Inside the Stars, IAU Colloquium 137
ASP Conference Series, Vol. 40, 1993
Werner W. Weiss and Annie Baglin (eds.)

DETECTING CONVECTIVE OVERSHOOT IN SOLAR-TYPE STARS

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INTRODUCTION

It is important for understanding stellar evolution to constrain observationally how overshoot occurs for stellar conditions. Simplified models of the dynamics (eg. Zahn 1991) indicate that overshoot results in a slightly subadiabatic region beyond the convectively unstable layers, followed by an almost discontinuous transition to radiative stratification. Abrupt changes such as this contribute with a characteristic periodic signal to the frequencies $\omega_{n,l}$ of modes of low degree l (Gough 1990). This signature may therefore be detectable for distant stars. Here we show that the signal is sensitive to the "severity" of the overshoot and, of practical importance for the solar case, how it may be extracted from modes of higher degree. Finally we apply our method to solar data.

To analyze the applicability of the method, we consider four stellar models, $Z_1 - Z_4$, with solar mass, radius (R) and luminosity; of these, Z_2 and Z_4 have overshoot. The bases of the nearly adiabatically stratified region in the models are at radii r_d/R = .729, .713, .713 and .700 respectively.

METHOD AND RESULTS

The sound speed and its gradient change rather abruptly at the base of the adiabatically stratified envelope of a solar model. Consider then the difference δc^2 between the actual sound speed squared (c^2) and a smoothed version, c_0^2. Let ω be the true frequency and ω_0 the frequency of the same mode of a star in which the sound speed is c_0. Consider the second derivative of ω with respect to n at constant l:

$$S_d = \left(\frac{d^2\omega}{dn^2}\right)_l = \left(\frac{d^2\omega_0}{dn^2}\right)_l + \left(\frac{d^2(\omega-\omega_o)}{dn^2}\right)_l \equiv S_d^0 + \delta S_d \ . \qquad (1)$$

https://doi.org/10.1017/S0252921100018431 Published online by Cambridge University Press

It may then be shown that

$$\delta S_d \sim -4A\tau_d^2 \left(\frac{d\omega_0}{dn}\right)_l^2 \omega \cos\left(2\tau_d\omega - \gamma\frac{L^2}{\omega} + 2\phi\right) . \qquad (2)$$

Here the acoustic depth $\tau_d = \int_{r_d}^{R} c^{-1} dr$; $\gamma = \int_{r_d}^{R} r^{-2} c\, dr$; $L^2 = l(l+1)$; ϕ is the phase (treated here as a constant, though more generally it is a function of ω); and A is proportional to $\delta c^2(r_d)$ and is thus increased by overshoot (Fig. 1a). For low l this signal is periodic in frequency (the period determined by τ_d if ϕ is constant).

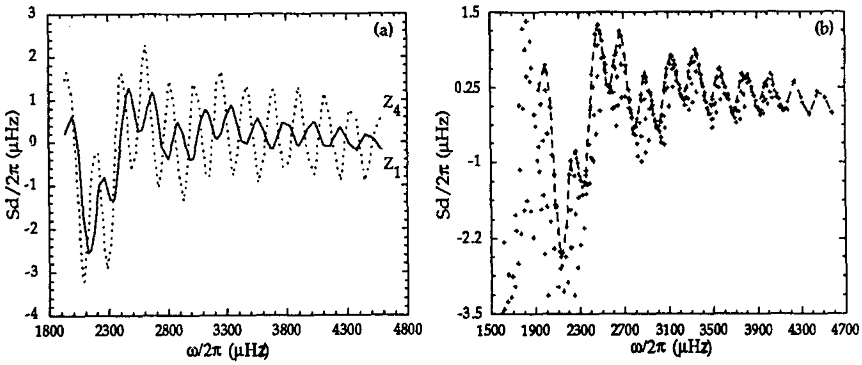

Fig. 1. Second derivatives S_d (a) for Models Z_1 (solid line) and Z_4 (dotted line), for $l = 0-4$; and (b) for Model Z_1 including higher degree data ($l = 5-20$) (the dashed line joins $l = 0-4$).

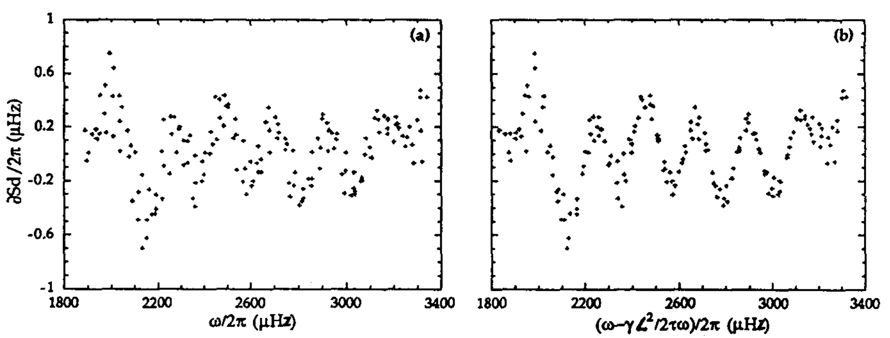

Fig. 2. (a) Second derivatives of frequency, for Model Z_1, after S_d^0 has been removed. (b) The same as (a), but now the ordinate for each point has been modified (after we have found τ_d and γ by our procedure) to remove the first-order correction to the argument of 'cos' in eq. (2). The higher-degree data also now exhibit a regular oscillatory signal.

The coherent behaviour is destroyed if modes with larger l are included (Fig. 1b). Two major contributions can be identified. One affects S_d^0 (eq. 1) by moving downwards the points for larger l. This component is explicitly removed by our method of isolating δS_d (see below). The second contribution is a phase shift which affects δS_d and which is accounted for by the term in γ (Fig. 2).

We determine S_d by constructing the cubic spline interpolating the points $\omega(n)$ (with the same l). S_d^0 is obtained similarly, but with a cubic spline that is forced to be smoother and hence cannot fit the frequencies exactly. Finally we make a fit, in which expression (2) is the leading contribution, to $\delta S_d = S_d - S_d^0$ to obtain A, τ_d, γ and ϕ. The results for model data and 1986 solar data (Libbrecht *et al.* 1990) are summarized in Fig. 3. Only $l = 5 - 20$ were used: the available solar data for $l = 0 - 4$ are too noisy, destroying the signal we are isolating.

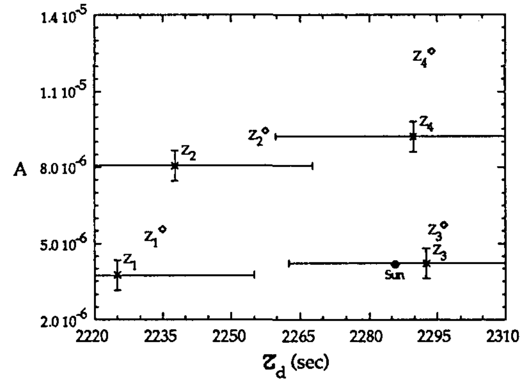

Fig. 3. The inferred amplitude A and acoustic depth τ_d for the Sun and for the models. Diamonds show results for artificial data with no errors added; crosses indicate results for data with errors, assumed gaussian with standard deviations given by Libbrecht *et al.* Error bars were derived from 15 realizations of the random errors for Model Z_3.

The solar data appear to favour models Z_1 and Z_3 which have no overshooting, rather than models Z_2 and Z_4 which have overshoot layers of depth $0.015R$ and $0.027R$ respectively. This is our principal conclusion. Amongst models with the same degree of overshoot, the relative depth determination is also fairly reliable (though note that it has rather large uncertainties in this method due to data errors). We note with interest that the Sun appears to resemble most closely Model Z_3, which has the same depth of convection zone as was inferred for the Sun by Christensen-Dalsgaard *et al.* (1991).

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