SOME ASPECTS OF FULL SCALE EXPERIMENTS.

Paper read by Mr. C. Howarth A.F.R.Ae.S. (Member) before the Institution in the Lecture Room of the Junior Institution of Engineers, 39, Victoria Street, London, S.W.1, on Tuesday, 12th January 1926. Mr. Lawrence Wingfield in the Chair.

INTRODUCTORY.

This paper is intended to show some of the problems in full scale aerodynamical research and the methods used in attacking these problems; the details of the experiments, the instruments used, the observations taken and the method of reduction of the results are dealt with. It is not intended to consider either the application or the analysis of the results. The report is divided into three sections, illustrating different types of experimental work :—

(1) The standard method of obtaining the total lift and drag of an aeroplane, with the method of reduction of the results.
(2) An application of the Cinema Camera to full scale experiments.
(3) Full scale experiments on Spinning, with the method of reduction of the results and a description of the instruments used.

Before turning to the detailed consideration of these special problems, two more general points will be touched on: (*a*) the weather conditions necessary for carrying out experimental work with accuracy; and (*b*) the method of measuring the air-speed of the aeroplane.

(a) WEATHER CONDITIONS.

It is essential that the air be steady, as the presence of bumps interferes with the accuracy of the work. If during a steady unstalled glide the aeroplane is disturbed by bumps, there will be variations in speed and altitude, and the steady conditions essential for accurate work cannot be attained. But if the aeroplane is disturbed by bumps during a stalled glide, the case is even more serious, as loss of control results, and considerable lateral and longitudinal oscillations are set up which take a long time to damp out, so the results obtained are worthless. The results obtained from glides carried out in doubtful weather are thus often misleading, and show considerable scattering of points when plotted, for which often no satisfactory explanation can be found.

In addition to the unsteadiness introduced by bumps, up and down currents constitute a serious source of inaccuracy. For low speeds the effect of an up or down current of the order of $\frac{1}{2}$ ft. per second would be appreciable on the corresponding point of a lift and drag curve, and a current of 1 ft. per second would cause serious error. For high speeds, however, the effect of currents up to 2 ft. per second would not be very serious. It is usually very difficult to detect these small variations in the atmosphere.

It is important to carry out full scale flying during the best part of the day, this is usually from 5 a.m. until 9 a.m. in summer, before the sun has much power, or else from sunset until dark. The rest of the day can be used for calibration of instruments, preparation for future flights, and reductions of results.

(b) NOTE ON MEASURING AIRSPEED

In the past most of the instruments used were either designed or developed by the person in charge of the experiment, now, however, greater precision and accuracy are needed and this calls for careful detail design This is now becoming an important department in connection with full scale work.

An important development has been the introduction of the suspended static head to deduce the true indicated airspeed, this has proved of enormous value, both from the points of view of convenience, economy in staff, and time previously taken in calibrating machines over the speed course to obtain the interference factor due to the wings on the standard pitot static system. As this instrument is used during most experiments, a few notes on it will be of use.

The static head is suspended by a rubber tube 40 ft. long, having a thin steel cable running through its bore to take the load. The suspended static head used in conjunction with a swivelling pitot head gives a correct determination of the true indicated speed up to the stalling angle. This combination has been tested over the Upper, Lower and Railway Speed Courses at Farnborough as well as against other calibrated aeroplanes, and gives very consistent results.

To use the suspended static head, it is only necessary to lower the instrument over the side of the aeroplane as soon as it is reasonably clear of the ground, to fix the end to a clip on the machine and connect to the static side of the airspeed indicator. This involves no extra staff, whereas obtaining the speed course factor over the Upper or Lower Speed Course, necessitates an additional two ground observers.

There is no absolute check that the combination of suspended static head and swivelling pitot head gives true speeds above the stalling angle, however, during tests on an Avro, two different types of suspended static heads were used, which, when corrected for lag in the system, showed close agreement. The static heads used were approximately 40 ft. long. Further, as another relative check on whether interference was present at this distance from the aeroplane, a stalled glide was carried out with the suspended static head 32 ft. out for the first half of the glide, and full out (40 ft.) for the remainder of the glide, and the readings compared with the standard fixed pitot static system. No change was observed in the correction deduced for the standard system, and it is therefore concluded that there is no interference present below 32 ft.

A correction must be applied to the measured air speed for lag in the system when using the suspended static head. This is due to the changing pressure during a glide or climb and is important, owing to the high rate of descent at large angles of incidence. This correction is determined experimentally by subjecting the system to similar conditions on the bench.

The instruments used in the other branches of the work will be dealt with in those sections of the paper.

1.—TOTAL LIFT AND DRAG OF AN AEROPLANE.

The lift and drag of an aeroplane are usually determined by carrying out a series of glides, with the propeller stopped, over the range of incidence required. The aeroplane is usually fitted with a band brake, to stop the propeller and a locking device to keep the propeller at the same position throughout the experiment—(the propeller is held at the same position during each glide). When comparison is required with the test on a similar model, the model propeller is fixed in the same position as the one on the full scale aeroplane.

An airspeed indicator (or a velometer which has a graduated glass tube containing alcohol) with connections to the suspended static head and swivelling pitothead, and also an adjustable bubble inclinometer are fitted in the observer's cockpit. A small-lag alcohol thermometer is fitted on to a wing strut in a convenient position to be read by the observer. A sensitive aneroid graduated every 20 ft. and a stop watch are carried by the observer.

Before the engine is run up on the ground the observer notes the amount of petrol, oil and water in the various tanks. During the climb the temperature at each 1,000 ft. is noted, and the suspended static head is lowered. When sufficient height has been reached to allow a steady glide of approximately 5,000 feet, the pilot switches off the engine and stops the propeller, locking it in its proper position. The observer notes the time taken during the climb (to allow for change in weight due to petrol and oil consumption). The pilot then settles down to a pre-arranged speed and maintains as constant a speed as is possible. Over the normal flying range the total variation in speed should be kept within ½ to 1 m.p.h. When doing stalled glides the variation may be from 2—3 m.p.h. As soon as the speed is steady, the observer starts his stop watch and notes the height; adjusts the longitudinal bubble until at a convenient zero ; and takes as many simultaneous readings of airspeed and inclinometer as possible, noting the height every minute, throughout the glide. The propeller locking device is now released, and the pilot dives the aeroplane to regain the engine. The aeroplane is then climed for further glides, three or four glides from 9,000 or 10,000 ft. being carried out on an average each flight.

On landing, a check is taken of the petrol and oil consumed and the loss of water in the radiator, the airspeed indicator is calibrated against a standard water manometer and the setting of the inclinometer to the lower chord at root is taken.

It is customary to carry out glides every 10 m.p.h. at high speeds, viz., 100 m.p.h. and over, every 5 m.p.h. over normal flying range and every 2 m.p.h. around the stalling speed.

REDUCTION OF RESULTS.

From the observations the following data are obtained : The mean height, speed, temperature, inclinometer reading and height drop per minute. The weight of the aeroplane, and the quantity of petrol oil and water carried during the experiment are known. The weight of the pilot and observer, suspended static head, and any other extra load carried is added to the weight of the aeroplane, giving the total weight at the commencement of flight.

The equations of motion are as follows :—

(1) $k_{\epsilon}\rho_{\circ}\mathrm{S}\,\mathrm{V}i^2=\text{Total weight}\cos\gamma$

(2) $(k\mathrm{D}+k\mathrm{R})\,\rho_{\circ}\mathrm{S}\,\mathrm{V}i^2=\text{Total weight}\sin\gamma$

From 1 and 2 $\dfrac{k\mathrm{D}+k\mathrm{R}}{k_{\epsilon}}=\tan\gamma$

It follows that if $\mathrm{V}i$ is constant throughout the glide, α and γ must also be constant. Thus, $\mathrm{V}c$ must be proportional to $\dfrac{1}{\sigma}$ and the curve of the glide, which consists of height plotted against time, is concave upwards. For the height range usually used, the concavity is small, and it is sufficiently accurate to take the mean value of the angle of glide, density, indicated speed and inclinometer to give a single determination of k_{ϵ} (or total $k\mathrm{L}$), $k\mathrm{D}+k\mathrm{R}$ (or total $k\mathrm{D}$) and α

The step by step method of getting these equations is as follows :—

Weight in lbs.	W=	Total weight at commencement of flight.—Weight of fuel consumed.
Mean height		taken from observations and corrected for standard Barometer.
Mean temp.		taken from observations and corrected for calibration of instrument.
Mean density	σ	from density chart.
$\mathrm{V}i$ obs. m.p.h.		mean observed airspeed.
$\mathrm{V}i$ corrected	$\mathrm{V}i$	corrected for calibration of airspeed indicator and lag in suspended static system.

V	$\frac{Vi}{\sqrt{\sigma}}$	true speed.
Vc		height drop in m.p.h.
Vc/V	$\sin\gamma$	
γ=angle of glide.		
Inclinometer reading		mean of observed inclinometer readings.
$\alpha - \gamma$		inclination of chord to horizontal corrected for mean wing incidence.
α		angle of incidence.
k_{ϵ}		total lift coeff. $\frac{W \cos\gamma}{\rho_0 S Vi^2}$ from equation (1)
$k_D + k_R$		total drag coeff. $\frac{W \sin\gamma}{\rho_0 S Vi^2}$ from equation (2)

ACCURACY.

There are two serious errors that can be introduced into "propeller stopped" glides. The first is in the height drop Vc due to up and down currents, which cause an error in γ and hence in the drag coefficient and α (which is determined as $(\alpha - \gamma) + \gamma$, the presence of up and down currents does not effect $\alpha - \gamma$, since this is the measured inclination of the aeroplane to the horizontal). To avoid up and down currents the experiments should be done in good weather and the glides carried out over a considerable height range to minimise the effect of small local disturbances.

The second source of error is the difficulty of determining the true speed especially at low speeds, but this has been practically eliminated by the use of the latest type of suspended static head, which has large bore tube and large holes in the static head, in order to reduce the lag to a very small amount.

GLIDES AT LARGE ANGLES OF INCIDENCE.

To enable the pilot to fly at large angles of incidence, a ballast tank is sometimes fitted in the rear bay of the fuselage, into which a known quantity of water is run back from a tank in the observer's cockpit, before the commencement of the glide. If the aeroplane is flown at the same angle of incidence in each case, the effect of adding a weight on the tail is simply to increase the tail lift, the wing lift being unaltered. As the total k_L is equal to $\frac{\text{total weight}}{\rho_0 S Vi^2}$ $\cos\gamma$ the value of k_L plotted against incidence will not give a consistent curve, but will be scattered an amount depending on the variation in the centre of gravity of the aeroplane. A standard centre of gravity is therefore chosen (x=some definite distance from the leading edge of the lower chord at root) and a correction is applied to the weight on the tail in each experiment to bring the centre of gravity to the standard position. The corresponding correction to the drag curve involves a knowledge of the tail incidence and cannot therefore in general be applied.

2.—AN APPLICATION OF THE CINEMA CAMERA TO FULL SCALE EXPERIMENTS.

By the use of a Cinema Camera it is possible to define the axes of an aeroplane in space. The camera is mounted on an aeroplane which flies straight towards the sun and the manœuvring is carried out by a following aeroplane.

DETAILS OF THE CAMERA.

The Cinema Camera, which is made to slide into a wooden box and held in place by three positioning screws, is suspended from a rigid support mounted on an ordinary scarf ring. This mounting is easily transferable from one aeroplane to another; it is only necessary to remove the existing scarf ring and bolt the one with cinema mounting in its place. (Fig. 1.)

On the earlier camera, the power was transmitted through a flexible drive from a windmill situated above the mounting and the speed of the film was dependent on the speed of the aeroplane. Now the cinema is driven by a small electric motor enclosed in the camera and the speed of the film can be varied through a range of from 14 to 36 pictures per second. A switch is fitted on the right-hand side of the box, while on the left-hand side is situated the sight, and the whole is controlled by handles on the base and side.

The camera is free to be tilted or pitched up and down, and turned or yawed sideways; with this method of attachment it is possible to record the pitch and yaw angles by means of indicators which move across the top and bottom edges of the film, these are photographed when the camera is in use. Attached to the pitch indicator is a small chain which runs over a pulley and is attached by a system of levers to a pin offset from the point of suspension.

A similar chain attaches the yaw indicator to a bell crank lever mounted on the camera, this lever is operated by a Bowden Control fixed to the rotating table on the mounting.

A wind driven escapement clock is mounted on one of the uprights of the main support. This makes contact every half second, and operates a solenoid situated in the camera, which in turn throws a small indicator on to the edge of the film so that it is photographed. By counting the number of pictures with the time mark showing and those when it is off, the speed of the film can be measured in number of pictures per second.

CALIBRATIONS.

The base of the scarf ring is placed horizontal; in this position the camera swings approximately horizontal, and a zero is taken. The camera is then tilted or pitched through ±5°, ±10°, ±15°, ±20° and records taken of each position.

The camera is next placed at right angles to the support and a zero taken; it is then yawed through ±5°, ±10°, ±15°, etc., and records taken. The films are then measured up, and the distance from the edge of the film to the indicator marks are noted. These distances are plotted against the angles at which the camera was calibrated.

PROCEDURE DURING FLIGHT.

Flights should be carried out on days when there is good light, and be high enough to be clear of bumps. The aeroplane carrying the cinema camera should fly straight towards the sun, and at such an angle of incidence, that the base of the scarf ring is horizontal (as this is the position of calibration). The following aeroplane makes a signal, by flashing a mirror if one is fitted or by some pre-arranged movement of the wings. The cinema operator then trains the camera on to the following aeroplane and starts the electric motor. The following aeroplane then performs some evolution, as turns to port or starboard, stalled turns or rolls. It is essential to have as large a picture of the aeroplane as possible The best distance is obtained by trial, but it is possible for the cinema operator to signal to the pilot of the following aeroplane to take position nearer or further away

MEASUREMENT OF FILMS.

The film is either measured direct, by passing over a glass plate illuminated from beneath, and having a line engraved at right angles to the direction of the film (all measurements being made with respect to this line), or the film may be passed through an enlarging lantern and the measurements taken as before. (Fig. 3.)

The data obtained are :—

(1) The angles μ_2, μ_2 and μ_3 which the projections upon the plane of the film of the normal axes ox, oy, oz, in the aeroplane make with the direction of the length of the film.

(2) ψ_o and θ_o the angles of yaw and pitch of the camera relative to the aeroplane it is mounted in.

(3) The number of pictures per second. Convenient axes in the following aeroplane are :—

ox—backwards from C.G. to tail.
oy—along port wing.
oz—upwards.

The axes should be chosen at right angles if possible ; on a biplane the following can be used, "ox" from propeller boss to fin or rudder, "oy" along the leading or trailing edge of the port wing, "oz" from wing tip to wing tip if the wings are of equal span, and the tips clearly defined, or the struts if at right angles to the wing, or the rudder post. If the axes are not at right angles this must be taken into account when reducing the results.

ψ_o and θ_o are taken from the calibration curve by measuring the distance of the indicator marks from the edge of the film.

METHOD OF REDUCTION.

In the early work, the reductions were carried out by mathematical analysis. This was a long and tedious method, and was soon superseded by a more mechanical one of direct measurement on a graduated sphere. (Fig. 5.)

The sphere is a hollow casting of bronze about 12 in. in diameter, supported at the bottom, and graduated by meridians running between two poles which are on a horizontal diameter. The meridians are 1° apart round the centre and at less frequent intervals near the poles.

A flat bronze ring with a groove running round it is supported on two arms from the support at the base. This just fits around the sphere and is capable of being tilted. It is mounted on adjustable pivots with its axis horizontal and intersects the meridians joining the poles at right angles. This is called the " Yaw Circle " or " equator."

A spherical triangle *xyz*, with sides and angles of 90°, which is made of thin steel strips and graduated in degrees fits closely over the sphere.

A spherical set square, called the Pitch triangle, slides in the groove round the Yaw Circle and is graduated along its side perpendicular to the Yaw Circle ; this is used for measuring the angle of pitch

TO USE THE SPHERE.

Let SS_1 be the line of sight of the observer. Then the angles μ_1 μ_2 and μ_3 are the angles that the planes containing the *x*, *y* and *z* axes respectively through the line of sight, make with the vertical plane through the line of sight. Therefore, the spherical triangle can be placed with its vertices on the meridians, so that the lines joining the centre to the vertices define the axes of the aeroplane in space. (Fig. 2.)

If possible use axes that are at right angles, then a right angled spherical triangle is used. If, however, this is not possible, a spherical triangle must be made so that the lines from the centre to the vertices still define the measured axes of the aeroplane

The axes used: *oz* the upward vertical, *ox* horizontal in plane of symmetry of observing aeroplane measured towards the tail, *oy* horizontal along port wing of observing aeroplane, are opposite to the conventional axis ; this enables the spherical triangle to be placed on the top of the sphere.

The position of the aeroplane is then obtained from these by :—

(1) A yaw ψ about *oz*.
(2) A pitch θ about the new *oy*.
(3) A roll ϕ about the final *ox*.

Yaw is about the vertical axes in space and can be measured as the displacement of the points where the desired *zy* plane cuts the Yaw Circle.

Pitch is measured from the desired *x* vertex perpendicular to the Yaw Circle.

Roll is the displacement of the *y* vertex from the Yaw Circle, measured in the *zy* plane.

If the camera is pitched up or down, the Yaw Circle can be pitched an equal amount, thus giving an artificial horizontal plane. Thus, the position of the spherical triangle relative to the tilted Yaw Circle, still corresponds exactly with the position of the three axes of the aeroplane relative to the true horizontal. The Yaw of the cinema camera is included in the yaw of the following aeroplane, as read off from the Yaw Circle scale, the reading $\psi_0 + \psi$.

Having found the angles of pitch, yaw and roll, these can be plotted against a time scale, from this the rates of pitch, yaw and roll may be determined.

This method may be used for comparing the manœuvreability of different aeroplanes, or for comparison of different pilots manœuvring the same aeroplane.

3.—FULL SCALE EXPERIMENTS ON SPINNING.

One of the important aspects of Full Scale experiments on spinning, is the difficulty under which the observer has to work. Many of those who are very capable when doing ordinary work, are entirely unsuited for these experiments. In some aeroplanes the conditions are very bad; take the B.E.2c, when the floor boards of the observer's cockpit contained the accelerometer there was no leg room left, and the observer had to sit cross-legged or tailor fashion on the seat. This was unpleasant enough in itself, for the B.E. had a very poor climb, but during the spin it was most uncomfortable and writing under these circumstances was a task.

In a Bristol one has much more room, but on the other hand, the observer is much further behind the centre of gravity and suffers from more " g " effect. A spin of five turns takes 12—15 seconds. During this time the observer has to record the time for a given height drop, note the lateral bubble, and take as many readings as possible of the side-slip indicators and longitudinal bubble. Doing this work, whilst being spun round under a " g " effect from 2—3 times normal, is entirely different from any other type of full scale observation. The effect on the head is quite different if looking out, especially along the lower wing or straight ahead, but, with the head in the cockpit all the time, taking readings and not bothering about direction, the conditions are very exacting, and it is hard work to tell what the altitude of the aeroplane is, especially when the pilot unstalls and dives out of the spin.

The Bristol Fighter which is now used for investigation of spinning, is fitted with the latest types of instruments, to record the following, the accelerations in the normal and lateral directions, the position of all control surfaces during the spin, the angle of sideslip, and also a lateral and longitudinal bubble. A description of, and the method of calibrating these instruments is given later.

It is important to know the position of the control surfaces, during the spin, as considerable change in the angle of incidence, and rate of turn can be obtained by varying the position of the controls, *i.e.*, on a B.E.2c, the range covered gives approximately 25° incidence and 1.55 radians per second rate of roll with stick back and central, and rudder three-quarters full over, while we get 40° incidence and 2.35 radians per second with stick back and fully over in opposite direction to spin, and the rudder fully over.

INSTRUMENTS USED ON FULL SCALE WORK.

A Control Movement Recorder is mounted on the rudder, each elevator and each aileron. They consist of a constant-speed-drum driven by a windmill, through the medium of an electrical clutch and centrifugal governors, and carrying a photographic film on which is recorded the movement of a tilting mirror by means of a simple optical system. A small rectangle is cut out of the bottom half of the recording mirror and a fixed mirror is placed in this opening. This gives a zero line through the centre of the film. The pivot arm of the tilting mirror is connected to the control surface through a system of levers. For stalled work, the instruments are mounted on the under surface, if mounted on the top surface, they are so much shielded that there is not enough flow to drive the windmill fast enough.

Sideslip Indicators are situated on outriggers fixed near the port and starboard inner struts—these are of the weather vane type—the vane moves round a graduated scale which has been previously calibrated in the wind channel.

Two recording accelerometers are mounted as near the C.G. position as possible. The pilot's dash-board has been cut away to fit these instruments. These instruments are the normal R.A.E. type, and consist of a fine semicircular glass fibre mounted on an adjustable base and reflect the light from a small electric bulb through suitable lenses on to a film drum which is driven by clockwork.

The accelerometer used for measuring the normal forces is mounted with its fibre horizontal and has a range of —2g to +5g. The one for measuring the lateral force is mounted with its fibre vertical. Owing to the very small forces which have to be measured a very sensitive glass fibre was specially drawn out to about 0.0005 in. diameter and carefully mounted on a base of about 1.4 in. diameter; this gives a range of about —.15g to +.15g for the full width of the film.

A Standard Lateral Bubble is placed in the cockpit, also an adjustable sensitive inclinometer is mounted on the longitudinal axis of the aeroplane. A small-lag thermometer is fitted.

A sensitive aneroid with graduations every 20 ft., and a stop watch are carried by the observer.

OBSERVATIONS TAKEN DURING AN EXPERIMENT.

Prior to the experiment proper, the pilot flies straight without sideslip and the zero of the sideslip indicator is noted, at the same time the sensitive accelerometer is run to get the lateral zero.

When sufficient height is attained, the accelerometers are started and the master switch to the electrical clutches in the control movement recorders is switched on—[a tapping key which causes a break in the recorded line in all the instruments is placed in the circuit, this synchronises the time]—the pilot then stalls the machine and places the controls in a pre-arranged order, viz. :—

(*a*) Stick right back, full rudder and stick right over in direction of spin.
(*b*) Stick right back, full rudder and stick right over against direction of spin.
(*c*) Stick right back, full rudder and stick central.
(*d*) Stick right back, rudder three-quarters over.

As soon as the spin is properly developed, the stop-watch is started, the height noted and the synchronising key is pressed simultaneously, the watch is stopped and the key again pressed after 2,000 or 3,000 ft. height drop. During this height drop, as many readings as possible are taken of the sideslip indicator, the lateral and longitudinal bubbles. The pilot usually takes the time of five complete turns and maintains the spin until the observer has obtained the readings he requires. He then flies straight without sideslip and check calibrations are taken to observe if any zero change has occurred. An average of three spins of eight to ten complete turns is carried out on each flight.

After the flight, the control movement recorders are calibrated by placing the control surfaces at definite angles and taking a record on the film. The distance of this line from the zero line against the angle of the controlling surface to some datum line is then plotted.

The normal force accelerometer is calibrated by placing it on its front, side and back (*i.e.*, —1G, O, +1G) and taking records in these positions. This instrument has been calibrated on a whirling arm and the scale is found to be linear beyond the range obtained when spinning. The lateral force accelerometer is calibrated by tilting from the vertical (or zero) through ±5°, ±10°, ±°15 [the sine of the angle gives the gravity force] and taking records in these positions.

The film records are numbered, then developed and measured up in an enlarging apparatus, from these measurements the positions of the control surfaces and the normal and lateral forces are obtained.

OBSERVATIONS.

(1) *Accelerations.*—(*a*) The normal accelerometer measures the acceleration "ag" in a direction normal to the chord.

(*b*) The longitudinal bubble gives the direction of the resultant acceleration in the plane ozx let i=reading of bubble.

(*c*) The lateral accelerometer measures the acceleration "bg" to port or starboard dependent on direction of spin.

The three measurements a, b and i suffice to give the resultant acceleration in magnitude and direction.

(2) *Velocities.*—The measurements taken give: (*a*) the vertical velocity, h; (*b*) the angle of sideslip β; and (*c*) the rate of rotation ω. (Fig. 4.)

If we take wind axes and choose for convenience :—

x—backwards.
y—to port.
z—upwards.

Referring to the sphere, looked at from above, $o\xi$, $o\eta$, $o\zeta$ are axes fixed in space. Then $\xi_o \; \xi_1$ is a horizontal circle, and $\zeta\,\xi$, $\zeta\;\eta$, etc., are vertical circles, all through the centre of the sphere.

Then we get a pitch θ about $O\eta$, and a roll ϕ about ox. If there is no sideslip, the velocity would be along xo. If, however, there is sideslip β to port, the resultant velocity will be along Po, where the arc xP is β.

The resultant angular velocity ω or rate of spin is about the vertical $o\zeta$.

The centre of the spin is in a direction perpendicular to the direction of horizontal velocity, along OR.

The acceleration due to the spin is ωW in the direction OR, where W= horizontal velocity.

The air forces must provide the following accelerations :—

(1) g along $o\zeta$.
(2) ωW along OR.

The accelerometer readings must be equivalent to these components. The observations give us a, b, i, β, ω, h and from these we have to deduce V, α , θ , ϕ.

This is done by resolving the accelerations and the velocities in the following way.

ACCELERATIONS.

Resolve along Ox, Oy and Oz respectively.

The observations give $ag\sec i\sin(\alpha+i)$ along Ox.
$-bg$ along Oy
$ag\sec i\cos(\alpha+i)$ along Oz.

These must be equivalent to g along $O\zeta$.
ωW along OR.

The expressions obtained by equating these may be reduced to the form :—

$$\omega^2W^2=g^2(a^2\sec^2 i+b^2-1)=g^2(k^2-1) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$$ag\sec i\sin(\alpha+i)=g\sin\theta-\omega W\cos\theta\sin\delta \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

$$-bg=g\sin\phi\cos\theta+\omega W(-\cos\phi\cos\delta+\sin\phi\sin\delta\sin\theta) \quad (3)$$

VELOCITIES.

The resultant velocity V along PO is equivalent to the horizontal velocity (W) along QO and the vertical velocity (h) along ζO.

These are resolved along $O\xi$, $O\eta$ and $O\zeta$ respectively.

The equations obtained are :—

$$W\cos\delta=V(\cos\theta\cos\beta+\sin\theta\sin\beta\sin\phi) \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

$$-W\sin\delta=-V\cos\phi\sin\beta \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

$$h=V(\sin\theta\cos\beta-\cos\theta\sin\beta\sin\phi) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

Squaring these equations and adding we should get

$$\underline{V^2=h^2+W^2} \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

which may be used in place of one of the above.

TO DEDUCE W, V, θ, ϕ AND α.

Equation (1) gives $W^2=\dfrac{g^2(a^2\sec^2 i+b^2-1)}{\omega^2}$

This gives W

Equation (7) now gives V.

Eliminate $\sin\delta$ and $\cos\delta$ from equation (3) by means of (4) and (5).

This gives

$$bg=\cos\theta(\omega V\cos\phi\cos\beta-g\sin\phi)$$

also (6) = $\dfrac{h}{V}=\sin\theta\cos\beta-\cos\theta\sin\beta\sin\phi$.

These two equations give θ, ϕ in terms of known quantities. The first can be written

$$b \sin \lambda = \cos \theta \cos (\phi + \lambda)$$

$$\text{where } \tan \lambda = \frac{g}{\omega V} \sec \beta.$$

Then α is given by (2) when $\sin \delta$ has been eliminated by means of (5), viz.

$$ag \sec i \sin (\alpha + i) = g \sin \theta - \omega V \cos \theta \cos \phi \sin \beta.$$

The radius of spin $r = \frac{W}{\omega}$

Before any discussion takes place, I wish to express my appreciation of the Air Ministry, for permitting me to read this paper, also for allowing the use of lantern slides and the privilege of reproducing the illustrations.