BULL. AUSTRAL. MATH. SOC. 46L05, 46L20, 47A10
VOL. 7 (1972), 307-308.

On numerical range and its application to Banach algebras

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The spatial numerical range of an operator on a normed linear space and the algebra numerical range of an element of a unital Banach algebra, as developed by Lumer and Bonsall, are considered and the theory of such numerical ranges applied to Banach algebra.

The first part of the thesis is largely expository, as in it we introduce the basic results on numerical ranges. For an element of a unital Banach algebra, the question of approximating its spectrum by numerical ranges has been considered by Bonsall and Duncan. We give an alternative proof that the convex hull of the spectrum of an element may be approximated by its numerical range defined with respect to equivalent renormings of the algebra. In the particular case of operators on a Hilbert space this leads to a sharper version of a result by Williams.

An element is hermitian if it has real numerical range. Such an element is characterized in terms of the linear subspace spanned by the unit, the element and its square. This is used to characterize Banach*-algebras in which every self-adjoint element is hermitian. From this an elementary proof that such algebras are B^*-algebras in an equivalent norm is given. As indicated by Palmer a formula of Harris is then used to show that the equivalent renorming is unnecessary, thus giving a simple proof of Palmer's characterization of B^*-algebras among Banach algebras.

The closure properties of the spatial numerical range are studied. A construction of Berberian is extended to normed linear spaces; however

Received 29 March 1972. Thesis submitted to the University of Newcastle, New South Wales, February 1972. Degree approved, March 1972. Supervisor: Dr J.R. Giles.

because the numerical range need not be convex, the result obtained is weaker than that of Berberian for Hilbert spaces.

A Hilbert space or an l_p-space $(1 \leq p < \infty)$ is seen to be finite dimensional if and only if all the compact operators have closed spatial numerical range. The spatial numerical range of a compact operator on a Hilbert space, or a l_p-space $(1 < p < \infty)$, is shown to contain all the non-zero extreme points of its closure. So for a compact operator on Hilbert space the spatial numerical range is closed if and only if it contains the origin.

Operators which attain their numerical radius are also considered. A result of Hilbert is extended to a class of Banach spaces. In a Hilbert space the hermitian operators which attain their numerical radius are shown to be dense among all the hermitian operators. This leads to a stronger form of a result by Lindenstrauss in the special case of operators on a Hilbert space.

https://doi.org/10.1017/S0004972700045081 Published online by Cambridge University Press