A NOTE ON THE REPRESENTATION THEORY OF THE HECKE ALGEBRA OF TYPE F_4

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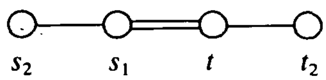
Introduction. In [**4**] Dipper and James investigated the representation theory of Hecke algebras of type B_n, $H(B_n)$. Using the results in [**4**] and exploiting the fact that the Hecke algebra of type F_4, denoted by $H(W)$, contains two copies of $H(B_3)$ certain right ideals of $H(W)$ will be constructed in this paper. These right ideals will be proved to be irreducible on the assumption that $H(W)$ is semisimple.

$H(W)$ will be defined as a vector space over a field K of arbitrary characteristic and its definition will depend on q and Q where $q, Q \in K$. From the right ideals constructed in this paper we will get some 8-dimensional and some 9-dimensional matrix representations of $H(W)$. The characters of these representations agree with the characters given in Geck [**5**]. In [**5**], Geck calculated the characters of the generic Hecke algebra of type F_4 by constructing matrix representations of degree at most 8. His representing matrices for the generators involve denominators. The method used in this paper to construct representations is different from Geck's and the representing matrices obtained here do not involve any denominators.

In the special case where char $K = 0$ and $q = 1 = Q$, the right ideals constructed in this paper will give all the 8-dimensional and 9-dimensional ordinary irreducible representations of the Weyl group of type F_4, the character table of which is given in [**1**, p. 413].

1. The Weyl group of type F_4. The Weyl group of type F_4, which we denote by W, is generated by t_2, t, s_1, s_2 where

$$t^2 = 1 = t_2^2 = s_1^2 = s_2^2,$$
$$t_2 s_i = s_i t_2 \text{ for } i = 1, 2, \quad ts_2 = s_2 t \quad \text{and} \quad (t_2 t)^3 = 1 = (s_1 s_2)^3 = (ts_1)^4.$$



In particular W has order 1152 and has two parabolic subgroups which are isomorphic to the Weyl group of type B_3, namely W_1 which is generated by $\{t, s_1, s_2\}$ and W_2 which is generated by $\{s_1, t, t_2\}$.

1.1. Remark. It is easy to see that there exists an automorphism ρ of W where $\rho(s_1) = t$, $\rho(t) = s_1$, $\rho(s_2) = t_2$ and $\rho(t_2) = s_2$.

1.2. Definition. Each element w of W is a product of the generators s_1, s_2, t, t_2. The length of w, $\ell(w)$, is defined to be the minimal length of any expression for w in this form. Hence if $w \in W$, then $w = v_1 v_2 \ldots v_\ell$ where each v_i belongs to $\{s_2, s_1, t, t_2\}$ and $\ell = \ell(w)$. Such an expression for w is called a reduced expression.

1.3. Definition. If H is a subgroup of a group G, we call Hg a right coset of H in G.

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If $\ell(hg) = \ell(h) + \ell(g)$ for all $h \in H$, we say that g is a distinguished right coset representative of H in G. Distinguished and coset representatives are defined in a similar way.

1.4. Lemma. *The set S given below is a complete set of distinguished right coset representatives of W_1 in W.*

$$\begin{aligned} S = \{s^* = {} & t_2ts_1tt_2s_2s_1ts_1s_2t_2ts_1tt_2,\ t_2ts_1tt_2s_2s_1ts_1s_2t_2ts_1t,\ t_2ts_1tt_2s_2s_1ts_1s_2t_2ts_1,\\ & t_2ts_1tt_2s_2s_1ts_1s_2t_2t,\ t_2ts_1tt_2s_2s_1ts_1t_2t,\ t_2ts_1tt_2s_2s_1ts_1s_2t_2,\ t_2ts_1tt_2s_2s_1ts_1s_2,\\ & t_2ts_1tt_2s_2s_1ts_1t_2,\ t_2ts_1tt_2s_2s_1ts_1,\ t_2ts_1tt_2s_2s_1tt_2,\ t_2ts_1ts_2s_1tt_2,\ t_2ts_1ts_2s_1t,\\ & t_2ts_1tt_2s_2s_1t,\ t_2ts_1tt_2s_2s_1,\ t_2ts_1tt_2s_2,\ t_2ts_1ts_2s_1,\ t_2ts_1tt_2,\ t_2ts_1ts_2,\ t_2ts_1t,\\ & t_2ts_1s_2,\ t_2ts_1,\ t_2t,\ t_2,\ 1\}. \end{aligned}$$

Proof. First note that $u = ts_1ts_1s_2s_1ts_1s_2t_2ts_1tt_2s_2s_1ts_1s_2t_2ts_1tt_2$ is the longest element in W with $\ell(u) = 24$, hence the expression given above for u is a reduced expression. Note also that $v = ts_1ts_1s_2s_1ts_1s_2$ is the longest element in W_1. The proof of the lemma now follows easily once we notice that $u = vs^*$.

1.5. Remark. In [**4**, 2.2], a canonical way for listing the elements of W_1 is given. Combining this result with Lemma 1.4, we get a canonical way for listing the elements of W in reduced form.

1.6. Remark. A complete set of distinguished left coset representatives of W_1 in W can easily be deduced from symmetry. Moreover, using the automorphism ρ of W given in Remark 1.1, we can easily find complete sets of distinguished right or left coset representatives of W_2 in W.

2. The Hecke algebra of type F_4.

2.1. Definition. Let K be a field and let q and Q be non-zero elements of K. The Hecke algebra of type F_4, $H(W)$, over K with respect to q and Q is defined to be a vector space over K with basis $\{T_w : w \in W\}$ with the following multiplication structure:

T_1 is the multiplicative identity of H;

$T_xT_x = q + (q-1)T_x$ if $x \in \{s_1, s_2\}$;

$T_yT_y = Q + (Q-1)T_y$ if $y \in \{t, t_2\}$;

if $w = v_1v_2 \ldots v_\ell$ is a reduced expression for $w \in W$ where each v_i belongs to $\{s_2, s_1, t, t_2\}$, then $T_w = T_{v_1}T_{v_2} \ldots T_{v_\ell}$.

(Here and later we write r to denote rT_1 where $r \in K$.)

2.2. Remarks.

(i) Alternatively, the multiplication structure of $H(W)$ can be defined using the defining relations given for example in [**5**, p. 51].

(ii) If we set $q = Q = 1$ in Definition 2.1 we recover the group algebra of W.

2.3. Remark. We can form the Hecke algebras $H(W_1)$ and $H(W_2)$ in the obvious way. Then $H(W_1)$ and $H(W_2)$ are subalgebras of $H(W)$ and they are both isomorphic to the Hecke algebra of type B_3. The representation theory of Hecke algebras of type B_n is given in [**4**].

2.4. REMARK. There is an automorphism θ of $H(W)$, the action of which on the generators is given by

$$\theta(T_x) = (q-1) - T_x, \qquad x \in \{s_1, s_2\},$$
$$\theta(T_y) = (Q-1) - T_y, \qquad y \in \{t, t_2\}.$$

The ordinary irreducible representations of W_1 (and of W_2) are indexed by pairs of partitions $(\alpha)(\beta)$ where α is a partition of a, β is a partition of b $(a+b=3)$. (In [**4**, 5.6] it is shown that the ordinary irreducible representations of $H(B_3)$ are indexed in the same way.) We therefore have that the ordinary irreducible representations of W_1 are indexed by

$$\phi_1:(3)(0),\ \phi_2:(21)(0),\ \phi_3:(1^3)(0),\ \phi_4:(2)(1),\ \phi_5:(1^2)(1),$$
$$\phi_6:(1)(2),\ \phi_7:(1)(1^2),\ \phi_8:(0)(3),\ \phi_9:(0)(21),\ \phi_{10}:(0)(1^3)$$

where for $i = 1, \ldots, 10$, ϕ_i denotes the character of the irreducible representation which corresponds to the given partition.

Let $\hat{\phi}_i$ $(1 \le i \le 10)$ be a representation of W_1 with character ϕ_i. We can then find a representation $\hat{\psi}_i$ of W_2 where $\hat{\psi}_i(s_1) = \hat{\phi}_i(t)$, $\hat{\psi}_i(t) = \hat{\phi}_i(s_1)$, $\hat{\psi}_i(t_2) = \hat{\phi}_i(s_2)$. Let ψ_i be the character of the representation $\hat{\psi}_i$. Then ψ_i, $1 \le i \le 10$, give all the ordinary irreducible characters of W_2. We will need the right ideals of $H(W_1)$ with generators

$$x_3 = (1+T_t)(q+T_{s_1ts_1})(q^2+T_{s_2s_1ts_1s_2})\left(1 - \frac{1}{q}T_{s_1} - \frac{1}{q}T_{s_2} + \frac{1}{q^2}T_{s_1s_2} + \frac{1}{q^2}T_{s_2s_1} - \frac{1}{q^3}T_{s_1s_2s_1}\right),$$
$$x_4 = (1+T_t)(q+T_{s_1ts_1})T_{s_2s_1}(Q-T_t)(1+T_{s_2}),$$
$$x_5 = (1+T_t)(q+T_{s_1ts_1})T_{s_2s_1}(Q-T_t)(q-T_{s_2}),$$
$$x_6 = (1+T_t)T_{s_1s_2}(Q-T_t)(Qq-T_{s_1ts_1})(1+T_{s_1}),$$
$$x_7 = (1+T_t)T_{s_1s_2}(Q-T_t)(Qq-T_{s_1ts_1})(q-T_{s_1}),$$
$$x_8 = (Q-T_t)(Qq-T_{s_1ts_1})(Qq^2-T_{s_2s_1ts_1s_2})(1+T_{s_1}+T_{s_2}+T_{s_1s_2}+T_{s_2s_1}+T_{s_1s_2s_1})$$

and the right ideals of $H(W_2)$ with generators

$$y_1 = (1+T_{s_1})(Q+T_{ts_1t})(Q^2+T_{t_2ts_1tt_2})(1+T_t+T_{t_2}+T_{tt_2}+T_{t_2t}+T_{tt_2t}),$$
$$y_3 = (1+T_{s_1})(Q+T_{ts_1t})(Q^2+T_{t_2ts_1tt_2})\left(1 - \frac{1}{Q}T_t - \frac{1}{Q}T_{t_2} + \frac{1}{Q^2}T_{tt_2} + \frac{1}{Q^2}T_{t_2t} - \frac{1}{Q^3}T_{tt_2t}\right),$$
$$y_4 = (1+T_{s_1})(Q+T_{ts_1t})T_{t_2t}(q-T_{s_1})(1+T_{t_2}),$$
$$y_7 = (1+T_{s_1})T_{tt_2}(q-T_{s_1})(Qq-T_{ts_1t})(Q-T_t),$$
$$y_8 = (q-T_{s_1})(Qq-T_{ts_1t})(Q^2q-T_{t_2ts_1tt_2})(1+T_t+T_{t_2}+T_{tt_2}+T_{t_2t}+T_{tt_2t}),$$
$$y_9 = (q-T_{s_1})(Qq-T_{ts_1t})(Q^2q-T_{t_2ts_1tt_2})(1+T_t)T_{t_2}(Q-T_t),$$
$$y_{10} = (q-T_{s_1})(Qq-T_{ts_1t})(Q^2q-T_{t_2ts_1tt_2})\left(1 - \frac{1}{Q}T_t - \frac{1}{Q}T_{t_2} + \frac{1}{Q^2}T_{tt_2} + \frac{1}{Q^2}T_{t_2t} - \frac{1}{Q^3}T_{tt_2t}\right).$$

2.5. REMARK. The method of constructing the generators x_i and y_j given above is described in [**4**, 5.7] and these generators were constructed so that in the special case

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char $K=0$ and $q=Q=1$, $x_iH(W_1)$ gives a representation for W_1 with character ϕ_i ($i=3, 4, 5, 6, 7, 8$) and $y_jH(W_2)$ gives a representation of W_2 with character ψ_j ($j=1,3,4,7,8,9,10$).

2.6. REMARK. The ordinary character table of W, the Weyl group of type F_4, is given in [**1**, p. 413]. We will refer to this table for the labelling of the characters (that is the ith character, χ_i, in our notation is the ith character in that table). Inducing up some of the characters of W_1 and W_2 we get the following table:

i	j	The unique common constituent of $\phi_i\uparrow^W_{W_1}$ and $\psi_j\uparrow^W_{W_2}$
6	1	χ_{10}
7	8	χ_{11}
8	7	χ_{12}
5	10	χ_{13}
4	8	χ_{21}
7	3	χ_{22}
8	4	χ_{23}
3	7	χ_{24}
6	9	χ_{25}

In the next section we will make use of these results to construct certain right ideals of $H(W)$ which will be proved to be irreducible on the assumption that $H(W)$ is semisimple. In the special case char $K=0$, $q=Q=1$, the representations of W corresponding to these right ideals have characters χ_m for $10\le m\le 13$ and $21\le m\le 24$ (that is, we will get all the 8-dimensional and all the 9-dimensional ordinary irreducible representations of W).

3. Certain right ideals of $H(W)$. In this section we denote $H(W)$ by H. In what follows we will need the following preliminary lemma the proof of which is standard.

3.1. LEMMA. *Suppose $e,f\in H$ where $e^2=ke$ ($k\in K$, $k\neq 0$). Then $\mathrm{Hom}_H(eH,fH)\cong fHe$ as vector spaces over K.*

Proof. The proof is easy once we notice that any H-homomorphism from eH to fH is given by left multiplication with fhe for some $h\in H$.

3.2. COROLLARY. *In addition to the hypothesis of Lemma 3.1, assume that H is semisimple and that fHe is 1-dimensional as a vector space over K. Then we can conclude that $(fHe)H$ is a simple H-module.*

Proof. In view of Lemma 3.1 we can deduce that $\mathrm{Hom}_H(eH,fH)$ is 1-dimensional as a vector space over K. It now follows from the fact that H is assumed to be semisimple that eH and fH have exactly one common composition factor. Now $fHe\neq 0$ by assumption, so we can find $h^*\in H$ such that $fh^*e\neq 0$. Since fHe is 1-dimensional we get that $(fh^*e)H=(fHe)H$. Defining $\alpha: eH\to fH$ such that $eh\mapsto (fh^*e)h$ ($h\in H$), we get that

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$\operatorname{Im}\alpha = (fh^*e)H \neq 0$ and that every composition factor of $(fh^*e)H$ is a composition factor of fH and of eH.

It follows that the unique common composition factor of eH and of fH is isomorphic to $(fh^*e)H$. Hence $(fHe)H = (fh^*e)H$ is simple.

For the remainder of this section assume that H is semisimple.

Define $P(Q,q)$, the Poincare polynomial for $H(W)$, by $\left(\sum_{w\in W} T_w\right)^2 = P(Q,q)\left(\sum_{w\in W} T_w\right)$. It follows from the assumption that H is semisimple that $P(Q,q)\neq 0$. In particular $(1+q)(1+Q)\neq 0$ since $(1+q)(1+Q)$ divides $P(Q,q)$.

We are now going to use Corollary 3.2 in order to construct a right ideal of H which will turn out to be a simple H-module. Let $f = x_7$ and $e = y_8$. We consider fHe as a vector space over K. First note that $e^2 = ke$ where

$$k = Q^3(1+q)(1+Q)(1+Q+Q^2)(q+Q)(q+Q^2)\neq 0.$$

(If $k=0$, we could then find a nilpotent ideal of H, namely $\sum_{w\in W} q^{-\nu(w)}T_w$, where $\nu(w)$ is defined to be the total number of s_1's and s_2's in the canonical expression for w given in Remark 1.5. This would then contradict the fact that H is semisimple.)

In view of the fact that $\{f, fs_2, fs_2s_1\}$ is a K-basis for $fH(W_1)$ (see [**4**, proof of 5.7]), we get that fH is spanned by $\{fT_w : w\in S\}\cup\{fs_2T_w : w\in S\}\cup\{fs_2s_1T_w : w\in S\}$, where S is the set of distinguished right coset representatives of W_1 in W given in Lemma 1.4.

Suppose now that $x\in W_2$. It is easy to see that $T_xe = \lambda e$ for some $\lambda\in K$. As a consequence of that, eliminating the elements of S which end in t_2, t or s_1, we get that fHe is spanned by $\{fT_re : r\in R\}$ where

$$R = \{1, s_2, s_2s_1, t_2ts_1s_2, s_2t_2ts_1s_2, s_2s_1t_2ts_1s_2, t_2ts_1tt_2s_2s_1ts_1s_2, s_2t_2ts_1tt_2s_2s_1ts_1s_2,\\ s_2s_1t_2ts_1tt_2s_2s_1ts_1s_2\}.$$

Consider for example fT_re where $r = t_2ts_1tt_2s_2s_1ts_1s_2$. Using the properties of multiplying inside the Hecke algebra, we get (recall $f = x_7$, $e = y_8$):

$$\begin{aligned} fT_re &= (1+T_t)T_{s_1s_2}(Q-T_t)(Qq-T_{s_1ts_1})(q-T_{s_1})T_ry_8 \\ &= (q-T_{s_2})(1+T_t)T_{s_1s_2}(Q-T_t)(Qq-T_{s_1ts_1})T_ry_8 \quad \text{(see [4, 3.10])} \\ &= (q-T_{s_2})(1+T_t)T_{s_1s_2}(Qq-T_{s_1ts_1})T_r(Q-T_t)y_8 \\ &\quad ((Q-T_t) \text{ commutes with } T_r \text{ and with } (Qq-T_{s_1ts_1})) \\ &= 0 \end{aligned}$$

since $(Q-T_t)y_8 = 0$. If we now assume that $r = s_2t_2ts_1tt_2s_2s_1ts_1s_2$ we get that $fT_re = 0$ using exactly the same argument as above and noting that $(Q-t)$ commutes with T_{s_2} as well.

In fact, using similar argument as in the examples above we can show that $fT_re = 0$ for all $r\in R - \{s_2s_1t_2ts_1s_2\}$.

Consider now fT_re where $r = s_2s_1t_2ts_1s_2$. We get,

$$fT_{s_2s_1t_2ts_1s_2}e = Q^5q^6(1+Q)(1+q)T_{t_2ts_1s_2} + \sum_{\ell(w)>4}\alpha_wT_w \ (\alpha_w\in K).$$

Since $(1+Q)(1+q)\neq 0$ (we have assumed that H is semisimple), we can conclude that $fT_{s_2s_1t_2ts_1s_2}e\neq 0$. We have thus shown that fHe is 1-dimensional as a vector over K. If we now define $z_{11} = x_7T_{s_2s_1t_2ts_1s_2}y_8$, we get that $z_{11}H$ is a simple H-module in view of

Corollary 3.2. In passing we remark that

$$z_{11}H = ((1+q)(1+T_t)T_{s_1s_2}(Q-T_t)(Qq-T_{s_1ts_1})T_{s_2s_1t_2ts_1s_2}y_8)H.$$

As a vector space over K, $z_{11}H$ is 9-dimensional and has basis

$$\{z_{11}, z_{11}s_2, z_{11}s_2s_1, z_{11}s_2s_1t, z_{11}s_2s_1tt_2, z_{11}s_2s_1ts_1, z_{11}s_2s_1ts_1t_2, z_{11}s_2s_1ts_1s_2, z_{11}s_2s_1ts_1s_2t_2\}.$$

With respect to this basis of $z_{11}H$ we get the following 9-dimensional matrix representation σ of H. It follows from the discussion above that σ is an irreducible representation (provided that H is semisimple). The characters of this representation agree with the characters given in Geck [**5**]. In [**5**], Geck calculated the characters of the generic Hecke algebra of type F_4 by constructing matrix representations of degree at most 8. (The notation {{row 1}, {row 2}, . . . , {row 9}} to present the matrices is used here.)

$$\begin{aligned}\sigma(T_{s_1}) = \{&\{-1,0,0,0,0,0,0,0,0\}, \{0,0,1,0,0,0,0,0,0\}, \{0,q,q-1,0,0,0,0,0,0\},\\ &\{0,0,0,0,0,1,0,0,0\}, \{0,0,0,0,0,0,1,0,0\}, \{0,0,0,q,0,q-1,0,0,0\},\\ &\{0,0,0,0,q,0,q-1,0,0\}, \{0,0,0,0,0,0,0,-1,0\}, \{0,0,0,0,0,0,0,0,-1\}\},\end{aligned}$$

$$\begin{aligned}\sigma(T_{s_2}) = \{&\{0,1,0,0,0,0,0,0,0\}, \{q,q-1,0,0,0,0,0,0,0\}, \{0,0,-1,0,0,0,0,0,0\},\\ &\{0,0,0,-1,0,0,0,0,0\}, \{0,0,0,0,-1,0,0,0,0\}, \{0,0,0,0,0,0,0,1,0\}, \{0,0,\\ &0,0,0,0,0,0,1\}, \{0,0,0,0,0,q,0,q-1,0\}, \{0,0,0,0,0,0,q,0,q-1\}\},\end{aligned}$$

$$\begin{aligned}\sigma(T_t) = \{&\{Q,0,0,0,0,0,0,0,0\}, \{0,Q,0,0,0,0,0,0,0\}, \{0,0,0,1,0,0,0,0,0\},\\ &\{0,0,Q,Q-1,0,0,0,0,0\}, \{0,0,0,0,Q,0,0,0,0\}, \{0,0,0,0,0,Q,0,0,0\},\\ &\{(q-1)q(q+Q), -1+q-q^2, -1+q, -1+q, -1+q, -1, -1, 0, 0\},\\ &\{0,0,0,0,0,0,0,Q,0\}, \{q(-1+q-q^2), (-1+q)(-1+q+qQ),\\ &1-q, 1-q, 1-q, 0, 0, -1, -1\}\},\end{aligned}$$

$$\begin{aligned}\sigma(T_{t_2}) = \{&\{Q,0,0,0,0,0,0,0,0\}, \{0,Q,0,0,0,0,0,0,0\}, \{0,0,Q,0,0,0,0,0,0\}, \{0,0,\\ &0,0,1,0,0,0,0\}, \{0,0,0,Q,Q-1,0,0,0,0\}, \{0,0,0,0,0,0,1,0,0\}, \{0,0,0,\\ &0,0,Q,Q-1,0,0\}, \{0,0,0,0,0,0,0,0,1\}, \{0,0,0,0,0,0,0,Q,Q-1\}\}.\end{aligned}$$

3.3. Remarks. (i) We can easily show by checking the set of defining relations for H that the matrices $\sigma(T_{s_1})$, $\sigma(T_{s_2})$, $\sigma(T_t)$, $\sigma(T_{t_2})$ still define a representation of H even in the case when no restrictions on q, Q are made. (Hence, if H is not semisimple, σ is still a representation of H through not necessarily an irreducible one.)

(ii) Using the automorphism θ of H given in 2.3 we can get another 9-dimensional matrix representation of H from the one given above.

(iii) In the special case $q = Q = 1$, char $K = 0$, the character of the matrix representation σ corresponding to $z_{11}H$ is χ_{11}. This is expected from the way f and e were chosen (compare Remark 2.6 (the case $i = 7, j = 8$)).

3.4. The results (compare Remarks 2.5 and 2.6). With suitable choices of f and e, and considering fHe, generators z_m (see the table below) were constructed ($10 \le m \le 13, 21 \le m \le 24$) such that, provided that H is semisimple, the right ideal z_mH is a simple H-module.

The proof that z_mH is simple is very similar to the proof given above for the case $m = 11$. Note that for the choices of f and e given below, $e^2 = ke$ for some $k \in K$, $k \neq 0$, and fHe was proved to be 1-dimensional as a vector space over K (compare Corollary 3.2 and the case $m = 11$).

e	f	Basis of fHe as a vector space over K
y_1	x_6	$x_6T_{s_2s_1t_2ts_1s_2}y_1 = z_{10}$
y_8	x_7	$x_7T_{s_2s_1t_2ts_1s_2}y_8 = z_{11}$
x_8	y_7	$y_7T_{t_2ts_2s_1tt_2}x_8 = z_{12}$
y_{10}	x_5	$x_5T_{t_2ts_1s_2}y_{10} = z_{13}$
y_8	x_4	$x_4T_{s_1t_2ts_1s_2}y_8 = z_{21}$
y_3	x_7	$x_7T_{t_2ts_1s_2}y_3 = z_{22}$
x_8	y_4	$y_4T_{ts_2s_1tt_2}x_8 = z_{23}$
x_3	y_7	$y_7T_{s_2s_1tt_2}x_3 = z_{24}$

In the special case $\operatorname{char} K = 0$, $Q = q = 1$, z_mH gives a representation of W with character χ_m ($10 \le m \le 13$, $21 \le m \le 24$). (Compare Remark 2.6.)

From these right ideals matrix representations of $H(W)$ can be constructed by fixing a K-basis for z_mH as in the example given above for $m = 11$. For example, for the case $m = 24$, we get the following 8-dimensional matrix representation τ of $H(W)$:

$$\tau(T_{s_1}) = \{\{-1, 0, 0, 0, 0, 0, 0, 0\}, \{0, -1, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 1, 0, 0, 0, 0\}, \{0, 0, q, q-1, 0, 0, 0, 0\}, \{0, 0, 0, 0, -1, 0, 0, 0\}, \{0, 0, 0, 0, 0, -1, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 1\}, \{0, 0, 0, 0, 0, 0, q, -1+q\}\},$$

$$\tau(T_{s_2}) = \{\{-1, 0, 0, 0, 0, 0, 0, 0\}, \{0, -1, 0, 0, 0, 0, 0, 0\}, \{0, 0, -1, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, q, q-1, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 1, 0\}, \{0, 0, 0, 0, 0, q, q-1, 0\}, \{0, 0, 0, 0, 0, 0, 0, -1\}\},$$

$$\tau(T_t) = \{\{Q, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 1, 0, 0, 0, 0, 0\}, \{0, Q, -1+Q, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 1, 0\}, \{0, 0, 0, Q, 0, -1+Q, 0, 0\}, \{0, 0, 0, 0, Q, 0, -1+Q, 0\}, \{q^3+qQ+q^2Q-q^3Q^2, q^3, q^3, -q^2, q, -q^2, q, -1\}\},$$

$$\tau(T_{t_2}) = \{\{0, 1, 0, 0, 0, 0, 0, 0\}, \{Q, -1+Q, 0, 0, 0, 0, 0, 0\}, \{0, 0, Q, 0, 0, 0, 0, 0\}, \{0, 0, 0, Q, 0, 0, 0, 0\}, \{0, 0, 0, 0, Q, 0, 0, 0\}, \{q, q, q, -1, 0, -1, 0, 0\}, \{-q, -q, -q, 0, -1, 0, -1, 0\}, \{q, q, 0, -q, 1, 0, 0, -1\}\}.$$

3.5. Remarks. (i) Again we can easily show by checking the set of defining relations for H that τ is still a representation of H even when no restrictions on q or Q are being made.

(ii) Note that the representing matrices of τ for the generators involve no denominators. (Compare [**5**, p. 66] where the representing matrices of the corresponding representation involve denominators.)

(iii) Comparing the expressions for z_{22} and z_{24} we see that z_{22} can be obtained from z_{24} by interchanging T_{s_1} with T_t, T_{s_2} with T_{t_2}, and also interchanging the roles of Q and q. Thus a matrix representation for $z_{22}H$ can be obtained directly from τ in the obvious way. (Compare Remark 1.1 and [**5**, p. 67].) In the same way we can obtain z_{23} from z_{21} and z_{12} from z_{11}.

Finally we consider the case $i = 6, j = 9$ in the table in Remark 2.6. Now $x_6H(W_1)$ is 3-dimensional and $y_9H(W_2)$ is 2-dimensional as vector spaces over K, so it is not necessarily true that either of $x_6^2 = kx_6$ or $y_9^2 = ky_9$ holds. However, if H is semisimple,

$y_9H = \varepsilon H$ for some idempotent ε. Thus, considering $x_6H\varepsilon$ as a vector space over K, we expect to be able to construct a 16-dimensional representation of H which, upon specializing q, Q to 1 has character χ_{25}.

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REFERENCES

1. R. W. Carter, *Finite groups of Lie type. Conjugacy classes and complex characters* (Wiley, 1985).

2. C. W. Curtis and I. Reiner, *Methods of representation theory*, *vols I*, *II* (Wiley, 1981/1987).

3. R. Dipper and G. James, Representations of Hecke algebras of general linear groups, *Proc. London Math. Soc.* (3) **52** (1986), 20–52.

4. R. Dipper and G. James, Representations of Hecke algebras of type B_n, *J. Algebra* **146** (1992), 454–481.

5. M. Geck, On the character values of Iwahori-Hecke algebras of exceptional type, *Proc. London Math. Soc.* (3) **68** (1994), 51–76.

6. C. Pallikaros, Representations of Hecke algebras of type D_n, *J. Algebra* **169** (1994), 20–48.

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