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GROUPS WITH PRESCRIBED AUTOMORPHISM GROUP: A CLARIFICATION

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In Theorems 1 and 2 of **[1]** necessary and sufficient conditions were given for a group G to have a finite automorphism group $\operatorname{Aut} G$ and a semisimple subgroup of central automorphisms $\operatorname{Aut}_c G$. Recently it occurred to us, as a result of conversations with Ursula Webb, that these conditions could be stated in a much simpler and clearer form. Our purpose here is to record this reformulation. For an explanation of terminology and notation we refer the reader to **[1]**.

Theorem 1*. *Let G be a group such that $\operatorname{Aut} G$ is finite and $\operatorname{Aut}_c G$ is semisimple. Then one of the following holds: here Q is a finite group with trivial centre and $q = |Q_{ab}|$.*

(i) $G \simeq \bar{G}(Q, F, \varepsilon) \equiv G(Q, F, \varepsilon)/M(Q)_{q'}$ *where F is a torsion-free abelian group, $\varepsilon \in \operatorname{Ext}(Q_{ab}, F)$ and $C_{\operatorname{Aut} F}(\varepsilon) = 1$.*

(ii) $G \simeq (G(Q, 1, 0)/K) \times D$ *where $K \leqq M(Q)_{q'}$, D is an elementary abelian 2-group of order different from 4 and*

$$q \cdot |M(Q)_{q'} \cdot K| \text{ is prime to } |D|.$$

The point here is that if G is infinite, then $G \simeq \bar{G}(Q, F, \varepsilon)$, which is a central extension of F by Q, the cohomology class being determined by ε. If G is finite, its structure is given by (ii); note that $G(Q, 1, 0)$ is a central extension of $M(Q)_{q'}$ by Q whose cohomology class is determined by the canonical projection $M(Q) \to M(Q)_{q'}$.

Proof of Theorem 1*. We know that G has the structure described in Theorem 1 of **[1]**. Assume that G is infinite, so that $F \neq 1$. Now F is divisible by $l = |D| \cdot |M(Q)_{q'} : K|$. The mapping $x \mapsto x^l$ is an automorphism of F, say α; this induces an automorphism in $E = \operatorname{Ext}(Q_{ab}, F)$ which is just multiplication by l. Since $(q, l) = 1$, there is a positive integer n such that $l^n \equiv 1 \bmod q$. Also $qE = 0$. Hence α^n operates trivially on E. But $C_{\operatorname{Aut} F}(\varepsilon) = 1$, so in fact $l = 1$. Therefore $D = 1$ and $M(Q)_{q'} = K$, as required.

Theorem 2 of **[1]** provides an immediate converse of Theorem 1*.

Theorem 2*.

(i) *If $G = \bar{G}(Q, F, \varepsilon)$ as in Theorem* 1* (i), *then* $\operatorname{Aut} G \simeq \operatorname{St}_{\operatorname{Aut} Q}(\varepsilon^{\operatorname{Aut} F})$ *and* $\operatorname{Aut}_c G = 1$.

(ii) *If* $G = (G(Q,1,0)/K) \times D$ *as in Theorem* 1*(ii), *then* $\operatorname{Aut} G \simeq N_{\operatorname{Aut} Q}(K) \times \operatorname{Aut} D$ *and* $\operatorname{Aut}_c G \simeq D$.

Finally, as a result of these simplifications we may refine Theorem 7 of [**1**] by replacing statement (v) by

(v)* *An infinite group* G *satisfies* $\operatorname{Aut} G \simeq S_4$ *if and only if* $G \simeq G(F,\varepsilon)/\mathbb{Z}_2 \equiv \bar{G}(A_4, F, \varepsilon)$ *where* F *is a non-trivial torsion-free abelian group, and* ε *in* F/F^3 *is such that* $C_{\operatorname{Aut} F}(\varepsilon) = 1$.

REFERENCE

1. D. J. S. Robinson, Groups with prescribed automorphism group, *Proc. Edinburgh Math. Soc.* **25** (1982), 217–227.

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