Second Meeting, December 14th, 1883.

THOMAS MUIR, Esq., M.A., F.R.S.E., President, in the Chair.

The Circles associated with the Triangle, viewed from their Centres of Similitude.

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[*Abstract.*]

The notation adopted in this paper for the triangle ABC is:

G the centroid.

I the centre of the inscribed circle.

I_1, I_2, I_3 the centres of the escribed circles within angles A, B, C.

O the centre of the circumscribed circle.

D, E, F; D_1, E_1, F_1; D_2, E_2, F_2; D_3, E_3, F_3 the points of contact of the inscribed and escribed circles with BC, CA, AB. The Ds lie all on BC, the Es on CA, and the Fs on AB.

H, K, L the mid points of BC, CA, AB.

X, Y, Z the feet of the perpendiculars from A, B, C, on BC, CA, AB.

A′, B′, C′ the vertices, opposite to A, B, C, of the triangle formed by drawing through A, B, C parallels to BC, CA, AB.

1. *a.* AX, BY, CZ are concurrent at O′.
   b. AD_1, BE_2, CF_3 „ „ I′.
   c. AD, BE_3, CF_2 „ „ I_1'.
   AD_3, BE, CF_1 „ „ I_2'.
   AD_2, BE_1, CF „ „ I_3'.

2. *a.* O′, orthocentre of △ABC, is circumscribed centre of △A′B′C′.
   b. I′, ... centre* of △ABC, is inscribed centre of △A′B′C′.
   c. I_1', first ... centre* of △ABC, is first escribed centre of △A′B′C′. Similarly for I_2' and I_3'.

3. *a.* Circumscribed centre of △ABC is orthocentre of △HKL.
   b Inscribed centre of △ABC is ... centre of △HKL.
   c. First escribed centre of △ABC is first ... centre of △HKL.

4. *a.* Centre of similitude of △s ABC, HKL is found by joining AH and O′O, which intersect at G the centroid.

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* Two words are wanted to denote I′ and I_1' with respect to △ABC.

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b. Centre of similitude of $\triangle$s ABC, HKL is found by joining AH and I′I, which intersect at G the centroid.

c. Centre of similitude of $\triangle$s ABC, HKL is found by joining AH and $I_1'I_1$, which intersect at G the centroid.

5. *a.* AO′ is parallel to HO and $=2$HO; $O'G = 2OG$.

b. AI′ „ „ HI and $=2$HI; $I'G = 2IG$.

c. AI_1' „ „ HI_1 and $=2HI_1$; $I_1'G = 2I_1G$.

6. *a.* To find the circumscribed centre of $\triangle$HKL.
From GO′ cut off $GM = \frac{1}{2}$ GO. M is the point required.

b. To find the inscribed centre of $\triangle$HKL.
From GI′ cut off $GJ = \frac{1}{2}$ GI. J is the point required.

c. To find the first escribed centre of $\triangle$HKL.
From GI_1' cut off $GJ_1 = \frac{1}{2} GI_1$. J_1 is the point required.

7. *a.* M is the mid point of O′O, and O′ the external centre of similitude of the circumscribed circles of $\triangle$s ABC, HKL.

b. J is the mid point of I′I, and I′ the external centre of similitude of the inscribed circles of $\triangle$s ABC, HKL.

c. J_1 is the mid point of $I_1'I_1$, and I_1' the external centre of similitude of the first escribed circles of $\triangle$s ABC, HKL.

8. *a.* If on O′O there be taken to the right and left of G segments successively = half of those on the left and right, the points so determined will be circumscribed centres of successive median triangles. Process reversible.

b. If on I′I there be taken to the right and left of G segments successively = half of those on the left and right, the points so determined will be inscribed centres of successive median triangles. Process reversible.

c. If on $I_1'I_1$ there be taken to the right and left of G segments successively = half of those on the left and right, the points so determined will be first escribed centres of successive median triangles. Process reversible.

9. *a.* HM, KM, LM produced bisect O′A, O′B, O′C at U, V, W.

b. HJ, KJ, LJ „ „ I′A, I′B, I′C „ U, V, W.*

c. HJ_1, KJ_1, LJ_1 „ „ $I_1'A$, $I_1'B$, $I_1'C$ „ U, V, W.

10. *a.* Circumscribed circle of $\triangle$HKL is circumscribed circle of $\triangle$UVW.

b. Inscribed circle of $\triangle$HKL is inscribed circle of $\triangle$UVW.

c. First escribed circle of $\triangle$HKL is first escribed circle of $\triangle$UVW.

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* These three triads of points are all different, though denoted by the same letters.

11. *a.* Six parallelograms, whose diagonals intersect at M are HOUO′, KOVO′, LOWO′; HKUV, KLVW, LHWU.

b. Six parallelograms whose diagonals intersect at J are HIUI′, KIVI′, LIWI′; HKUV, KLVW, LHWU.

c. Six parallelograms whose diagonals intersect at J_1 are HI_1UI_1', KI_1VI_1', LI_1WI_1'; HKUV, KLVW, LHWU.

12. *a.* HWKULV is a hexagon whose opposite sides are parallel, and respectively $=\frac{1}{2}O'A$, $\frac{1}{2}O'B$, $\frac{1}{2}O'C$.

b. HWKULV is a hexagon whose opposite sides are parallel, and respectively $=\frac{1}{2}I'A$, $\frac{1}{2}I'B$, $\frac{1}{2}I'C$.

c. HWKULV is a hexagon whose opposite sides are parallel, and respectively $=\frac{1}{2}I_1'A$, $\frac{1}{2}I_1'B$, $\frac{1}{2}I_1'C$.

13. *a.* AO′, BO′, CO′ pass through the points where the circumscribed circle of △HKL cuts the sides of △ABC.

b. AI′, BI′, CI′ pass through the points where the inscribed circle of △HKL touches the sides of △HKL.

c. AI_1', BI_1', CI_1' pass through the points where the first escribed circle of △HKL touches the sides of △HKL.

On Determinants with *p*-termed elements.

By Thomas Muir, M.A., F.R.S.E.

This paper will be found in the *Messenger of Mathematics* for January 1884, Vol. xiii, New Series.

Construction for Euclid II. 9, 10.

By R. W. M'Arthur.

Take line AB divided in C and D as in Euclid. On AD describe the rectangle AEFD having AE, DF each equal to AC or CB. According as D is in AB, or in AB produced, from DF or DF produced cut off FG equal to DB; and join CG, GE, EC.

Mr James Taylor gave a proof of the known theorem :—"If two sides of a skew quadrilateral ABDC inscribed in a circle be produced to meet in E, and FEG be drawn perpendicular to the diameter passing through E, the two other sides produced make equal intercepts on FEG." Mr Taylor's object was to call attention to the desirability of obtaining a simpler mode of demonstration.