PUZZLES AND PROSPECTS IN PLANETARY RING DYNAMICS

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Abstract. I outline some of the main processes that shape planetary rings. Then I focus on two outstanding issues, the role of self-gravity in the precession of narrow rings and the dynamics of Neptune's arcs. By airing these well-defined but unsolved problems, I hope to encourage others to join me in the quest for their solutions.

1. Introduction

The 15 years just past have witnessed an explosion in our knowledge of planetary rings. Jupiter, Uranus, and Neptune have joined Saturn as members of the family of ringed planets. We have come to appreciate that a few simple physical mechanisms account for the bewildering variety of structures displayed by ring systems. With the completion of the Voyager missions, the level of activity in this field will decline. Major new discoveries probably must await the arrival of the Cassini spacecraft at Saturn a decade hence.

Planetary rings were the focus of my research for several years. Although this has not been the case for some time, and may never be again, they still hold my interest. After struggling to understand various issues, I feel well-situated to offer a commentary on unsolved problems. That is my aim here. After a brief review of some of the processes at work in rings, I shall focus attention on two outstanding puzzles. The first involves the precession of narrow rings and, in particular, the role played by the self-gravity of the ring material. The second concerns the mechanisms responsible for the unusual morphology of Neptune's ring arcs.

2. Physical Mechanisms

Rings are only found close to planets. They owe their existence to tidal gravity which inhibits small particles from collecting into satellites. The manner in which the particle size distribution is established, especially its upper and lower cutoffs, remains a matter of speculation. Some of the ongoing processes in rings were of importance during the relatively brief period of initial growth of planets and satellites (Borderies, Goldreich, and Tremaine 1984).

Planetary rings are not forever. Collisions among ring particles dissipate mechanical energy and transfer angular momentum outward. The gross effect is that most rings, like most people, tend to spread as they age. Estimates of angular momentum transfer rates suggest that rings may evolve on timescales significantly shorter than the 4.6×10^9 year age of the solar system. Some ring material may not be left over from the epoch of satellite formation, but may represent more recent accretion by the planet (Goldreich and Tremaine 1982).

The bewildering variety of structures seen in ring systems demands explanation. Given the tendency of ring material to diffuse radially, the sharp edges displayed by many rings are especially noteworthy. Satellites are responsible for some of this structure. Gravitational perturbations between satellites and ring material, coupled

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with collisions among ring particles, give rise to an outward transfer of angular momentum between rings and satellites. This effect is analogous to the tidal transfer of angular momentum from the earth's spin to the moon's orbit. Because their masses are large in comparison with those of ring particles, satellites can absorb or emit relatively large amounts of angular momentum without significantly changing their orbits (Goldreich and Tremaine 1978, 1979a).

Satellite torques are localized near resonances where ring particles are forced at either their epicyclic or vertical frequencies; these are referred to as Lindblad and vertical resonances, respectively. If the angular momentum transferred is small in comparison to the total flowing through the ring (the angular momentum luminosity), the gross structure of the ring is little affected, and the disturbance propagates away from the resonance as a density or bending wave (Shu 1984). Many trains of density waves, and a few of bending waves, are seen in Saturn's rings. Perturbations strong enough to remove or supply the entire angular momentum luminosity can open gaps and maintain ring edges. Examples of satellite maintained edges include the outer edges of Saturn's A and B rings, and the inner and outer edges of the epsilon ring of Uranus. The Encke gap in Saturn's A ring is held open by a small satellite that orbits within it.

At the sharpest edges the optical depth drops abruptly on a scale of tens of meters, a distance much smaller (by up to a factor 10^3) than that over which the angular momentum transfer between satellite and ring takes place. This remarkable behavior is a consequence of torque reversal near satellite resonances. Briefly stated, torque reversal occurs if the perturbed streamlines of particle flow are sufficiently distorted so that the angular velocity increases outward over a range of azimuth leading to the inward transport of angular momentum by collisions (a negative angular momentum flux). Torque reversal enables the angular momentum luminosity (the angular momentum flux integrated over azimuth) to vanish at a finite value of the surface density (Borderies, Goldreich, and Tremaine 1984).

3. Precession Of Narrow Elliptical Rings

The discovery of a system of narrow rings encircling Uranus preceded the Voyager encounters with Saturn (Elliot, Dunham, and Mink 1977). These provided our first examples of narrow planetary rings. Later, analysis of data returned by the Voyager spacecraft revealed the presence of similar narrow rings in the Saturnian system.

Further analysis showed that a number of these narrow rings have shapes that deviate from equatorial circles. Some are eccentric with the planet located at one focus of the ellipse (a Keplerian ellipse) and/or inclined to the planet's equatorial plane. Others exhibit less familiar distortions. For example, the delta ring of Uranus is elliptical, but the planet lies at the center of the ellipse rather than at a focus (a planet centered ellipse). In all cases, the eccentricities and inclinations are small ($\lesssim 0.01$), but that is not the end of the story.

The apsidal lines of the Keplerian ellipses precess, and the nodal lines of the inclined rings regress. The precession and regression rates are those expected for test particle orbits with semi-major axes intermediate between the inner and outer ring edges. The planet centered shape of the delta ring also rotates, but much faster;

its pattern speed is slightly faster than one-half the orbital rotation rate of the ring particles. The gamma ring of Uranus is another special case. It has two separate modes of distortion. The first is a Keplerian ellipse which precesses in the normal fashion. The second is a circularly symmetric pulsation at a period comparable to the orbital period of the ring particles. This is not the place for a detailed discussion of all the remarkable properties of the modes of narrow rings (cf. French et al. 1991). However, it is an opportunity for me to describe the essence of a single crucial issue, the role of self-gravity in the precession of narrow rings. In so doing, I shall begin by focusing on the epsilon ring of Uranus, the largest of the Uranian rings, and the best studied of all narrow rings. Later, I shall be more concerned with problems posed by the masses of the alpha and beta rings.

The epsilon ring has semi-major axis, $a \approx 50,000$ km, eccentricity, $e \approx 0.008$, and precesses at $\dot{\varpi} \approx 1.4$ degrees per day. Its inner and outer edges are aligned Keplerian ellipses. However, the eccentricity of the outer edge is approximately 10 percent larger than that of the inner edge. As a consequence, the ring's radial width varies from about 20 km near periapse to about 100 km near apoapse. The rigid precession of the ring poses a dynamical problem. What interactions prevent differential precession from spoiling the apse alignment between the ring's inner and outer edges? The initial answer to this question singled out the self-gravity of the ring material (Goldreich and Tremaine 1979a,b). The idea is a simple one. Self-gravity is most effective near periapse where the ring is at its narrowest. Near periapse the acceleration due to self-gravity points radially outward in the inner half of the ring and radially inward in the outer half of the ring. This is the correct sense to counter the differential precession produced by the gravitational field of the planet. From the ring geometry and planet's gravitational field, the mass of the ring was predicted to be $m_\epsilon \approx 5 \times 10^{18}$gm, corresponding to a mean surface density of $\Sigma_\epsilon \approx 25$ g cm^{-2}. The self-gravity model for rigid ring precession received a boost when several other elliptic rings were found to be narrowest at periapse and widest at apoapse.

Gravitational perturbations by nearby satellites and stresses resulting from particle collisions were investigated as alternative mechanisms for maintaining rigid ring precession. The required combination of satellite size and separation from the ring was viewed as implausible. Today, after the Voyager encounter, we know that a suitable satellite does not exist. There are two arguments against collisional stresses. The first is that they are dissipative and therefore unlikely to be consistent with apse alignment. The second is that they are too weak.

Prior to the Voyager encounter with Uranus, the case for self-gravity as the agent responsible for rigid ring precession seemed unassailable. In retrospect, however, there were a few hints of trouble. Several Keplerian ellipse shaped rings in the Saturnian and Uranian ring systems were found to have a width-radius relations that did not conform to the standard self-gravity model. That is, they were not narrowest at periapse and widest at apoapse. Also, the model yielded surprisingly low surface densities, $\Sigma \approx 1$ g cm^{-2}, for the alpha and beta rings of Uranus. However, the self-gravity model did have some notable successes. It predicts reasonable values for the surface densities in some narrow rings in the Saturnian system; that is, values comparable to those deduced from density and bending waves, and from diffraction peak scattering of the spacecraft radio signal, in broader regions of the main rings.

During the Voyager encounter with Uranus, the spacecraft's radio signals were occulted by the rings. The attenuation at X-band ($\lambda \approx 3.6\,\mathrm{cm}$) and S-band ($\lambda \approx 13\,\mathrm{cm}$) by the alpha and beta rings greatly exceeded that which material with $\Sigma \approx 1\,\mathrm{g\,cm^{-2}}$ could produce (Gresh et al. 1989). Apparently the self-gravity model grossly (by an order of magnitude or more) underestimates the masses of these rings. Voyager also found that Uranus possesses an extended hydrogen atmosphere that envelopes the rings. Unless the alpha and beta rings are more massive than originally estimated, atmospheric drag would render their lifetimes very short (French et al. 1991). Moreover, shepherding by satellites cannot help here. Voyager did not discover any shepherds for the alpha and beta rings; presumably these exist but have sizes and albedos that put them below the limit of detectability. Small shepherds could hold the alpha and beta rings in place against atmospheric drag, but only if the rings are more massive than the self-gravity model implies. Of course, the shepherds must also prevent the rings from spreading under the action of their internal collisional stresses. This sets upper limits to the masses of the alpha and beta rings. Given the considerable uncertainties in atmospheric density, satellite density and albedo, and the relation between ring surface density and radio opacity, there are probably narrow ranges of viable masses for the alpha and beta rings.

The situation regarding the epsilon ring is less clear. There is some indication that the shapes of the streamlines internal to the ring are inconsistent with the predictions of the self-gravity model. However, this depends on an assumed proportionality between surface density and optical depth, and is a more subtle issue than I can deal with here. The mass deduced from the self-gravity model also seems a bit too small to account for the radio opacity, but the contradiction is not as striking as for the alpha and beta rings. The advantage we have in discussing the epsilon ring is that we know precise orbits and approximate masses for both its inner and outer shepherds. We can say with confidence that these satellites could not shepherd the epsilon ring if it were as much as a few times more massive than the self-gravity model predicts (Porco and Goldreich 1987, Goldreich and Porco 1987).

I wish to emphasize that the self-gravity of ring material plays an essential role in the density and bending waves observed in the broader parts of Saturn's rings (Shu 1984). The relative importance of collisional stresses is much smaller. The observed wavelengths yield values for the local ring surface density. These values are in good agreement with those determined in similar regions by the technique of radio occultation. Nothing is amiss with the application of self-gravity here. It is then surprising that self-gravity seems to fail as an explanation of the rigid precession of narrow rings. After all, the distortions of these rings are most naturally viewed as due to standing bending and density waves trapped within them. This view also explains why the most common modes are the Keplerian ellipse and the inclined circular ring. These modes are special because, to support them, self-gravity must only overcome differential precession. For other modes, such as the planet centered ellipse and the radially pulsating circle, self-gravity must compensate for the much larger differential rotation across the ring (Borderies, Goldreich, and Tremaine 1985). We are not sure how these modes are excited in narrow rings, but parametric forcing by shepherd satellites is the leading candidate.

Suppose that the alpha and beta rings are more massive than the self-gravity model implies. What prevents the particles near their outer edges from precessing more rapidly than those near their inner edges? I wish that I could offer an answer to this question, even one that I could not justify, but might at least think plausible. Unfortunately, I cannot.

4. Neptune's Ring Arcs

Following the discovery of the Uranian ring system, it was natural to look for rings about Neptune. After several occultation observations failed to find evidence for material near Neptune, it was concluded that Neptune did not possess a ring system (Elliot et al. 1981). With this background, the 1984 discovery of incomplete arcs orbiting Neptune came as a great surprise (Hubbard et al. 1986). Further occultation observations showed that these arcs cover a small fraction (about 10 percent) of the circumference at a planocentric radius of about 60,000 km. These observations also established that individual arcs are rather short; an arc might be detected at one observatory during a particular occultation but be missed at another. The arcs have radial width's of about 15 km and modest optical depths ($\tau \approx 0.1$) at infrared wavelengths. Pre-Voyager attempts to determine the geometry of Neptune's ring arcs led to the belief that the system of arcs spanned a range in orbital radius. However, Voyager observations found that all of the arcs are contained within a single, diffuse, complete, circular ring (Smith et al. 1989). In hindsight, it is apparent that the incorrect deduction of multiple arc radii was a consequence of an inaccurate value of Neptune's rotational pole (Nicholson et al. 1990).

Neptune's arcs challenge the ingenuity of theorists. Three models have been proposed for this system. In each case azimuthal confinement is associated with a corotation resonance (CR). A corotation resonance occurs at an orbital radius where the angular velocity matches the pattern speed of a perturbation potential. Equilibrium positions for test particles are located at azimuths where the perturbation potential is maximal. For example, the Trojan asteroids librate about corotation resonances created by gravitational perturbations from Jupiter. Because a corotation resonance is located at a potential maximum, energy lost during inelastic collisions causes trapped particles to drain away from it. A source of energy input is necessary to maintain a collection of ring particles in libration about a corotation resonance. Lindblad resonances (LR) are assumed to provide the energy input in models for the Neptune arcs. A Lindblad resonance occurs at an orbital radius where a perturbation potential forces an orbiting test particle at its epicyclic frequency. Particle orbital eccentricity is excited near a Lindblad resonance. Torques produced by satellites at Lindblad resonances are responsible for shepherding particles near ring edges.

Competing models for Neptune's arcs differ in the manner in which the corotation resonances are assumed to arise. In the first model, the corotation resonances are taken to be located at the equilateral, triangular, Lagrange points of hypothetical satellites (Lissauer 1985). The Lindblad resonances are provided by additional satellites. The second model suggests that the corotation resonances are associated with a satellite that moves on an inclined orbit (Goldreich, Tremaine, and Borderies 1986). We refer to these as corotation inclination resonances (CIR). The orbital inclination implies that the satellite's perturbation potential possesses components whose pattern speeds differ from the satellite's orbital angular velocity. There is a Lindblad resonance located near the radius of each circle of corotation resonances. Thus a single satellite might confine the arcs in azimuth and shepherd them in radius. The third model shares with the first the proposal that the corotation potential and Lindblad potential have different sources (Lin, Papaloizou, and Ruden 1987). However, it has the unique feature that the corotation potential is assumed to arise from non-axisymmetric perturbations of the planet's **gravitational** potential (perhaps associated with excited modes of oscillation). In common with the first model, the third model relies on a satellite to provide the Lindblad resonances required for shepherding. That combinations of corotation and Lindblad resonances could act to confine and maintain arcs has been convincingly demonstrated by numerical simulations of inelastically colliding particles orbiting a planet while being perturbed by one or more satellites (Sicardy 1991).

Each of the three models for the Neptune arcs was published prior to the Voyager encounter with Neptune. I am sure that each investigator believed that data returned when Voyager flew by the planet would reveal the true nature of this mysterious system. Unfortunately, this hope was not realized. Although much was learned from the Voyager images, we are still at a loss to explain the origin of the planet's arcs. A measure of our confusion is that subsequent to the Voyager flyby no papers were published either claiming that a previously proposed explanation was correct or advancing a new one. This situation is about to change.

I shall now offer a brief commentary on the difficulties that the Voyager observations present for each of the three arc models. As mentioned previously, Voyager found that Neptune's arcs were embedded in a diffuse ring. The satellite, Galatea, with radius $R \approx 80$km, moves on a circular orbit about 1,000 km inside the location of the ring. The relative inclination of the arc ring to Galatea's orbital plan is small, $i \lesssim 0.03$ degrees. None of the other satellites of Neptune have any resonances close to the arc ring. These facts appear to doom all three arc models. The arcs are not associated with the equilateral triangular points of satellites, since satellites of the required size, $R \gtrsim 100$km, sharing the orbit of the arcs are not seen and could not have been missed. The suggestion that the arcs are located at the corotation resonances of an inclined satellite suffers because the relative inclination of Galatea to the ring plane is so small. For the widths to be interpreted as the spread in semi-major axes of trapped particles, the relative inclination would have to be about 25 times larger than the observationally determined upper limit. Finally, the idea that the corotation potential might be due to the planet's gravitational harmonics fails because the arcs are so short. Very high harmonics, corresponding to azimuthal separation parameter $m \approx 80$, would be needed. Since these decline at least as fast as $(R/r)^{(m+1)}$, incredibly large distortions of the planet would be implied.

Given this pessimistic assessment of previous theoretical models, new inputs from the observational side offer the best hope for progress. These have now been supplied. In a paper currently in the press at Science, Porco (1991) claims that

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the ring arcs are associated with the corotation inclination resonances of Galatea (previously known as 1989N4). I shall devote most of the remainder of this lecture to reviewing her claim.

Porco's analysis of Voyager images reveals the presence of a 42 lobe, 30 km amplitude, radial distortion of the arcs that rotates at a pattern speed equal to the orbital angular velocity of Galatea. This distortion possesses the correct phase and appropriate amplitude (given a reasonable density for Galatea) for forcing of particles whose semi-major axes lie just outside (by $\sim$ 1.6 km) Galatea's 42:43 outer Lindblad resonance (OLR). The detection of perturbations forced by Galatea at its 42:43 OLR which lies within the arcs is satisfying, but not unexpected. What is surprising is the distortion's large amplitude and consistent phase given the substantial ($\sim$ 15 km) radial width of the arcs. We shall return to this point later.

Porco goes on to link the arcs to the inclination corotation resonances of Galatea. This association rests on precise determinations of the mean motions of Galatea and the arcs. Since Galatea is a compact object, its mean motion is determined to considerable accuracy by Voyager observations alone. The arcs are extended objects but, as they were detected 5 years prior to the Voyager flyby, their mean motion is more tightly constrained than that of Galatea (Nicholson et al. 1990). By combining the mean motions determined for Galatea and the arcs with the coefficients of the planet's gravity field, Porco establishes that the orbit of the arcs coincides with the radius of the 42:43 CIR to within an uncertainty of 0.13 km. In addition to their location at particular radii, corotation resonances pick out those azimuths at which the perturbation potential is maximal. Unfortunately, these azimuths cannot be accurately located for the 42:43 CIR because the small relative inclination between the orbits of Galatea and the arcs results in a large uncertainty in the angular position of the relative line of nodes.

Up to here, Porco's arguments rely on direct deductions from observation. That they point to an association of the arcs with Galatea's 42:43 CIR is surprising given the small relative orbital inclination of the arcs and Galatea. Here, Porco introduces a bold hypothesis. She assumes that the radial width of the arcs reflects a spread in orbital eccentricity rather than semi-major axis. The orbital eccentricity is taken to arise from forcing by Galatea at the 42:43 OLR that lies 1.6 km inside the 42:43 CIR. The spread in arc particle semi-major axis is assumed to be given by the width of the corotation resonance, about 0.6 km given reasonable estimates for the mass of Galatea and its relative orbital inclination to the plane of the arcs. Thus the semi-major axes of the arc particles lie between 1.3 and 1.9 km outside the 42:43 OLR of Galatea. The problem posed by the large amplitude and consistent phase of the Lindblad distortion, to which we had referred earlier, is resolved because the semi-major axes of all of the particles lie close to but outside the OLR. The resolution of an outstanding problem raised by the observations lends credence to Porco's hypothesis. However, her picture is not free of difficulties. Perhaps the most serious is that it implies the crossing of orbits of arc particles having different semi-major axes. Given the substantial optical depths of the arcs, there must be concern that collisions will eject arc particles on an untenably short timescale.

Porco goes on to discuss the azimuthal positions of the arcs, including a new leading arc that she has discovered. She proposes that not all consecutive corotation sites are filled, and that some arcs span more than one corotation site. A substantial discussion of this aspect of her work is not possible in the time allotted to my lecture. For more details, I refer you to her paper in Science.

Next, I raise a new possibility. The self-gravity of the arc material may play an important role given the weakness of the 42:43 CIR potential. In particular, satellites with radii of several kilometers would produce gravitational perturbations that are competitive with those due to Galatea's CIR potential. That bodies of this size might be trapped in the corotation sites is not beyond belief. They would be undetectable in the Voyager images.

I must confess that I cannot decide whether Porco's picture of the arcs is correct. Perhaps enough information is buried in the Voyager data set so that this issue will be settled before the next spacecraft visits Neptune sometime during the 21'st century. However, I would not bet on it. Finally, even if Porco's model stands the test of time, we are still faced with understanding the origin of this remarkable system.

After this talk was delivered, I received a preprint from Sicardy and Lissauer (1991) in which they discuss most of the issues involving the Neptune arcs that I covered here, and additional ones as well.

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Discussion

P.K.Seidelmann – Considering the sequencing of images taken by Voyager and the examination of these images to date, is it possible that satellites with inclined orbits may exist, but have not been discovered?
P.Goldreich – I am not well-qualified to answer this question. It is my *impression* that a satellite with radius greater than a few tens of kilometers would have been seen irrespective of inclination if its semi-major axis is less than about 3 planet radii.
K.Aksnes – In Porco's model for retaining the Neptune ring arcs, how critical is the value of the inclination of Galatea's orbit?
P.Goldreich – It is critical that the satellite and arc ring have a small but *finite relative* inclination i. The observational upper bound of i is 0.03°
R.A.Broucke – Is there a possibility that particles in a ring may avoid colliding with each other due to the same type of gravitational interaction which prevents the collision of the coorbital satellites?
P.Goldreich – Collisionless rings have been advocated by Von Eshleman and Michel. However, I think they are, at most, a remote possibility. No one has been able to demonstrate how a collection of particles dense enough to provide the optical depth observed in planetary rings can avoid collisions. Only for the special case in which the planet's gravitational field is taken to be pure $1/r^2$ (so, test particle orbits are exactly periodic) and the self gravity of the particles is ignored, do collisionless rings seem viable.
S.F.Dermott – It may help with these problems to consider how rings are formed. If a narrow ring is formed from the breakup of a small satellite, then one would expect the products to have a size-frequency distribution where most of the mass is in a comparatively few particles and all the area in the small particles, the stuff that we see has very little mass. Have you looked into the dynamics of this more realistic system?
P.Goldreich – I agree with your comment concerning the size distribution of the debris produced when a satellite breaks up. However, I believe it is likely that the size distribution in rings is set by ongoing processes more than by initial conditions. Also, large particles are more difficult to detect (per unit mass) than small ones. The problem with the α and β rings is that their optical depths at radio wavelengths are too high to be consistent with the masses deduced from the self-gravity model for ring precession. Adding large particles would exacerbate this problem.