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Letter to the Editor

Waves in Rydberg plasmas

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Abstract. We define as the Rydberg plasma the weakly ionized gas produced in magneto-optical traps. In such a plasma, the neutral atoms can be excited in Rydberg states. Wave propagation in Rydberg plasmas and the mutual influence of plasma dispersion and atomic dispersion are considered. New dispersion relations are established, showing new instability regimes and new cut-off frequencies.

1. Introduction

In recent years, there has been an increasing interest in the physics of very low temperature plasmas, in the temperature range of 1 K, resulting from the ionization of cold atomic molasses confined and laser cooled in a magneto-optic trap. This extends the domain of application of plasma physics into a quite new direction, which contrasts with the traditional view of a plasma medium as a very high temperature gas, which electron temperatures of the order of or higher than the energy of ionization, typically above an electronvolt.

It has been experimentally demonstrated that ultra-cold plasmas can be produced simply by direct photo-ionization of an ultra-cold gas [1], or as an alternative, by spontaneous evolution from neutral to ionized gas, when the atoms are excited in highly excited Rydberg quantum states [2]. Such a spontaneous ionization is possibly attained after a cascading process occurring inside the Rydberg atomic spectrum, from higher to lower energies, where the loss of internal energy of the highly excited neutral atoms provides the additional energy for ionization to occur. However, we should notice that Rydberg states are also excited in the case of direct ionization. The result is a low ionized plasma where the remaining neutral atoms have highly excited energy states [3–5].

The physics of this new area of ultra-cold plasmas has been reviewed recently [6], where they are called 'neutral plasmas'. As an alternative, here we propose to call them 'Rydberg plasmas', because the term 'neutral plasmas' can be misleading, for several different reasons. Firstly, most plasmas are electrically neutral, over dimensions larger than their typical scale, the Debye length. This means that, inside a Debye sphere, the total electron charge is equal to the total positive ion charge, on average. Second, in quite exceptional conditions where such a charge neutrality is not exactly fulfilled, we talk about 'non-neutral plasmas' [7]. Extreme examples of such non-neutral plasmas are electron or positron beams. Finally, the ultra-cold neutral gas confined in a magneto-optical trap manifests itself as a 'non-neutral plasma', due to the existence of an effective electric charge of the neutral atoms [8]. As a consequence, neutral atoms repel each other, as if they had the same electric charge, which explains the observation of Coulomb-type explosions [9], and the possible occurrence of many collective processes [10].

Here we discuss a new aspect of Rydberg plasmas, and examine the properties of wave dispersion in this medium. A new dispersion relation is derived, which contains contributions associated with both the electrons and the neutral atoms. The resulting wave refractive index will depend on the electron susceptibility as well as on the nearly resonant atomic transitions. It will be shown that low-frequency electromagnetic waves can be destabilized due to energy transfer between atoms and plasma electrons. Another important qualitative change is the possible occurrence of wave propagation below the plasma cut-off frequency.

2. Basic formulation

Let us consider electromagnetic wave propagation in a weakly ionized plasma, where the neutral atoms have highly excited energy states. We can state the wave equation as

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E} = \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} - \mu_0 \frac{\partial \mathbf{J}}{\partial t} \tag{1}$$

where $\mathbf{J} = -en_0\mathbf{v}_{\mathrm{e}}$ is the electron plasma current, and $\mathbf{P}$ is the polarization vector associated with the neutral atoms. Here, e is the electron charge, n_0 is the mean electron density, and $\mathbf{v}_{\mathrm{e}}$ the electron velocity. We know that, for transverse waves, the amplitude of density perturbations is equal to zero. We then consider infinitesimal wave perturbations with frequency ω and wavevector $\mathbf{k}$ of the form

$$(\mathbf{v}_{\mathrm{e}}, \mathbf{E}, \mathbf{P}) = (\mathbf{v}, \mathbf{E}_\omega, \mathbf{P}_\omega)\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t). \tag{2}$$

From the electron equations of motion, we get

$$\mathbf{v} = -i\frac{e}{m_{\mathrm{e}}}\frac{\mathbf{E}_\omega}{(\omega + i\nu_{\mathrm{e}})} \tag{3}$$

where m_{e} is the electron mass. We retain the electron collision frequency ν_{e} because very-low-frequency electromagnetic waves can eventually be considered. We can also use the relation

$$\mathbf{P}_\omega = N_{\mathrm{a}}\chi_{\mathrm{a}}(\omega)\mathbf{E}_\omega \tag{4}$$

where N_{a} is the density of the neutral atoms, and $\chi_{\mathrm{a}}(\omega)$ is the atomic susceptibility. From the above equations we can then derive the dispersion relation

$$\frac{k^2c^2}{\omega^2} = \epsilon(\omega) \equiv 1 + \chi_{\mathrm{e}}(\omega) + N_{\mathrm{a}}\chi_{\mathrm{a}}(\omega) \tag{5}$$

where $\epsilon(\omega)$ is the dielectric function of the medium, and the electron susceptibility of the plasma is determined by

$$\chi_{\mathrm{e}}(\omega) = \chi'_{\mathrm{e}}(\omega) + i\chi''_{\mathrm{e}}(\omega) \tag{6}$$

where the real and the imaginary parts are

$$\chi'_{\mathrm{e}}(\omega) = -\frac{\omega_{\mathrm{pe}}^2}{(\omega^2 + \nu_{\mathrm{e}}^2)}, \quad \chi''_{\mathrm{e}}(\omega) = -\frac{\nu_{\mathrm{e}}}{\omega}\chi'_{\mathrm{e}}(\omega) \tag{7}$$

where $\omega_{\rm pe} = (e^2 n_0/\epsilon_0 m_{\rm e})^{1/2}$ is the usual electron plasma frequency. In order to establish the atomic susceptibility $\chi_{\rm a}(\omega)$, we first notice that most of the Rydberg states of the neutral atoms inside the ultra-cold partially ionized plasma are significantly populated. The corresponding energy levels can be written in hydrogenic form as

$$E_\nu = -\frac{E_0}{\nu^2} \tag{8}$$

where E_0 is a positive constant, and $\nu \gg 1$ is an integer. The dominant contribution to $\chi_{\rm a}(\omega)$ will come from the closest radiative transition, such that $\hbar\omega \simeq E_{\nu+1} - E_\nu$. This can be approximately determined by

$$\omega \simeq \frac{2E_0}{\hbar \nu^3} \tag{9}$$

Using the well-known theory for the radiative transitions in a two-level atom, with quantum states $|\nu+1\rangle$ and $|\nu\rangle$, and defining $\hbar\omega_{\rm a} \equiv E_{\nu+1} - E_\nu$, we can write

$$\chi_{\rm a}(\omega) = \chi_{\rm a}'(\omega) + i\chi_{\rm a}''(\omega) \tag{10}$$

where the real and the imaginary parts are

$$\chi_{\rm a}'(\omega) = -\frac{f_{\rm a}}{n_0}\frac{\omega_{\rm pe}^2 \Delta}{(\Delta^2+\gamma^2)} D, \quad \chi_{\rm a}''(\omega) = \frac{\gamma}{\Delta}\chi_{\rm a}'(\omega). \tag{11}$$

Here $D = [\rho_{\nu+1}^{(0)} - \rho_\nu^{(0)}]$ is the unperturbed population difference between the two states, γ is the natural linewidth, and $\Delta = (\omega_{\rm a} - \omega)$ is the frequency detuning. We have also defined the oscillator strength $f_{\rm a}$ as

$$f_{\rm a} = \frac{m_{\rm e}}{\hbar}|\langle \nu+1|z|\nu\rangle|^2 \tag{12}$$

where the wave electric field $\mathbf{E}_\omega$ is assumed to be linearly polarized along the z-axis. This is smaller by a factor of $m_{\rm e}/M_{\rm a}$, with respect to the usual definition. An alternative way of writing (11) would be to use the electric dipole momentum of the transition $p = e\langle \nu+1|z|\nu\rangle$, instead of $f_{\rm a}$. It should be noted that the population difference D is nearly equal to zero over a considerable part of the Rydberg energy spectrum, which tends to become homogeneously populated due to atomic collisions [11,12]. However, there is also a significant part of this spectrum where D can be significantly different from zero, and can take positive or negative values around the state excited by the external ionization field [3].

3. Dispersive properties

We first neglect the imaginary part of both susceptibilities, $\chi_{\rm e}(\omega)$ and $\chi_{\rm a}(\omega)$, which is an approximation that is valid in the limit $\nu_{\rm e} \ll \omega$, and $\gamma \ll \Delta$. Their influence will be discussed later. We can write the dispersion relation for the electromagnetic waves propagating in a Rydberg plasma in its simplest expression,

$$\omega^2 = k^2c^2 + \omega_{\rm pe}^2\left[1 + \beta\frac{\omega^2}{(\omega_{\rm a} - \omega)}\right] \tag{13}$$

where the quantity

$$\beta = f_{\rm a}\frac{N_{\rm a}}{n_0} D \tag{14}$$

has the dimensions of time, and can be positive or negative, according to the sign of the population difference D. In order to understand the effect of the resonance

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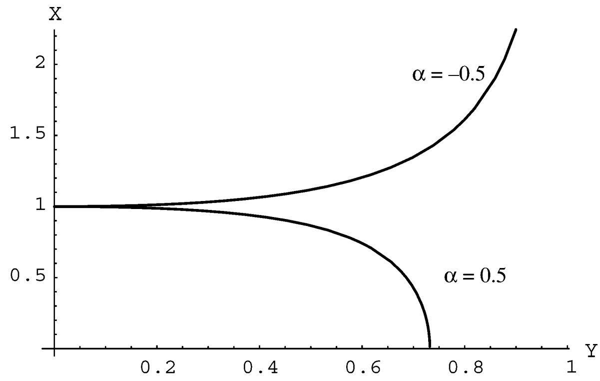

Figure 1. Cut-off frequency for electromagnetic waves propagating in a Rydberg plasma, for $\alpha = \pm 0.5$ or $\beta = \pm 1/2\omega_{\mathrm{pe}}$. Dimensionless parameters $X = \omega/\omega_{\mathrm{pe}}$ and $Y = \omega/\omega_{\mathrm{a}}$ are used.

term in the dispersion relation, let us consider $\omega = \omega' + i\omega''$, and assume the double resonance condition

$$\omega' = \omega_{\mathrm{a}} = (\omega_{\mathrm{pe}}^2 + k^2c^2)^{1/2}. \tag{15}$$

Replacing this in (13), and assuming that the imaginary part of the frequency is very small, $|\omega''| \ll \omega'$, we get

$$\omega'' = \pm\omega_{\mathrm{pe}}\sqrt{\beta\omega'/2}. \tag{16}$$

This shows that ω'' can have real and positive values for $\beta > 0$, which corresponds to an inversion of population, or $D > 0$. The resonant electromagnetic wave with frequency (13) can then become unstable. Such an instability results from the transfer of energy from the Rydberg atoms to the transverse electron oscillations of the plasma medium. Another interesting aspect of the dispersion relation (13) is the modification of the cut-off frequency. For $\beta = 0$ such a cut-off is simply determined by ω_{pe}, which is the usual plasma result. However, for $\beta \neq 0$, the cut-off is determined by the condition

$$(X^2 - 1) = \alpha\frac{Y^2}{(1 - Y)} \tag{17}$$

where we have used the dimensionless quantities

$$X = \frac{\omega}{\omega_{\mathrm{pe}}}, \quad Y = \frac{\omega}{\omega_{\mathrm{a}}}, \quad \alpha = \beta\omega_{\mathrm{a}}. \tag{18}$$

We can see that, when $Y \to 0$, we get $X = 1$, which is the usual plasma cut-off. In addition, for $Y \to 1$, we have a resonance $X \to \infty$. Noting that $X = Y(\omega_{\mathrm{a}}/\omega_{\mathrm{pe}})$, we conclude that such a resonance can only be approached for $\omega_{\mathrm{a}} \gg \omega_{\mathrm{pe}}$. Furthermore, there is no real cut-off for $Y > 1$, or $\omega > \omega_{\mathrm{a}}$. The properties of the solution $X = X(Y)$ are illustrated in Fig. 1, for both positive and negative values of α. Let us now go back to the dispersion relation (5) and retain the influence of the quantities ν_{e} and γ. With generality, we can write

$$\frac{kc}{\omega} = N(\omega) = \sqrt{1 + \chi_{\mathrm{e}}(\omega) + N_{\mathrm{a}}\chi_{\mathrm{a}}(\omega)}. \tag{19}$$

The refractive index is obviously complex, $N(\omega) = N'(\omega) + iN''(\omega)$, where $N'(\omega) \simeq \sqrt{1+\chi'(\omega)}$ for $|\chi'(\omega)| \geqslant |\chi''(\omega)|$, and the sign of the imaginary part $N''(\omega)$ is equal to the sign of $\chi''(\omega)$. It is therefore useful to represent the quantities $\epsilon'(\omega)$ and $\chi''(\omega)$, which determine the wave dispersion and the wave damping, respectively, and to compare them with the case of a purely neutral medium. Here it is convenient to introduce the following dimensionless quantities:

$$z = \frac{\delta}{\gamma}, \quad a = \frac{\omega_{\mathrm{a}}}{\omega_{\mathrm{pe}}} \tag{20}$$

where z represents the wave frequency detuning, and

$$b = \frac{f_{\mathrm{a}}}{n_0} \frac{\omega_{\mathrm{pe}}^2 D}{\gamma}, \quad \eta = \frac{\nu_{\mathrm{e}}}{\omega_{\mathrm{pe}}}, \quad g = \frac{\gamma}{\omega_{\mathrm{pe}}}. \tag{21}$$

This allows us to write the real part of the susceptibility as

$$\chi'(z) = -\frac{bz}{z^2+1} - \frac{1}{(a-zg)^2+g^2} \tag{22}$$

where the first term represents the contribution from the neutral atoms, and the second term that of the plasma. Similarly, we have for the imaginary part

$$\chi''(z) = -\frac{b}{z^2+1} + \frac{\eta}{(a-zg)^3+(a-zg)\eta^2}. \tag{23}$$

The quantity $1+\chi'(z)$ is represented in Fig. 2, for negative and positive inversion of populations, $b = \pm 1$, respectively. Notice that wave propagation is forbidden when this quantity is negative. These curves show a significant deviation with respect to the purely neutral gas. We can see the appearance of the plasma cut-off for both cases, with a significant qualitative difference for the case of $b = -1$ (no inversion of the population).

4. Conclusions

Here we have proposed the use of 'Rydberg plasmas' to designate the ultra-cold and partially ionized gas with highly populated Rydberg states of the neutral atoms, in a Kelvin temperature range. We have studied the wave propagation in Rydberg plasmas, established the corresponding dispersion relation, and discussed its main dispersive properties. The Rydberg energy spectrum provides an infinite set of nearly resonant transitions which contribute to wave dispersion. In particular, we have shown that wave propagation below the plasma cut-off is possible, and that wave instability can occur due to the energy transfer between the excited Rydberg states of the neutral atoms and the free electrons of the plasma.

In this work we have neglected the influence of external magnetic fields, which are very weak inside a magneto-optical trap. However, these results can easily be extended to a magnetized Rydberg plasma. Finally, we should mention the case of electrostatic waves. In principle, neutral atom susceptibility can also be added to the usual electron plasma term, leading to a change in the properties of the electron plasma waves in Rydberg plasmas, similar to that discussed here for transverse electromagnetic waves. The corresponding oscillator strength f_{a} would be formally identical, the only difference being that the wave electric field is now parallel to the direction of wave propagation. However, the electric dipole transition between

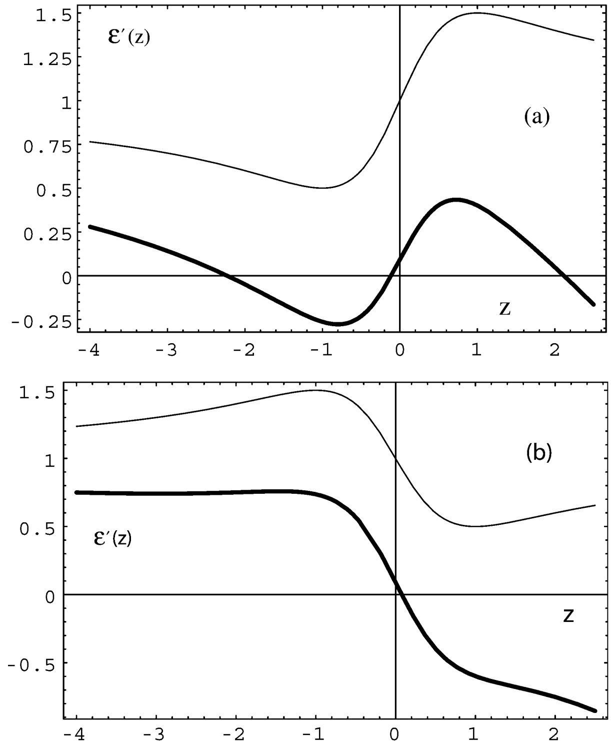

Figure 2. The dielectric function $\epsilon'(z) = 1 + \chi'(z)$ (in bold) as a function of the wave frequency detuning $z = \Delta/\gamma$, for $a = 1$, $g = \eta = 0.1$, and (a) $b = -1$ and (b) $b = 1$. For comparison, the curves of pure atomic dispersion corresponding to a non-ionized gas are also shown.

two adjacent Rydberg states is only non-zero for $\Delta l = l - l' = \pm 1$, where l and l' are the two quantum numbers of the hydrogen-like Rydberg states. This means that, in order to conserve angular momentum, we need to consider a finite angular momentum for the electron plasma wave. However, in contrast to the photons, the plasmons (or quanta of electron plasma waves) have no spin. Therefore, a finite oscillator strength $f_a \neq 0$ would only be possible if the plasmons could have a finite orbital angular momentum which is not inconceivable, but is highly speculative.

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