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So far the galaxy correlation analysis was the only quantitative method used to describe the distribution of galaxies in space. Here we consider other numerical methods to treat impersonally various aspects of the galaxy distribution.

1. CATALOGUES USED

Observational data are based on a compilation of all available redshifts by Dr. J. Huchra. Using Huchra's data, rectangular supergalactic coordinates and absolute magnitudes have been calculated for every galaxy, taking the recession velocity as a distance indicator. The relative velocity of galaxies in clusters has been reduced in order to remove the "god finger" effect. After reduction, the extent of clusters in radial and tangential direction is approximately the same.

All numerical studies have been made for galaxies within a cube of a certain size, L , and limiting absolute magnitude of galaxies, M_0 . The basic observational catalogue was centered on the Virgo cluster, has cell size $L = 4000$ km/s in redshift space (80 Mpc for $H = 50$ km/s/Mpc, used in this paper), and limiting absolute magnitude $M_0 = -19.5$. Other observational catalogues used have different cell sizes, limiting magnitude and center position X_0, Y_0, Z_0 . Data on catalogues used are given in Table 1.

Several theoretical catalogues have been used for comparison based on the adiabatic scenario of galaxy formation, hierarchical model of galaxy distribution, and a random Poisson distribution. All simulated catalogues have approximately the same number of objects (see Table 1) and equal size, corresponding to 80 Mpc.

In the following, the catalogues are denoted as follows:
O = observed (basic variant O₂); A = adiabatic; H = hierarchical;
P = Poisson.

2. QUANTITATIVE ANALYSIS

2.1. Correlation function. Correlation analysis has been widely used by Peebles (1980). We also have applied this method to study the behavior of our catalogues. In all cases, three-dimensional initial data have been used. The results for basic catalogues are displayed in

Table 1

Catalogue Name	X_0	Y_0 (Mpc)	Z_0 (Mpc)	M_0 (Mpc)	L	N	ν	r_c (Mpc)	B	β	γ	
Observed	0 ₁	0	24	0	-16.5	40	1009	1.5	2.4	0.9	0.24	0.61
	0 ₂	0	20	0	-19.5	80	866	1.4	4.7	0.8	0.43	0.42
	0 ₃	0	20	0	-21.0	80	191	1.5	12.0	2.7	0.23	0.67
	0 ₄	20	60	20	-21.0	160	924	1.5	12.3	1.8	0.19	0.71
	0 ₅	40	40	40	-21.0	200	1433	1.5	12.4	1.5	0.23	0.66
	0 ₆	60	60	60	-21.5	240	862	1.5	20.0	2.1	0.32	0.60
Adiabatic A					80	753	1.5	4.8	0.7	0.00	0.51	
Hierarch. H					80	819	1.8	13:	15:	1.00	0.00	
Poisson P					80	850	0.0	7.5	3.0	0.00	0.00	

Figure 1 and indicate that O, A and H catalogues are fairly similar in this respect. As expected, P catalogue has zero correlation. In Table 1 we give for catalogues studied the index ν of the correlation function:

$$\xi(r) = (r/r_0)^{-\nu}$$

2.2. Cluster analysis. New quantitative methods of the study of the galaxy distribution are based on cluster analysis. Similar methods are used in the percolation theory to study the electrical conductivity of semiconductors and in many other fields of physics (Shandarin 1982; Einasto et al. 1982).

In the cluster analysis the clustering tendency of test particles is studied as follows: Take a cube with size L and N test particles within the cube. The mean density of test bodies is $n = N/L^3$. Draw a sphere of radius r around each test particle. If within this sphere there are other test particles, all are considered as members of a connected system, called a "cluster" in the percolation theory. Thus, clusters consist either of single isolated test particles or of systems of test particles with each member having at least one neighbor with a distance r.

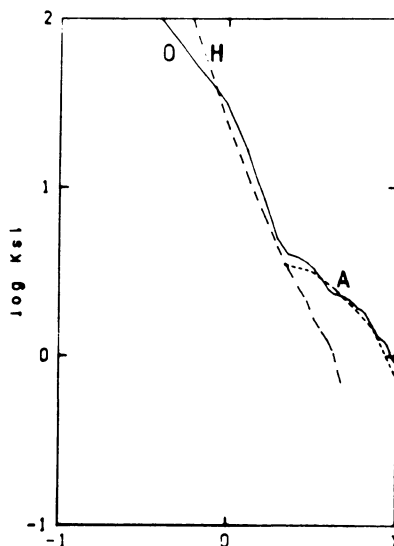


Figure 1. Correlation function for O, A and H catalogues.

It is evident that the richness and size of clusters depend on the neighborhood radius r . For small r almost all clusters have only one member. With increasing r , the size and richness of the largest clusters increase rapidly. At some critical $r = r_c$, the size of the cluster is just sufficient to bridge two opposite sidewalls of the cube. This critical radius plays an important role in the percolation theory. Properties of clusters depend on the mean number of test particles in the sphere of radius r_c :

$$B = \frac{4\pi}{3} N(r_c/L)^3 .$$

B is a stochastic variable and changes from one kind of distribution to another. For a random distribution of test particles, the mean value is $B_p = 2.7$.

It is evident that for a random but clumpy distribution of particles the radius r_c depends not on the total number of particles but on the number of clumps. For this reason in the hierarchical case $B_H > B_p$. These expectations have been confirmed by our calculations (Zeldovich, Einasto, Shandarin 1982; Shandarin 1982; Einasto *et al.* 1982); see Table 1. On the other hand, if test particles are evenly spaced along strings, the parameter B can be as low as $B_S = 10^{-3}$.

In a real clumpy case, the parameter should lie somewhere between B_S and B_H , as confirmed by our calculations. For catalogues less influenced by selection effects (O_1 and O_2), B is about 15 times smaller than the respective parameter for the random hierarchical catalogue with a similar clustering parameter (H). Other observed catalogues have B that is smaller by 5 to 10 times. This difference between the observed B values in the different observed catalogues may be due to various selection effects (for example, some galaxies may be in strings connecting superclusters but are not observed). Another possibility is that in galaxy strings galaxies have lower luminosity (Einasto, Jõeveer, Saar 1980). In any case, the parameter B is much smaller than for the random hierarchical case. This indicates the presence of galaxy strings which connect all superclusters to a single network.

2.3. Cluster multiplicity. The correlation function and the percolation parameter B say little about the distribution of galaxies according to the multiplicity of systems. Thus, cluster multiplicity is an additional factor to be included in the quantitative analysis.

Let us combine galaxies into clusters using the neighborhood method outlined above. The distribution of galaxies according to cluster multiplicity depends on the neighborhood radius r . At small r most clusters are single, whereas at very large r almost all galaxies join to single huge cluster, so we do not expect large differences between various catalogues at very small and very large neighborhood radii. The differences are largest at radii where catalogues with strong tendencies toward string-like structures reach conductivity, that is, for $r = r_c$. Respective histograms for all basic catalogues under study are given in Figure 2.

Clusters can be divided into three types: poor, medium, and rich. Poor clusters are represented by the multiplicity histogram for

the random P catalogue, which peaks at singles and has no clusters with multiplicity greater than $2^4 = 16$. Medium-size clusters are represented by the multiplicity histogram for the hierarchical H catalogue, which peaks at multiplicity $2^3 - 2^4$ and has practically no singles and no rich systems. Rich systems have $2^7 = 128$ or more members. Such clusters are present in all observed and in the adiabatic catalogues.

Let us denote the fractions of galaxies in poor, medium and rich systems by α , β , and γ , respectively. By definition:

$$\alpha + \beta + \gamma = 1.$$

Thus, the multiplicity distribution can be described by two independent parameters, say β and γ . Respective values are given for all catalogues in Table 1.

2.4. Shape and orientation of clusters. It is well known that clusters of galaxies are not spherical but triaxial. Statistically, the cluster shapes and orientations can be described by cluster axial ratios and by the deviation of the cluster's major axis from the direction towards nearby external clusters. It is well known (J  veer, Einasto, Tago 1978; Binggeli 1982) that rich clusters have a tendency to point their major axes toward neighboring clusters.

Preliminary results of the study of the shapes of clusters demonstrate the absence of large-scale sheets of galaxies. Around the Virgo cluster a small sheet has been observed (Zeldovich, Einasto, Shandarin 1982). It is too early to say whether this sheet can be identified with a Zeldovich pancake. The basic structural element seems to be a chain of galaxies or clusters of galaxies.

2.5. Morphology of galaxies in clusters. All properties studied so far represent different aspects of the geometry of galaxy distribution. It is well known (Einasto, J  veer, Saar 1980) that at different parts of the supercluster, galaxies have different mean absolute magnitudes, morphological types and other morphological properties. These properties play an important role for the theory of galaxy formation and evolution. However, it is difficult to describe them concisely in quantitative terms. No galaxy formation scenario is detailed enough to predict the behavior of these properties. Thus, in numerical simulations, morphological properties are neglected in most cases. Further

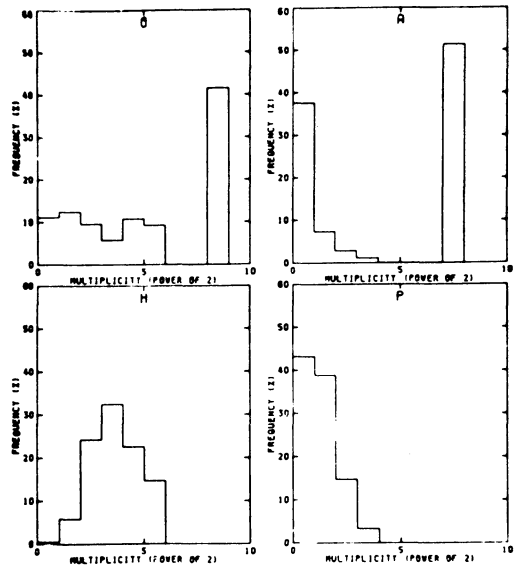


Figure 2. Distribution of galaxies according to the cluster multiplicity for neighborhood radius $R = 5$ Mpc.

observational and theoretical study of the morphology of galaxies in various parts of superclusters is badly needed.

3. MEAN DENSITIES

Data available allow us to derive the mean luminosity density in various parts of the universe.

As a measure of the luminosity density, we use the number of bright galaxies ($M \leq -21.0$). The total luminosity of a galaxy system can be expressed as follows:

$$L = n^* \times 3.6 \times 10^{11} L_0 ,$$

where n^* is the number of bright galaxies in the respective systems (Jõeveer, Einasto, Tago 1978). Here we assume that the luminosity function is the same everywhere. This is, of course, not the case, but for rough estimates it gives a good approximation.

The average luminosity density in the region bounded by $X, Y = \pm 5000$ km/s, $Z = \pm 7500$ km/s in redshift space is $5 \times 10^7 L_0/\text{Mpc}^3$. This may be a slight underestimate due to the exclusion of several rich clusters from the region. The density in superclusters exceeds the mean density by a factor of 3, and the density far from superclusters is about three times lower. Within galaxy strings and cluster chains, the density is 70 to 100 times higher than the mean density (Einasto *et al.*, 1982).

In all observed catalogues the relative number of poor clusters is $\alpha \approx 0.10$. A study of the distribution of these isolated galaxies shows that at least 2/3 of them are outlying members of other systems. So we conclude that, if a field population exists at all, it cannot contain more than approximately three percent of all galaxies. Thus, the mean density of luminous matter outside galaxy systems is lower than the mean density by a factor of at least 30, and the density contrast between voids and galaxy systems is more than three orders of magnitudes (Einasto *et al.* 1982).

Near superclusters, galaxy systems fill about ten percent of the total volume, and in the whole region under study, only one percent. Thus, the space outside superclusters is very empty indeed (Zeldovich, Einasto, Shandarin 1982).

4. DISCUSSION

As we have seen, catalogues of different kinds behave completely differently. All observed catalogues have similar properties. This indicates that statistical parameters found are stable and do not depend on the individual peculiarities of particular regions.

The hierarchical catalogue has only one common property with observed catalogues, the correlation function. All other parameters differ completely from observations. The present hierarchical catalogue was constructed artificially, following prescriptions of Soneira and Peebles (1978). It would be interesting to compare the observed distribution with results of numerical simulations, which also involve dynamical evolution. This study is under way. If numerical simulations are close to the hierarchical catalogue studied above, then we come to

the inevitable conclusion: the simple hierarchical clustering scenario contradicts observations.

The adiabatic catalogue has many properties in common with observations. However, in two points important differences are present. About one-half of all test particles of the A catalogue have multiplicity distribution like the random P catalogue. These particles can be considered a field population. The spatial density of particles of this population is lower than the average density. The observed catalogue has no appreciable field populations. This difference can be explained as follows: In regions of lower-than-average matter density, no galaxy formation takes place. The matter in low density regions remains in some pre-galactic form.

The second difference lies in the fact that the adiabatic catalogue has no systems of medium size. On the other hand, the observed catalogue has about one-half of all galaxies in systems of intermediate size. Medium-size systems are characteristic of the fine structure of superclusters. The presence of fine structure is an essential property of superclusters and should be incorporated in scenarios of galaxy formation.

Summarizing the results of the quantitative analysis of the galaxy clustering, we come to the following conclusions:

- 1) The basic structural element in the Universe, larger than clusters of galaxies, is a string of galaxies and clusters of galaxies;
- 2) No large-scale sheets of galaxies have been found so far. Small-scale sheets surround some clusters;
- 3) Galaxy strings connect all superclusters to a single intertwined lattice;
- 4) Superclusters consist of both large and medium-size strings;
- 5) A field population of galaxies, if it exists, contains at most three percent of all galaxies;
- 6) The luminosity density contrast between voids and galaxy strings exceeds three orders of magnitude;
- 7) Galaxy strings fill about one percent of space in the Universe; the rest is void of galaxies;
- 8) No theoretical scenario proposed so far explains all observed clustering properties.

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Discussion

Scott: What are the observational catalogues you are referring to: O_1 , O_2 , O_3 , ... ? The results you show are very different from the distributions obtained from known catalogues by others. For example, your distribution of multiplicity is very different from that obtained from the Lick survey by Neyman, Shane and Scott (and by others), namely, a distribution continually decreasing. I know of no catalogue that will give a high probability of huge multiplicity except, of course, a catalogue of clusters. To use such would not make sense.

Einasto: All catalogues are subsamples of Huchra's (CfA) compilation. They differ in cube size, center position and absolute magnitude limit.

The reason the multiplicity function differs from those derived earlier is because a completely different method has been used to define a multiple system. In our case, the method used in the percolation theory has been applied. Rich aggregates found by us are clusters plus string systems at the percolation radius (i.e., at radius where the system joins two opposite cube sides).

Szalay: Is the Coma/A1367 complex now considered as a string?

Einasto: The Coma-A1367 supercluster consists of a number of strings. The string connecting the Coma and A1367 clusters is the strongest.

Djorgovski: Is it true that the strings intersect? If so, what is the fraction of galaxies in the knots (intersections), and what is between them?

Einasto: Yes, strings do intersect. The fraction of galaxies in knots is not yet determined. As a very crude estimate, we can take the fraction of galaxies in clusters. In any case, the exact definition of both clusters and strings is difficult due to the continuous character of the structure.