DARK MATTER IN THE UNIVERSE

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Thirty years ago, observational cosmology consisted of the search for two numbers: H_o, the rate of expansion of the universe at the position of the Galaxy; and q_o, the deceleration parameter. Twenty years ago, the discovery of the relic radiation from the Big Bang produced another number, 3^oK. But it is the past decade which has seen the enormous development in both observational and theoretical cosmology. The universe is known to be immeasurably richer and more varied than we had thought. There is growing acceptance of a universe in which most of the matter is not luminous. Nature has played a trick on astronomers, for we thought we were studying the universe. We now know that we were studying only the small fraction of it that is luminous. I suspect that this talk this evening is the first IAU Discourse devoted to something that astronomers cannot see at any wavelength: Dark Matter in the Universe.

Because we are just at the beginning of the study of dark matter, we must ask simple questions, perhaps deceptively simple questions.

Does it exist? Another way to formulate this question is to ask: Does the distribution of luminosity trace the distribution of mass? I hope the discussion will convince you that dark matter does exist.

Where is it? This is the question we can answer best, for the gravitational attraction of the dark matter on the luminous matter permits us to trace its distribution. Much of the talk will be devoted to answering this question.

How much is there? At present, this question appears to have several alternative answers. All dynamical information leads to one answer, while the arguments of the theoretical physicist lead to a different one. And if we wish to disregard conventional Newtonian gravitational theory, we can produce still a third answer.

What is it? This is the question we can answer at present only by saying we don't know.

WHERE IS IT? It seems likely that dark matter is associated with structures of all sizes ranging upward from galaxies. Our own Galaxy, the Milky Way, contains dark matter. Oort (1960) showed that the

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distribution and motions of stars above and below the galactic plane imply a mass in the plane which is of the order of 0.185 $M_\odot$ pc^{-3}, while a count of all known or inferred stars, gas, and dust produces a surface mass density only one-half this amount. Recent work by Bahcall (1984) and coworkers has supported this discrepancy. However, the discussion this evening will be concerned principally with dark matter not in our Galaxy but dark matter associated with other spiral galaxies. We do not know if the dark matter in the plane of our Galaxy is of the same type as that now believed to surround spiral galaxies.

One major result from work during the past ten years is the understanding that dark matter is a property of individual field spiral galaxies. Because this is the area in which I have been working, I will describe the work in some detail. In a distant disk galaxy, all of the stars orbit in concert about the distant nucleus. Viewed at a proper angle, the orbital motion carries the stars toward the observer on one side of the major axis of the galaxy, and away from the observer on the other side. Because the gas and stars act as test particles in the gravitational potential of the galaxy, study of their motions tells us about the distribution of mass in the galaxy.

Over the past ten years, my colleague W. Kent Ford, Jr. and I have obtained long-slit spectra for spiral galaxies covering a wide range of Hubble types and luminosities. The observations have been carried out at Cerro Tololo, Kitt Peak, Las Campanas and Lowell Observatories. From the emission lines in the spectra (Fig. 1), especially Hα, we can measure velocities of high accuracy at successive radial distances, and hence map the velocity field of the galaxy. By analogy with the solar system, in which the orbital velocities are lower for more distant planets, astronomers had long expected that the rotational velocities would first increase with increasing radial distance from the nucleus, reach a maximum, and then fall to low velocities at large radial distances.

However, observations across the luminous disks of nearly 100 galaxies (Fig. 2) reveal several major facts: (1) Falling velocities are not observed; rotation curves are flat or slightly increasing even at large nuclear distances. (2) Rotation curve <u>amplitude</u> increases with galaxy luminosity. The larger, more massive, more luminous galaxies have higher rotational velocities. (3) Rotation curve <u>form</u> is similar for galaxies of very different optical morphologies.

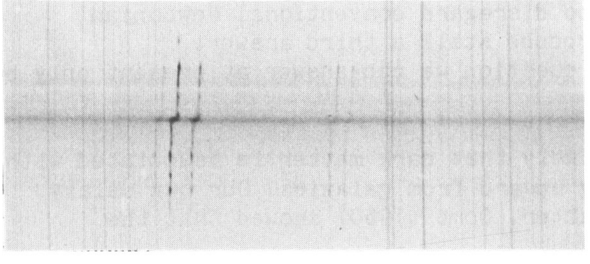

Fig. 1. Spectrogram along the major axis of NGC 7541, taken with the Kitt Peak 4-m spectrograph which incorporates a Carnegie image tube. Straight vertical lines arise in the earth's atmosphere. Tilted emission lines come from gas in the rotating galaxy disk.

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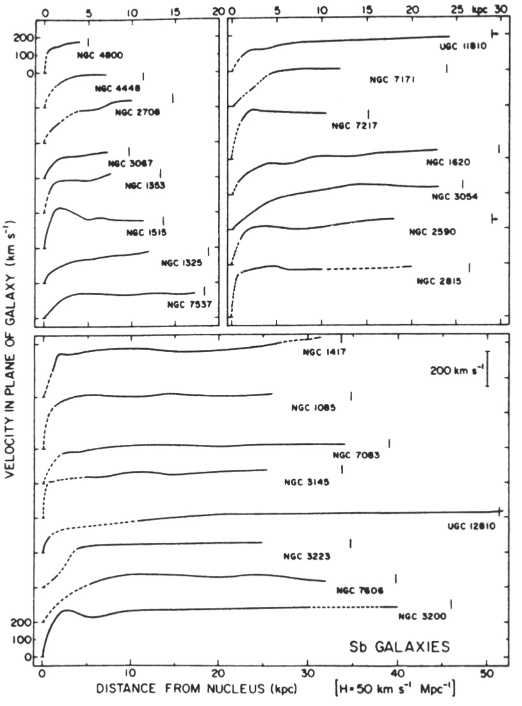

Fig. 2. Rotation curves for 23 Sb galaxies, arranged according to galaxy luminosity. With increasing luminosity, the galaxies get larger, the maximum rotational velocity increases, and the mass increases. Note that all galaxies have generally flat rotation curves; nowhere is a Keplerian decrease in velocity observed.

To emphasize the similarity of form for galaxies of very different optical morphology, I compare in Fig. 3 the rotation curves for an Sa, Sb, and Sc galaxy. The lower three curves, all of similar form, come from an Sa with a bulge-to-disk luminosity ratio of 4, and from an Sc with a bulge-to-disk ratio of 0.1. Thus, these two galaxies have rotational velocities and hence mass distributions of similar form, even though one galaxy is almost all bulge, and one is almost all disk. This fact alone implies that the form of the luminosity distribution is not a major factor in determining the form of the mass distribution: the distribution of luminosity is not mapping the distribution of mass.

We can deduce specific characteristics of the distribution of the dark matter from an application of Newton's laws to the orbital motion of the stars and gas. Assume that in a disk galaxy the stars rotate about the center in circular orbits, with velocities that result from the combined gravitational attraction of all matter (luminous and dark) in the galaxy. Then from the equality of the gravitational and centrifugal forces on a mass m at distance r from the center,

$$\frac{G\,M(r)m}{r^2} = k\,\frac{mV^2(r)}{r}\,, \qquad (1)$$

where M(r) is the luminous plus dark matter interior to r and G is the gravitational constant. The constant k is of order unity and depends on the geometry of the mass distribution. If we adopt units such that G=1 and ignore the small dependence on k, then

$$V(r) = [M(r)]^{1/2}/r^{1/2}. \qquad (2)$$

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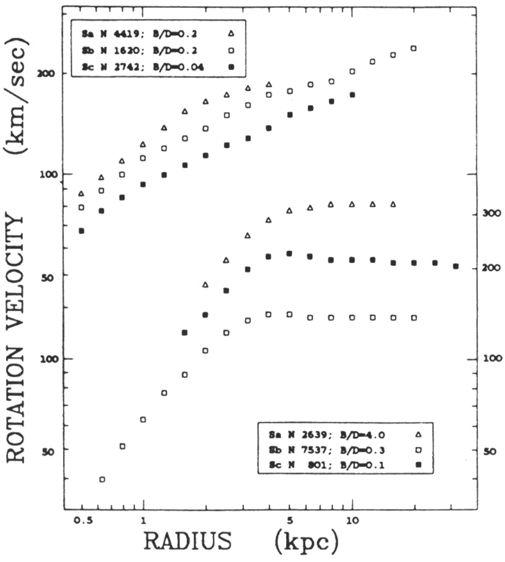

Fig. 3. Rotation curves for two sets of galaxies. Although the forms of the rotation curves are similar in each set, each set contains an Sa, Sb, and Sc galaxy of very different morphology. For the lower three rotation curves, the bulge-to-disk ratios differ by a factor of 40, yet the shapes of the rotation curves do not reflect this difference.

In the solar system, where virtually all of the mass is located in the sun, V consequently decreases with increasing r. However, the observation of constant velocity in the rotation of galaxies means (from Eq. 2) that mass rises linearly with radius, and does not approach a limiting mass at the edge of the optical galaxy. We conclude that orbital velocities remain high in response to the gravitational attraction of matter which we cannot see. Several conclusions follow from such observations.

(1) The dark matter is less centrally concentrated than is the luminous matter. With mass growing as radius, local mass density M/r^3 is falling as ρ^{-2}; but galaxy luminosity is falling faster. Thus the ratio of mass-to-luminosity, M/L, is increasing across a galaxy.

(2) The dark matter is clumped about galaxies. The value of the mass density at the limits of the optical galaxies is higher by about 4 orders of magnitude than the mean mass density in the universe.

(3) The dark matter is more extended than the luminous matter. In some galaxies, neutral hydrogen is distributed well beyond the optical galaxy, and 21-cm observations show that this gas too is orbiting with velocities which remain virtually constant at large radial distances. The 21-cm rotation curve for M31 (Roberts 1975, Fig. 4), an important milestone in revealing that rotation curves are flat, was followed by a series of extended rotation curves from 21-cm observations (Bosma 1978). These velocities also indicate that galaxy mass continues to rise virtually linearly with distance from the nucleus, even beyond the optical galaxy. Thus the neutral hydrogen, which acts as a probe of the potential arising from the dark plus luminous matter, confirms once more that optical luminosity does not trace mass.

Recent second-generation high-sensitivity 21-cm observations have determined rotation velocities extending to several times the optical

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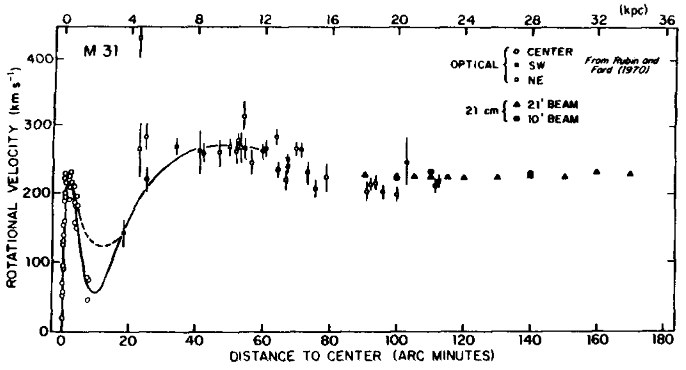


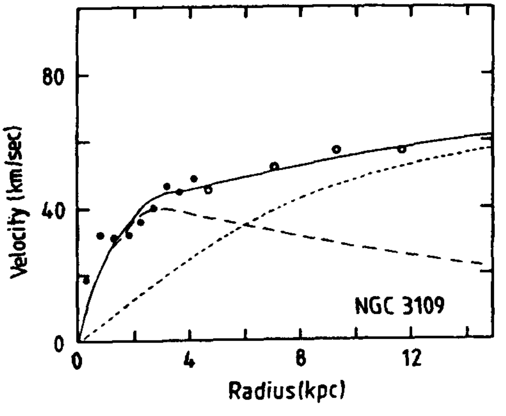
Fig. 4. Rotation velocities for M31 from optical and from radio observations of Roberts and colleagues. The most distant HII regions identified optically are 120' from the nucleus.

diameter, for late-type galaxies (van Albada et al. 1985; Carignan and Freeman 1985). In all cases, the velocities remain flat or increase (Fig. 5) beyond the optical image. From the optical surface-brightness profile, rotation velocities can be predicted for a disk of constant mass-to-luminosity ratio. As seen in Fig. 5, the predicted velocities rise to a maximum in a few kpc, and then fall well below the observed velocities. A component of dark matter is postulated, which produces the velocities indicated by the dotted curve. Note that with increasing radial distance the dark matter becomes a larger fraction of the total mass; at largest distances the M/L ratio locally exceeds several hundred.

To these conclusions concerning the distribution of dark matter we can add one more.

(4) The gravitational potential is more nearly spherical than flat. Prescient theoretical arguments relating to the stability of spiral disks led Ostriker and Peebles (1973) to predict that disk galaxies are imbedded in halos whose mass is a few times the disk mass. A similar conclusion comes also from analysis of warps in disks, and studies of velocity dispersions and thickness of the gas layer (van der Kruit and Freeman, 1984). And recent studies of polar ring galaxies, those curious

Fig. 5. Rotation curve of NGC 3109 from optical (filled circles) and radio observations, deconvolved into velocities produced by disk mass (dashed curve) and halo mass (dotted curve) from Carignan (1985). Note that the contribution from the halo equals that of the disk at about the limit of the optical disk; the halo is the dominant mass at greater radial distances.



disk galaxies with a ring of matter encircling the rotation axis (Schweizer, Whitford, and Rubin 1984) show that velocities at height z along the rotation axis are equal to velocities at r=z in the plane of the disk. This circumstance holds only in a spherical potential.

The above evidence convinces us that a component of dark matter is clumped around spiral galaxies, distributed in a mostly spherical halo; the fraction of dark-to-luminous matter becomes larger with increasing nuclear distance. But astronomers had earlier known that dark matter was present in clusters of galaxies. As early as 1933, a sufficient number of radial velocities were available for galaxies in the Coma cluster so that an analysis of the cluster dynamics could be made. Zwicky (1933) noted that the individual galaxies are moving so rapidly that their mutual gravitational attraction (calculated from their luminous mass) is insufficient to hold the cluster together. If clusters are not flying apart (and the evidence is that they are not), then dark matter must be present; its gravitational attraction binds the cluster together. Zwicky deserves credit for being the first astronomer to uncover evidence for the existance of non-luminous matter. Initially, astronomers were prone to think that this "missing mass" was an exotic property of clusters, unrelated to more isolated galaxies. The importance of the recent observational work is that it demonstrates that non-luminous matter is a property of single galaxies as well.

We have seen that gas and stars orbiting a galaxy can be used as test particles of the galaxy potential; similarly, galaxies in clusters can be used as test particles of the cluster potential. And recently, the hot x-ray gas surrounding cluster ellipticals has been used to determine the gravitational potential and mass distribution for a few elliptical galaxies.

The elliptical galaxy M87, the second brightest galaxy in the Virgo cluster, has two unique properties: it is located approximately at the center of the cluster, and it is virtually at rest with respect to the mean cluster motion. Observations show that M87 is enveloped by an enormous hot plasma which radiates X-rays. This X-ray gas has been detected as far as 1.5 degrees from M87, a distance which encompases several other Virgo galaxies. Fabricant, Leçar, and Gorenstein (1980) calculate that the mass in X-ray gas is 10^{12} $M_\odot$. Due to its high kinetic temperature, the gas will escape from the galaxy unless there is sufficient mass in the galaxy to retain gravitationally this corona. Calculations show that a mass for M87 of 3 to 6 x $10^{13} M_\odot$ is necessary to bind the hot gas. Mass-to-light ratios of several hundreds are implied at distances of order 100 kpc from the nucleus of M87. It is interesting to recall that many years ago de Vaucouleurs (1969) and Arp and Bertola (1969) produced evidence that optical images of M87 show a faint image extending several degrees.

M87 is the only elliptical galaxy in the Virgo cluster to exhibit such an extended massive X-ray halo. It is clear that the potential well at the position of M87 is enormous, and may represent the potential of the cluster as a whole. It is also possible that the M87 halo has been acquired from other galaxies in the cluster. Regardless of the evolutionary history, this observation is evidence that one giant elliptical galaxy contains dark matter extending to an enormous distance beyond the optical galaxy.

There is additional evidence that other X-ray galaxies contain dark matter too. Observations of about 50 elliptical galaxies located outside the cores of clusters by Forman, Jones, and Tucker (1985) reveal that each galaxy has a massive X-ray halo. Using arguments of hydrostatic equilibrium analogous to those used for M87, these authors show that each galaxy must contain a significant quantity of dark matter. Although there is at present some question of the applicability of the equilibrium arguments, it seems quite probable that elliptical galaxies, like their spiral counterparts, contain more dark than luminous matter.

HOW MUCH IS THERE? Let's start by discussing how much matter we can see. When you attempt to build a galaxy out of stars and gas and dust, as Larson and Tinsley (1978) and others have done, you find that the mean value of the ratio of mass-to-luminosity, M/L, is of the order of one or two. But the galaxy dynamics tell a different story. Calculated values for M/L are about twice those derived for the Tinsley-Larson models. And from the component mass deconvolution discussed above, it appears that the luminous disk mass is matched by an equal non-luminous halo mass (Fig 5). Moreover, beyond the optical galaxy out to the largest radii to which rotation velocities have been measured, the dark matter amounts to 5 or 10 times the luminous matter.

To pursue the answer for a larger region of the universe, we must define a few terms. We accept the following consistent set of values.

$$H_o = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

$$\rho_c = 3H_o^2/8\pi G$$

$$\sim 10^{11} M_\odot \text{ Mpc}^{-3}$$

$$\langle L \rangle = 10^8 L_\odot \text{ Mpc}^{-3}$$

$$\Omega = \rho/\rho_c$$

$$M/L = \Omega\rho_c/L \sim \Omega\, 1000\, M_\odot/L_\odot$$

The mean mass density which will just halt the expansion, i.e., close the universe, is designated ρ_c; Ω is the ratio of the observed density to the closure density. The mean luminosity density, $\langle L \rangle$, is a very uncertain number, for it comes from integrating the faint uncertain end of the luminosity function, and depends also upon the choice of values of extinction in our Galaxy. The listed value is a best guess. With these adopted values, a value of M/L=1000 will close the universe; a galaxy 1/10th the mass of our Galaxy or 1/10th the Andromeda galaxy located each cubic megaparsec will close the universe. But values as large as M/L=1000 are not found in any dynamical studies. Rather, the analysis of galaxy dynamics results in M/L values of 10's and 100's.

Order-of-magnitude results for various dynamical systems are shown in Table 1. While exact numbers are controversial, I think this set of values is a fair assessment of the current status of our knowledge. Details of the various analyses and results up to 1979 are reviewed in an important paper by Faber and Gallagher (1979), and in numerous surveys since that time.

As is apparent from the upper part of the table, various dynamical analyses give values of M/L less than about 200, corresponding to a value of Ω <0.2. Thus all of the dynamical studies would be satisfied with a universe whose mass is only about 1/5 the critical mass, but one in which the quantity of dark matter exceeds by a factor of ten the luminous matter.

Table 1. Values of M/L and Ω for Various Systems

System	Mass	Scale	M/L	Ω
Visible				
Galaxy	10^9–10^{11}	25 kpc	2	0.002
Dynamical				
Galaxy	10^{10}–10^{12}	25 kpc	10	0.01–0.02
Binary	10^{11}–10^{13}	50 kpc	50	0.05
Group	10^{13}	150 kpc	150	0.15
Cluster	10^{15}	250 kpc	250	0.25
Local Supercluster	10^{15}	20 Mpc	300:	0.15–0.30
Deuterium abundance				<0.2
Inflation				1
Closed				>1

A measure of the density of the universe also comes from the theory of nucleogenesis in the conventional Big Bang cosmology. Because Deuterium, produced in the inital Big Bang, can only be destroyed by subsequent evolution, its current abundance is a measure of the density of the universe. Early studies by Tinsley, Schramm and coworkers, and recent detailed analysis by Audouze (1985) and colleagues are all consistent in deducing a low universe density, i.e., Ω < 0.2. Thus, in agreement with the dynamical results, our universe can be one in which the dark matter is normal baryonic matter: matter which has been processed and which has evolved along with the universe.

Such a universe is consistent with the observations. However, it poses questions which theorists find difficult to answer. How did galaxies imitially form in such a world? Why is the microwave background radiation so smooth, given the lumpy distribution of matter, and a universe so large that regions of it could never have communicated with each other? And why is $\Omega=0.2$ so tantalizingly close to $\Omega=1$? Why, for example, is Ω observed not orders of magnitude smaller or orders of magnitude larger than unity?

Given these valid questions, theorists have postulated a universe in which $\Omega=1$. The amount of dark matter in such a universe is much larger than that required by the dynamical arguments; the dark matter cannot be baryonic. This model modifies conventional Big Bang cosmology to incorporate a time of rapid inflation during the initial universe, thus solving the smoothness or horizon problem.

We may simplify the current status of models of the universe into these two extremes: one with $\Omega=0.2$ which is derived from dynamics, and one with $\Omega=1$ which is derived from theory. There is, however, still a third model, which is presently favored by only a very few. In this model, "what you see is what you get." That is, the distribution of mass in a galaxy is described by the distribution of light, but Newtonian potential theory is assumed to be not valid for the very low values of acceleration encountered. Using a modified form for the gravitational attraction, the observed distribution of light gives rise to the observed velocities. In this model, Ω is very small, Ω 0.002. Bekenstein and Milgrom (1984) have been working on such models in Israel, and I am sorry that they are not here to discuss their work.

WHAT IS IT? Before we discuss this further, one relevant observation must be introduced: the large scale distribution of galaxies in the universe. If galaxies formed in a sea of exotic particles, these particles determined the dynamical evolution of the early universe, and hence the nature of the universe we inhabit today. A signature of this early universe must remain in the large scale distribution of galaxies which we presently observe.

Some years ago, Seldner et al. (1977) produced a plot of the 1,000,000 brightest galaxies (Fig. 6), using observations of Shane and Wirtanen (1967). A remarkable distribution of chains, strings, filaments, and voids, all connected in a lace-like pattern, gave visual evidence of the non-random distribution of bright galaxies. But more than the two dimensional distribution of galaxies is clumped. Radial velocity measures of galaxies in Bootes (Kirshner, Oemler, Schechter, and Shectman 1981) were the first to confirm that there is a region (in velocity space) void of galaxies, which extends over 50 Mpc. These observations, coupled with the identification of other large regions

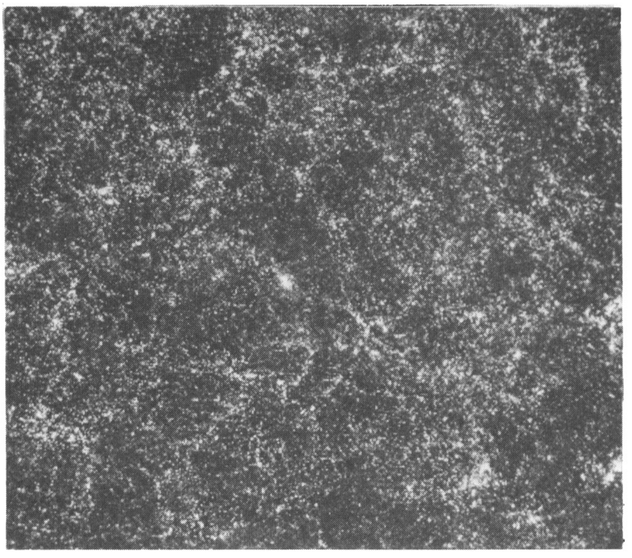


Fig. 6 The million brightest galaxies as they appear on the northern sky, from counts by Shane and Wirtanen (1967) at Lick Observatory, newly reduced by Seldner et al. (1977). The north galactic pole is at the center; the galactic equator is just off the edge of the figure. Note the striking lacelike pattern, and the conspicuous voids.

void of galaxies, have produced an enormous industry in N-body calculations, as astronomers attempt to identify the physical and dynamical conditions which would give rise to such clumpy distributions.

The deepest survey of galaxy distribution is that of Lapparent et al. (1986) who have obtained velocities for all galaxies brighter than magnitude 15.5 in a strip of sky 6^{o} by 120^{o} going through the Coma cluster. Assuming that the Hubble flow is smooth, an assumption which has yet to be seriously tested by independent distances to galaxies at large distances, these results show the galaxies arranged in gigantic strings and voids, forming structures as large as the scale of the observations. These enormous features may be coming close to conflicting with the smoothness implied by the microwave background, and to taxing gravitational models. They may instead suggest explosive hydrodynamical galaxy formation, as Ostriker and Cowie (1981) suggested. Because structure size has continued to grow with sample size, we may not yet have studied a large enough fraction of the universe to comprehend its structure.

Currently, we do not know if any matter IS present in the voids. Are they void of all matter? Or only void of baryonic matter? Are low luminosity galaxies present there? Is there a critical density, below which galaxies will not form? Does the formation of some galaxies inhibit the formation of others? These are the questions which follow from the apparently clumpy distibution of the most luminous galaxies.

I will conclude with only a few brief comments suggesting possible forms for the dark matter. Cowsik and McClelland (1973) were the first to point out that if the neutrino has a mass in the range of 10's of electon volts, this would have interesting cosmological implications. Since then, experimental evidence for the existance of a neutrino mass has been questioned, and cosmological implications of a neutrino mass have been examined. Neutrinos are hot, relativistically moving particles, and a universe dominated by neutrinos would form enormous structures early in its history. Such a universe would form "top-down." The largest structures separate out first, and later substructures form clusters and then galaxies. Fragmentation down to galaxy size would take an appreciable fraction of the age of the universe; galaxies would have only recently formed. Such a time scale seems to conflict significantly with our present ideas of galaxy evolution.

Many cosmologists now favor an alternative model of a universe dominated by cold particles. Such particles, photinos, gravitons, axions (and there are many more) have never been detected, but are permitted by the physics of the gauge theories from which they emerge. In such a universe, the cold axions, for example, form clumps early in the universe. These withstand the expansion, and merge in hierarchical fashion to form galaxies and ultimately clusters of galaxies. The major drawback is that these models are based on particles whose existances are presently only postulated.

Some of the possible dark matter candidates are listed in Table 2, column 1; the observations they must explain are listed in column 2. Only future work will tell if we are presently close to answering the question, "What is it?"

Table 2. Dark Matter in the Universe

Possible Candidates	To Explain
Massive object $>10^5 M_\odot$	Milky Way: gravitational attraction of stars
Massive object 200 $M_\odot$	Spiral Galaxies:rotation curves
Massive object	Elliptical Galaxies: X-ray halos
Neutron star	Clusters of Galaxies
White dwarf	Largest scale: distribution of clusters and superclusters
Low mass planet $< 0.1 M_\odot$	
Primordial black hole	
Monopoles	
inos $\begin{cases} \text{cold} \\ \text{warm} \\ \text{hot} \end{cases}$	$\Omega=1$

Tonight we have been telling a story, a story whose details we are just beginning to comprehend. We can understand a little concerning the early universe, and we can attempt to predict its future history. Most of the answers still elude us. We have a lot to learn.

In conclusion, I wish to use this opportunity to thank our Indian hosts for offering us this opportunity to meet with our colleagues in an atmosphere which is intellectually stimulating and culturally exciting. We will not forget this wonderful stay.

Ever since women and men have been looking at the stars, some have been asking questions like those asked by the Indian poets in the Ṛg Veda, in the 1st millenium, B.C.

"Then even nothingness was not, nor existance.
There was no air then, nor the heavens beyond it.
What covered it? Where was it? In whose keeping?
Was there then cosmic water, in depths unfathomed?

At first there was only darkness wrapped in darkness.
All this was only unillumined water.
That One which came to be, enclosed in nothing,
arose at last, born of the power of heat."

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