ON THE RELATIVE MOTION OF THE EARTH'S AXIS OF FIGURE AND THE POLE OF ROTATION

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Abstract. The motion of the Earth's axis of inertia has been derived, taking elastic deformation into account, from the polar coordinates determined by the BIH for the period from 1962.0 to 1975.0. Characteristics of the motion of both the pole of inertia and the pole of rotation have been examined. The secular displacement of these poles relative to the pole defined by the low order harmonics C_{21}, S_{21} determined from observations of satellites seems to confirm that the inertial reference axis has an apparent wandering motion within the deformable Earth.

1. INTRODUCTION

The fact that the Earth is a deformable body has been well known for a long time. As a result of several forces of varied nature acting on the Earth's mass (oceans and atmosphere, centrifugal forces, earthquakes, etc.), one must expect that the inertia tensor J is time dependent. Consequently also the Earth's axes of inertia, whose direction cosines are given by the characteristic system

$$(J - \lambda I)\,(\xi, -\eta, \zeta)^{-1} = 0 \qquad (1)$$

where I is the indentity matrix, cannot be considered as constant with respect to the "fixed" reference frame.

As outlined by Gaposchkin (1968) and Melchior (1972) the position of the Earth's axes of inertia could be determined by very exact artificial satellite observations, since the components of the inertia tensor are related to coefficients of the harmonics of the geopotential.

Unfortunately, as shown in table 1, the observed values of the tesseral low harmonics C_{21}, S_{21}, given by satellite observations are at present incapable of providing definitive information about the motion of the inertial pole since their values are comparable with observational errors. Moreover, we would like to have pole positions every

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five days in order to compare them with, for instance, the position of the pole of rotation. So we are obliged to try to calculate the ξ and η coordinates of the inertial pole directly from the x and y coordinates of the instantaneous pole of rotation. Attempts to deduce the motion of the inertial pole from the Eulerian equation of motion by taking into account the role of the Earth's deformations have been made by C. Dramba (1964, 1976). The difficulty of obtaining reliable results arises from the need to use accurate astronomical observations and appropriate approximations in the equations of motion.

Table 1. Unnormalized geopotential coefficients and components of inertia tensor ($Ma^2 = 1$)

	GEM 6	SE III	GRIM 2
$10^6.C_{20}$	-1082.6283	-1082.6370	-1082.6350
$10^6.C_{21}$	- 0.0012		
$10^6.S_{21}$	- 0.0041		
$10^6.C_{22}$	1.5654	1.5362	1.6059
$10^6.S_{22}$	- 0.8961	- 0.8815	- 0.8807
A	0.329697	0.329699	0.329699
B	0.329703	0.329706	0.329706
C	0.330783	0.330785	0.330785
$10^6.D$	- 0.0012		
$10^6.E$	- 0.0041		
$10^6.F$	- 1.7922	- 1.7630	- 1.7614

2. EQUATIONS OF MOTION

The motion of the free nutation of the deformable Earth about its centre of mass may be described by the Liouville equation (Munk and MacDonald, 1960)

$$\frac{d}{dt}\{(J.\bar{\omega}) + \bar{h}\} + \bar{\omega} \times \{(J.\bar{\omega}) + \bar{h}\} = 0 \qquad (2)$$

where $\bar{\omega}$ is the angular velocity of the reference frame x_i, $\bar{h}$ the relative angular momentum and J the inertia tensor.

Since the axis of instantaneous rotation is close to the axis of figure a perturbation scheme can be used to find approximate solutions of equation (2) (Volterra, 1895, 1898). For this purpose let us consider the right hand reference frame with x_1 axis along the Greenwich meridian, x_2 axis along 90^o East and x_3 axis pointing to the

CIO. In this system we put

$$\omega_1 = \Omega x \qquad \omega_2 = -\Omega y \qquad \omega_3 = (1 + m)\,\Omega$$

Conventionally x and y are the coordinates of the instantaneous rotation pole, Ω is the mean angular velocity of the Earth, 2π radians per sidereal day, and m is the relative change in the length of the day. After neglecting the term of order 10^{-9} and substituting the coordinates of the pole of inertia given by (1), namely,

$$\lambda = C$$
$$\xi = (-D(C-B) + EF)/((C-A)(C-B))$$
$$\eta = (E(C-A) - DF)/((C-A)(C-B))$$

according to the assumed approximation, equation (2) is reduced to the linearized system

$$\frac{\sigma_1}{\Omega} \cdot \frac{dx}{dt} - y + \eta = \varepsilon\,(\xi - x)\ ;\ \frac{\sigma_2}{\Omega} \cdot \frac{dy}{dt} + x - \xi = \varepsilon\,(y - \eta) \qquad (3)$$

where $\sigma_1 = A/(C-B)$, $\sigma_2 = B/(C-A)$ and $\varepsilon = F/(C-A) = F/(C-B)$ is a corrective term.

3. COORDINATES OF THE INERTIAL POLE FROM BIH DATA

Equations (3) are the differential equations of the polar motion. If one assumes the functions ξ and η to be known, solutions of (3) can easily be found. Vice-versa we may consider (3) as an algebraic system for the unknowns ξ and η and we can solve it by using the observed values of x and y. Dramba and Stanila (1969), using nearly similar equations, followed this procedure and resolved the systems by the least-squares method assuming σ_1, σ_2 and ε to be also unknown. In our opinion, however, the instability of such a system can cause large errors. On the other hand, parameters σ_1, σ_2 and ε vary more slowly and can be regarded as constants. From table 1 we have derived

$$\sigma_1 = 305.437 \pm 0.010 \quad \sigma_2 = 303.680 \pm 0.032 \quad \varepsilon = -0.00164 \pm 0.00007$$

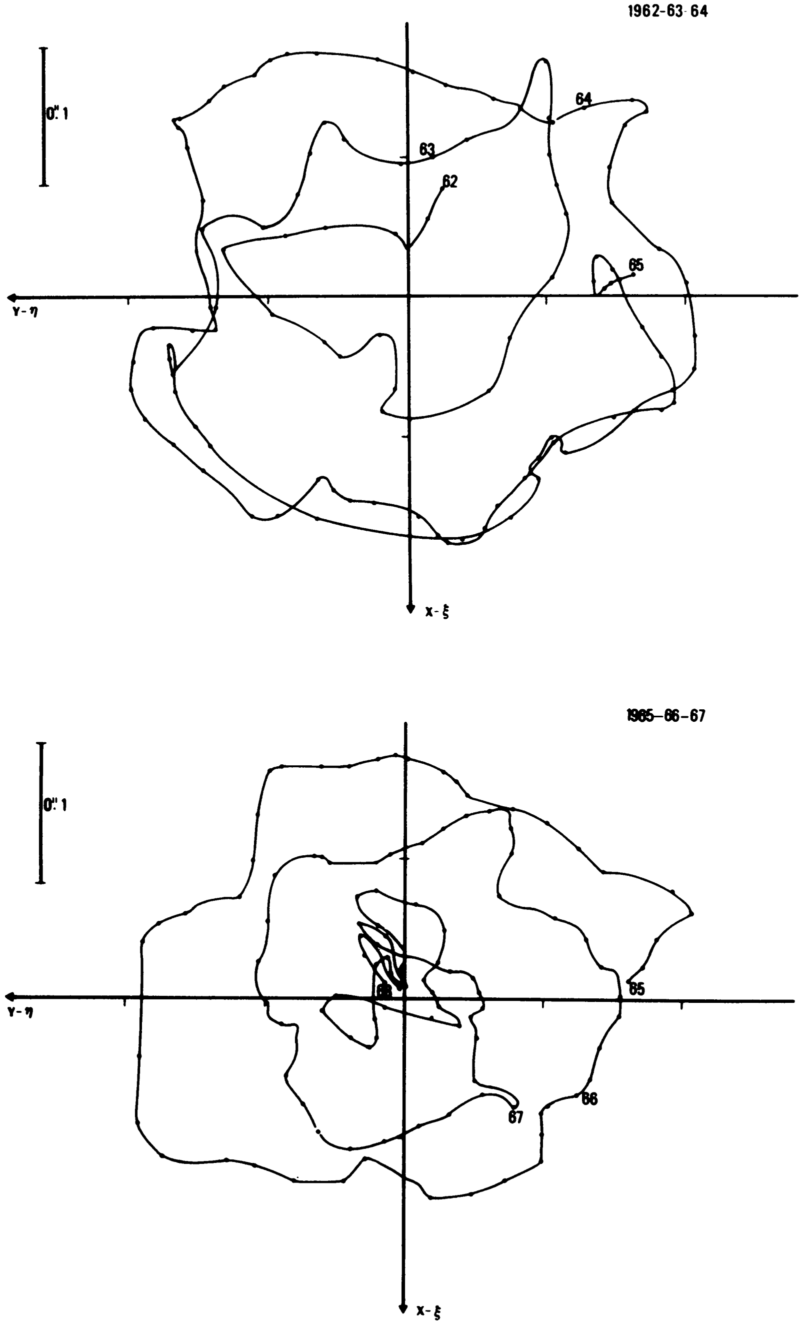
and we have solved each single equation by means of the iterative method

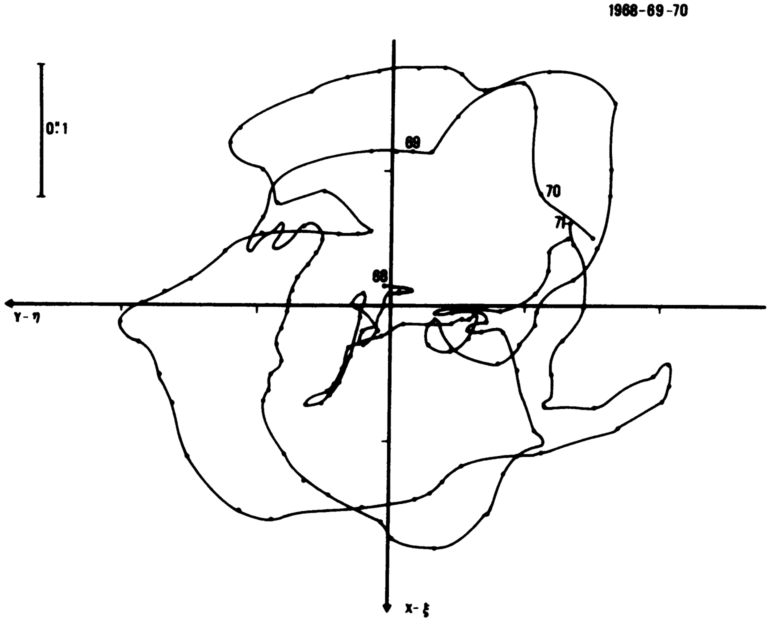
$$\xi = \xi_o - \varepsilon(y - \eta_o)\ ;\ \eta = \eta_o - \varepsilon(x - \xi_o) \qquad (4)$$

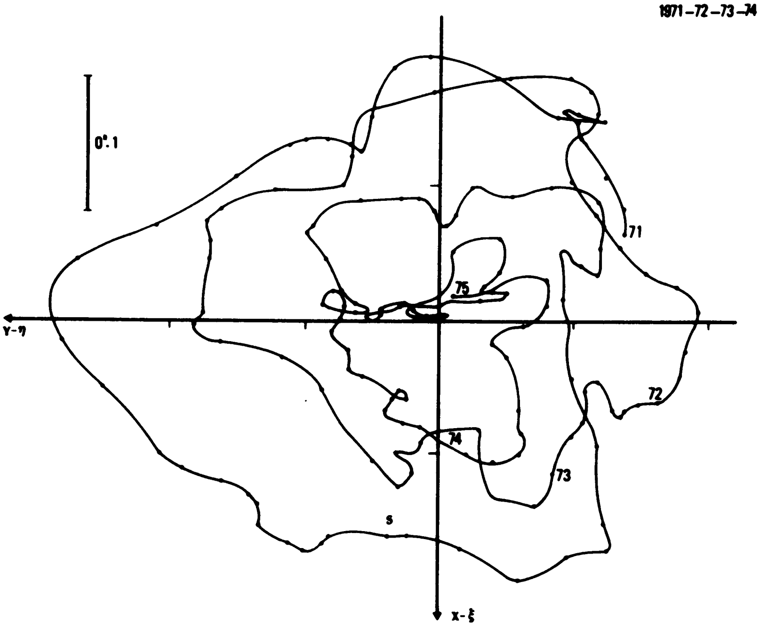
where ξ_o and η_o are the solution of (3) for $\varepsilon = 0$.

By using equations (4) the ξ, η coordinates have been derived from the smoothed x,y coordinates of the rotation pole. The latter were supplied by the BIH every five days for the period 1962.0 - 1975.0. The derivatives dx/dt and dy/dt were computed using the usual five-point Langrangian differentiation formula. The wobbles $P(x - \xi,\ y - \eta)$ of the instantaneous rotation pole with respect to the instantaneous inertial pole are plotted in Fig. 1. Irregular variations sometimes occur when $x - \xi$ and $y - \eta$ are small. This could result from errors inherent in the method, but could alterna-

Fig. 1. Wobbles of the instantaneous rotation pole with respect to the instantaneous inertial pole.




1968-69-70
0".1
69
70
71
68
Y - η
X - ξ


1971-72-73-74
0".1
71
75
Y-η
72
74
73
S
X-ξ

tively be a physical consequence of the fact that $x = \xi$ and $y = \eta$.

Spectral analyses have been carried out on the series F(t) of (x,y), (ξ, η) and ($x - \xi$, $y - \eta$) coordinates by means of the conditional equations

$$F(t) = A_a \sin(2\pi t/P_a + F_a) + A_c \sin(2\pi t/B_c + F_c).$$

Two principal periods were emphasized, namely the annual (368 days) and the Chandler period (432 days).

Table 2 Periodical components of the rotation and inertial poles

	P_a	P_c	A_a	F_a	A_c	F_c
x	368^d	432^d	0".103	209°	0".126	346°
y	368	432	0.091	302	0.133	76
ξ	368	432	0.027	199	0.032	346
η	368	432	0.008	300	0.044	77
$x-\xi$	368	432	0.074	212	0.094	346
$y-\eta$	368	432	0.084	301	0.089	76

The results, given in Table 2, are in good agreement with those found by other authors.

Finally a 6-year running filter was used to derive mean values of the coordinates, free from both annual and Chandler components, at intervals of 1 year; the results are shown in Table 3. It can be seen that both inertia and rotation poles have a similar secular motion. This result seems to confirm the existence of a secular wandering motion of the Earth's rotation axis; but, on the other hand, it could be only an immediate consequence of the equation of motion.

If the observed secular motion of the pole of rotation were really due to the secular drift of the pole of inertia, the results obtained by astronomical observations would be comparable with those derived by satellite observations. However, such a comparison today gives us poor results because, as has been said, few and inaccurate data are generally available. The comparison of the mean BIH pole of inertia for the epoch 1968 with the mean pole derived from C_{21} and S_{21} by GEM 6 (Smith et al., 1976) and GEM 8 (Wagner et al., 1976), given in table 4, shows that the derived secular variations are in very poor agreement.

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Table 3. Annual means of the coordinates of the rotation and inertial poles after 6 year running means.

Year	x	y	ξ	η	x-ξ	y-η
1965	-0".0031	0".2379	-0".0029	0".2378	-0".0001	0".0001
1966	- 0014	2364	- 0011	2363	- 0003	0001
1967	0024	2380	0020	2376	0004	0004
1968	0035	2372	0039	2365	- 0004	0007
1969	0064	2399	0071	2393	- 0007	0007
1970	0101	2418	0110	2416	- 0009	0002
1971	0142	2457	0145	2456	- 0003	0001

Table 4

	ξ	η
GEM 6	+ 0".213	- 0".725
GEM 8	+ 0".023	+ 0".069
BIH	+ 0".006	+ 0".240

The values of GEM 6 are one order of magnitude higher than those of GEM 8 while the latter are in but moderate agreement with the data derived from astronomical observations. So only a drastic improvement in the accuracy of satellite observations will confirm the existence or not of a secular trend in the position of the inertial reference axis.

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DISCUSSION

J.D. Mulholland: How can you separate the "secular" motion of the pole from secular errors in the orbit of the satellite?

E. Proverbio: We cannot.