + \int_{H_1}^{H_2} (z/\eta)\,dz$, $\alpha_1 = (\lambda)_1/(\eta)_1$, $\alpha_2 = (\lambda)_2/(\eta)_2$, $\beta_1 = (\delta)_1/(\eta)_1$ and $\beta_2 = (\delta)_2/(\lambda)_2$. Integrating the equation of continuity (2.2) w.r.t. z and taking limits from $z = H_1$ to $z = H_2$ gives

$$\int_{H_1}^{H_2} \frac{\partial \rho}{\partial t}\,dz + \int_{H_1}^{H_2} \frac{\partial}{\partial x}(\rho u)\,dz + \int_{H_1}^{H_2} \frac{\partial}{\partial y}(\rho v)\,dz + (\rho w)_{H_1}^{H_2} = 0. \tag{2.5}$$

The integrals of (ρu) and (ρv) are evaluated by partial integration. Introducing the expressions for (ρu) and (ρv) and their derivatives in (2.5) gives

$$\begin{aligned}
&\frac{\partial}{\partial x}\left\{(F_2+G_1)\frac{\partial P}{\partial x}\right\}+\frac{\partial}{\partial y}\left\{(F_2^1+G_1^1)\frac{\partial P}{\partial y}\right\}\\
&\quad = H_2\left\{\frac{\partial}{\partial x}(\rho u)_2+\frac{\partial}{\partial y}(\rho v)_2\right\}-H_1\left\{\frac{\partial}{\partial x}(\rho u)_1+\frac{\partial}{\partial y}(\rho v)_1\right\}\\
&\quad -\frac{\partial}{\partial x}\left\{\frac{(U_2-U_1)(F_3+G_2)}{F_0}+U_1G_3\right\}-\frac{\partial}{\partial y}\left\{\frac{(V_2-V_1)(F_3+G_2^1)}{F_0^1}+V_1G_3\right\}\\
&\quad +\int_{H_1}^{H_2}\frac{\partial\rho}{\partial t}\,dz+(\rho w)_{H_1}^{H_2}, \qquad (2.6)
\end{aligned}$$

where

$$F_2=\int_{H_1}^{H_2}\frac{\rho z}{\eta}\left[z-\frac{F_1}{F_0}\right]dz,\quad F_2^1=\int_{H_1}^{H_2}\frac{\rho z}{\eta}\left[z-\frac{F_1^1}{F_0^1}\right]dz,\quad F_3=\int_{H_1}^{H_2}\frac{\rho z}{\eta}\,dz,$$

$$G_1=\int_{H_1}^{H_2}\left[z\frac{\partial\rho}{\partial z}\left\{\alpha_1H_1+\int_{H_1}^{z}\frac{z\,dz}{\eta}-\frac{F_1}{F_0}\left[\alpha_1+\int_{H_1}^{z}\frac{dz}{\eta}\right]\right\}\right]dz,$$

$$G_1^1=\int_{H_1}^{H_2}\left[z\frac{\partial\rho}{\partial z}\left\{\beta_1H_1+\int_{H_1}^{z}\frac{z\,dz}{\eta}-\frac{F_1^1}{F_0^1}\left[\beta_1+\int_{H_1}^{z}\frac{dz}{\eta}\right]\right\}\right]dz,$$

$$G_2=\int_{H_1}^{H_2}\left\{z\frac{\partial\rho}{\partial z}\left[\alpha_1+\int_{H_1}^{z}\frac{dz}{\eta}\right]\right\}dz,\quad G_2^1=\int_{H_1}^{H_2}\left\{z\frac{\partial\rho}{\partial z}\left[\beta_1+\int_{H_1}^{z}\frac{dz}{\eta}\right]\right\}dz,$$

$$G_3=\int_{H_1}^{H_2}z\frac{\partial\rho}{\partial z}\,dz.$$

Equation (2.6) represents a generalised form of the Reynolds equation for compressible fluid film lubrication considering slip velocities at the bearing surfaces. The two sets of functions F and G depend upon the variation of fluid properties both along and across the film and on the slip conditions at the surfaces, that is, $(\lambda)_1=(\lambda)_2=(\delta)_1=(\delta)_2=0$ and $\alpha_1=\alpha_2=\beta_1=\beta_2=0$.

The viscosity of the lubricant can vary across the film and may be different near the bearing surfaces owing to the reaction of additives and surfactants with the surfaces [2, 6, 8, 9, 12, 23, 26].

Considering a reasonable case where the density and viscosity of the lubricant near the bearing surfaces may be different from the central region, we can have

$$\begin{aligned}
&\rho=\rho_1(x,y),\quad \eta=\eta_1(x,y), && H_1\le z\le H_1+h_1,\\
&\rho=\rho_2(x,y),\quad \eta=\eta_2(x,y), && H_1+h_1\le z\le H_1+h_1+h_2,\\
&\rho=\rho_3(x,y),\quad \eta=\eta_3(x,y), && H_1+h_1+h_2\le z\le H_1+h_1+h_2+h_3.
\end{aligned}$$

This introduces the concept of multiple-layer lubrication. By taking $U_1 = U$, $U_2 = V_1 = V_2 = 0$, $\alpha_1 = \beta_1$, $\alpha_2 = \beta_2$, $\partial \rho_i/\partial z = 0$, $i = 1, 2, \dots$, the generalised equation with slip reduces to the following form:

$$\frac{\partial}{\partial x}\left[F_2\frac{\partial P}{\partial x}\right] + \frac{\partial}{\partial y}\left[F_2\frac{\partial P}{\partial y}\right] = H_2\left\{\frac{\partial(\rho u)_2}{\partial x} + \frac{\partial(\rho v)_2}{\partial y}\right\} - H_1\left\{\frac{\partial(\rho u)_1}{\partial x} + \frac{\partial(\rho v)_1}{\partial y}\right\}$$
$$+ U\frac{\partial}{\partial x}\left[\frac{F_3}{F_0}\right] + [\rho w]_{H_1}^{H_2}, \tag{2.7}$$

where

$$\begin{aligned}
F_0 &= \alpha_1 + \alpha_2 + h_1/\eta_1 + h_2/\eta_2 + h_3/\eta_3, \\
F_1 &= \alpha_1 H_1 + \alpha_2 H_2 + h_1(2H_1 + h_1)/(2\eta_1) + h_2(2H_1 + 2h_1 + h_2)/(2\eta_2) \\
&\quad + h_3(2H_1 + 2h_1 + 2h_2 + h_3)/(2\eta_3), \\
F_2 &= \frac{\rho_1}{3\eta_1}\{(H_1 + h_1)^3 - H_1^3\} + \frac{\rho_2}{3\eta_2}\{(H_1 + h_1 + h_2)^3 - (H_1 + h_1)^3\} \\
&\quad + \frac{\rho_3}{3\eta_3}\{H_2^3 - (H_1 + h_1 + h_2)^3\} - \frac{F_1 F_3}{F_0}, \\
F_3 &= \frac{\rho_1 h_1}{2\eta_1}(2H_1 + h_1) + \frac{\rho_2 h_2}{2\eta_2}(2H_1 + 2h_1 + h_2) \\
&\quad + \frac{\rho_3 h_3}{2\eta_3}(2H_1 + 2h_1 + 2h_2 + h_3), \\
(\rho u)_1 &= \rho_1\alpha_1\left[H_1 - \frac{F_1}{F_0}\right]\frac{\partial P}{\partial x} + \rho_1 U\left[1 - \frac{\alpha_1}{F_0}\right], \\
(\rho u)_2 &= -\rho_3\alpha_2\left[H_2 - \frac{F_1}{F_0}\right]\frac{\partial P}{\partial x} + \rho_3 U\frac{\alpha_2}{F_0}, \\
(\rho v)_1 &= \rho_1\alpha_1\left[H_1 - \frac{F_1}{F_0}\right]\frac{\partial P}{\partial y}, \quad (\rho v)_2 = -\rho_3\alpha_2\left[H_2 - \frac{F_1}{F_0}\right]\frac{\partial P}{\partial y}, \\
[\rho w]_{H_1}^{H_2} &= (\rho u)_2\frac{\partial H_2}{\partial x} + (\rho v)_2\frac{\partial H_2}{\partial y} - (\rho u)_1\frac{\partial H_1}{\partial x} - (\rho v)_1\frac{\partial H_1}{\partial y} - V_s.
\end{aligned}$$

Here V_s is the resultant velocity towards the film.

To see the effect of slip consider three symmetrical incompressible layers between two solid boundaries: $\eta_1 = \eta_2$, $\rho_1 = \rho_2 = \rho_3$, $H_1 = 0$, $H_2 = (h + a) = h$, $h_1 = h_3 = a/2$, $h_2 = (h - a)$, $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1/\beta$. The Reynolds equation applicable to this case can be written from (2.7) as follows:

$$\frac{\partial}{\partial x}\left[F_4\frac{\partial P}{\partial x}\right] + \frac{\partial}{\partial y}\left[F_4\frac{\partial P}{\partial y}\right] = U\frac{\partial}{\partial x}(h) - V, \tag{2.8}$$

where $F_4 = (h - a)^3/(12\eta_2) + (h^3 - (h - a)^3)/(12\eta_1) + h^2/(2\beta)$, taking $\beta = \eta_1/\lambda$ to be the slip parameter.

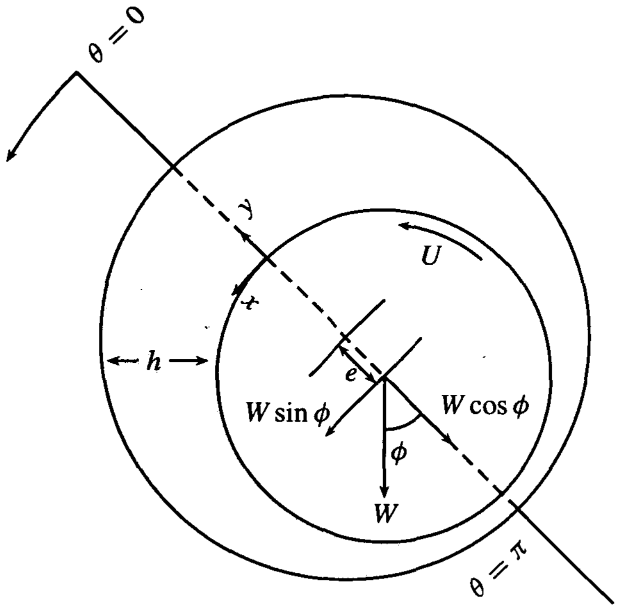

FIGURE 1. Journal bearing configuration.

3. Lubrication of two symmetrical cylinders in journal bearings

Consider the flow of an incompressible fluid between two eccentric cylinders in uniform relative motion. The gap or clearance c (the difference between the two radii) between the two cylindrical surfaces is small compared with the radius of the inner cylinder (journal). The journal bearing operates with a constant external load, W and speed, U. Under the physical conditions imposed, the journal operates at an eccentricity e as shown in Figure 1.

The one-dimensional form of the equation governing the pressure in the fluid film, taking $\eta_1 = k\mu$ and $\eta_2 = \mu$ from (2.8), is

$$\frac{d}{dx}\left\{\frac{h^3}{\mu}F_4\frac{dP}{dx}\right\} = \frac{U}{2}\frac{d}{dx}(h), \tag{3.1}$$

where

$$F_4 = \frac{(1 - a/h)^3(k-1)+1}{24k} + \frac{\mu}{4h\beta}.$$

Here β represents the slip parameter, k is the viscous layer parameter, a is the thickness of the peripheral layer, μ is the viscosity of the middle layer and the total film thickness

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of the lubricant is given by $h = c(1 + \epsilon \cos\theta)$.

Integrating once and using (3.1) gives

$$\frac{dP}{dx} = \frac{U}{2}\frac{\mu}{h^3 F_4}(h - h_2),$$

where h_2 is the film thickness corresponding to maximum pressure.

Using the half-Sommerfeld boundary conditions [31]

$$P = 0 \quad \text{at } \theta = 0 \text{ and } \theta = \pi, \tag{3.2}$$

and putting $x = R\theta$, the pressure gradient and the pressure become

$$\frac{dP}{d\theta} = \frac{\mu U R}{2h^3}\frac{(h - h_2)}{F_4} \tag{3.3}$$

and

$$P(\theta) = \frac{\mu U R}{2h^3}\int_0^\theta \frac{h - h_2}{F_4}\, d\theta$$

respectively. The pressure distribution is put into dimensionless form using $H = h/c$, $H_2 = h_2/c$ and $\overline{P} = P/(\mu U R/2c^2)$ and becomes

$$\overline{P}(\theta) = \int_0^\theta \frac{H - H_2}{H^3 \overline{F}_4}\, d\theta, \tag{3.4}$$

where

$$\overline{F}_4 = \frac{(1 - \bar{a}/\bar{h})^3(k - 1) + 1}{24k} + \frac{1}{4\bar{h}\bar{\beta}} \tag{3.5}$$

and $\bar{a} = a/c$, $\bar{h} = h/c$, $\bar{\beta} = c\beta/\mu$. Using the boundary conditions (3.2) we get

$$H_2 = \frac{\int_0^\pi \left(H^2 \overline{F}_4\right)^{-1} d\theta}{\int_0^\pi \left(H^3 \overline{F}_4\right)^{-1} d\theta}.$$

Load components. The load components per unit length along and perpendicular to the line of centres are obtained by integrating the pressure around the bearing from $\theta = 0$ to $\theta = \pi$. The load components normal to the line of centres per unit length is $W_{\pi/2} = W\sin\phi = \int_0^\pi P \sin\theta R\, d\theta$.

Integrating by parts and using (3.3) gives

$$W_{\pi/2} = \frac{\mu U R^2}{2}\int_0^\pi \frac{h - h_2}{h^3 F_4}\cos\theta\, d\theta.$$

The load W is put into dimensionless form using the relation

$$\overline{W} = \frac{W}{\mu R^2 U/2c^2}$$

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and hence

$$\overline{W}_{\pi/2} = \int_0^\pi \frac{H - H_2}{H^3 \overline{F}_4} \cos\theta \, d\theta,$$

where $\overline{F}_4$ is given by (3.5).

The load component per unit length along the line of centres is given by

$$W_0 = W\cos\phi = -\int_0^\pi P\cos\theta R\, d\theta = R\int_0^\pi \frac{dP}{d\theta}\sin\theta\, d\theta.$$

The corresponding dimensionless form is

$$\overline{W}_0 = \int_0^\pi \frac{H - H_2}{H^3 \overline{F}_4} \sin\theta \, d\theta$$

and the dimensionless load is given by

$$\overline{W} = \left(\overline{W}_{\pi/2}^2 + \overline{W}_0^2\right)^{1/2}. \tag{3.6}$$

Coefficient of friction. The shear stress along the surface [18] is

$$\tau = \eta\left(\frac{\partial u}{\partial y}\right) = \left[\frac{U}{F_0} + \left(z - \frac{h}{2}\right)\frac{\partial P}{\partial x}\right],$$

where $F_0 = \alpha_1 + \alpha_2 + \int_0^h dz/\eta$.

At $z = 0$, the shear stress on the journal is $\tau_0 = U/F_0 + (-h/2)(\partial P/\partial x)$, and at $z = h$, we have $\tau_h = U/F_0 + (h/2)(\partial P/\partial x)$. Now the frictional force is

$$F = \int \tau_x \, dx = \int_0^\pi \left(\frac{h}{2}\frac{dp}{dx} + \frac{U}{F_0}\right) dx.$$

Therefore

$$F = \int_0^\pi \left(\frac{h}{2R}\frac{dP}{d\theta} + \frac{U}{F_0}\right) R\, d\theta,$$

where $F_0 = (h - a)/\mu + a/(k\mu) + 2/\beta$.

Since it is assumed that the pressure gradient is zero beyond $\theta = \pi$, the contribution to the frictional drag beyond $\theta = \pi$ will be due only to the surface velocity. The frictional drag F per unit length is thus given by

$$F = R\int_0^\pi \left(\frac{h}{2R}\frac{dP}{d\theta} + \frac{U}{F_0}\right) d\theta + R\int_\pi^{2\pi} \frac{U}{F_0}\, d\theta. \tag{3.7}$$

Substituting $dP/d\theta$ from (3.3) into (3.7) gives

$$F = \frac{\mu U R}{2c}\left[\int_0^\pi \frac{(h - h_2)\, d\theta}{2h^2 F_4} + 2\int_0^{2\pi} \frac{d\theta}{(h - a) + a/k + 2\mu/\beta}\right].$$

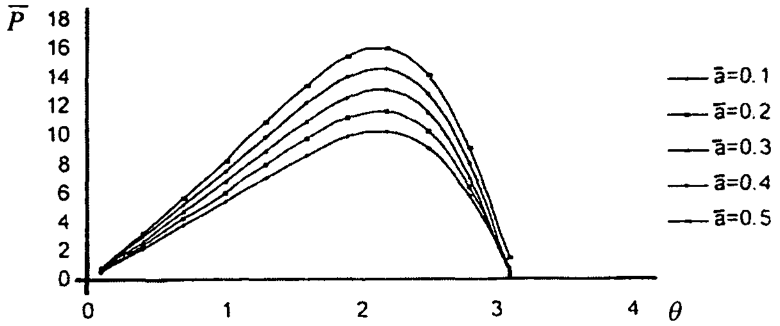

FIGURE 2. Variation of $\overline{P}$ versus θ for various values of $\bar{a}$ with $k = 2.0$, $\epsilon = 0.4$ and $\bar{\beta} = 25.0$.

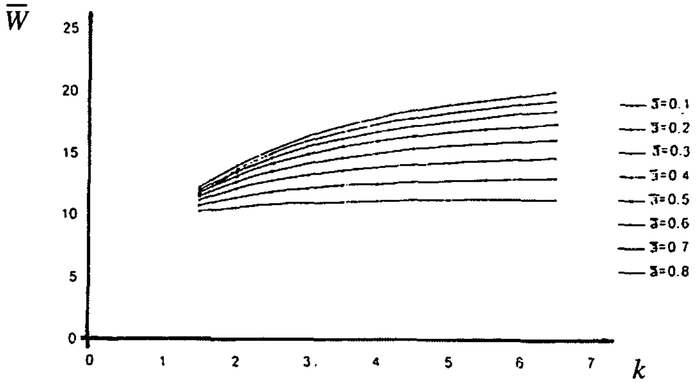

FIGURE 3. Variation of $\overline{W}$ versus k for various values of $\bar{a}$ with $\bar{\beta} = 10.0$ and $\epsilon = 0.4$.

This is put into dimensionless form using $\overline{F} = F/(\mu U R/2c)$ and becomes

$$\overline{F} = \frac{1}{2}\int_0^{\pi} \frac{H - H_2}{H^2 \overline{F}_4}\, d\theta + 2\int_0^{2\pi} \frac{d\theta}{(\bar{h} - \bar{a}) + \bar{a}/k + 2/\bar{\beta}},$$

where $\overline{F_4}$ is given by (3.5).

The friction coefficient, μ_f, is obtained by dividing the frictional drag by the load:

$$\mu_f(R/C) = \overline{F}/\overline{W}. \tag{3.8}$$

4. Results and discussion

(I) Dimensionless parameters. The bearing characteristics depend on the parameters $\bar{\beta}$, k and $\bar{a}$.

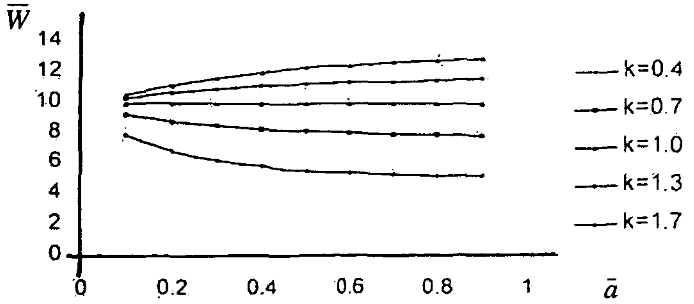

FIGURE 4. Variation of $\overline{W}$ versus $\bar{a}$ for various values of k with $\bar{\beta} = 10.0$ and $\epsilon = 0.4$.

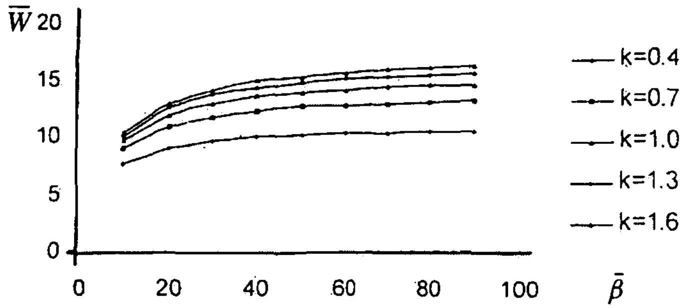

FIGURE 5. Variation of $\overline{W}$ versus $\bar{\beta}$ for various values of k with $\bar{a} = 0.1$ and $\epsilon = 0.4$.

(a) *Slip parameter.* The slip parameter is represented by $\bar{\beta}$. As $\bar{\beta}$ tends to infinity, this indicates no-slip at the surface. As $\bar{\beta}$ tends to zero, the slip will be maximum, that is, as $\bar{\beta}$ increases, the slip decreases. So lower values of $\bar{\beta}$ indicate high slip and higher values of $\bar{\beta}$ indicate less slip.

(b) *Viscous layer parameter.* We mentioned earlier that the viscosity near the surface is different to the viscosity at the middle layer. This is taken into account by the parameters k and $\bar{a}$. When $k > 1$, the viscosity near the periphery is more than the viscosity of the middle layer. If $k = 1$, this indicates that the viscosity is the same everywhere. When $k < 1$, the viscosity at the periphery is less than the viscosity of the middle region. Thus the difference between the viscosity of the middle and peripheral regions is indicated by the parameter k.

Another key parameter is $\bar{a}$, which indicates the thickness of the peripheral layer due to the presence of additives. When $\bar{a} = 0$, there is no peripheral layer. As the peripheral layer thickness is small, normally the values of $\bar{a}$ would be small.

(II) Bearing characteristics. Equations (3.4), (3.6) and (3.8) have been analysed numerically and appropriate graphs have been plotted with these parameters. In

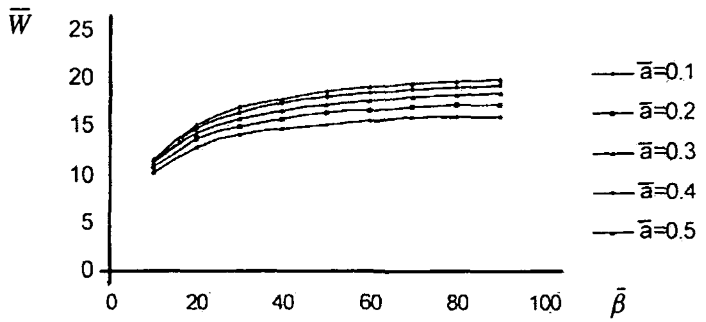

FIGURE 6. Variation of $\overline{W}$ versus $\bar{\beta}$ for various values of $\bar{a}$ with $k = 1.5$ and $\epsilon = 0.4$.

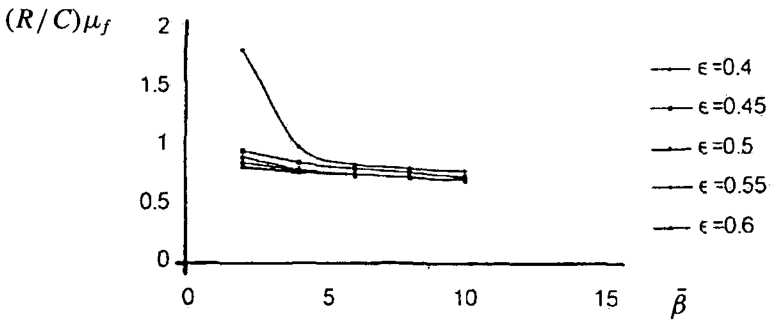

FIGURE 7. Variation of $(R/C)\mu_f$ versus $\bar{\beta}$ for various values of ϵ with $k = 1.0$ and $\bar{a} = 0.0$.

Figure 2 the pressure $\overline{P}$ is plotted against θ for various values of $\bar{a}$. In this figure, we find that the pressure increases as θ increases and reaches a maximum pressure which occurs at $\theta = 2.2$. It is independent of the parameters $\bar{\beta}$, $\bar{a}$ and k. It is also observed that the pressure is increased for higher values of $\bar{a}$ for $k > 1$ and decreases as $\bar{a}$ increases for $k < 1$ and it is also observed that the pressure increases as $\bar{\beta}$ increases, that is, as the slip parameter decreases.

The load capacity $\overline{W}$ versus various parameters has been plotted in Figures 3–6. It can be seen from Figure 3 that the load capacity increases as k increases, that is, the viscosity of the peripheral layer increases and it is higher for higher values of $\bar{a}$. It may also be seen from Figure 4 that $\overline{W}$ increases as $\bar{a}$ increases for $k > 1$ and decreases as $\bar{a}$ increases for $k < 1$ and it is parallel to the x-axis when $k = 1$, that is, when the thickness of the peripheral layer is greater than that of the middle layer. The load capacity increases as its thickness increases and also from Figures 5–6, we find that the load decreases as the slip increases.

The friction coefficient parameter $(R/C)\mu_f$ is plotted against $\bar{\beta}$ in Figure 7. It is found that $(R/C)\mu_f$ decreases as $\bar{\beta}$ increases, that is, as the slip decreases the friction coefficient decreases and it is lower for low values of ϵ.

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