and the one to go twelve times as fast as the other. It is easy to see that while the slower clock goes 12 hours the two clocks have the conditions satisfied 143 times. But all these combinations of readings are not different. Thus we have

A 7 hr. $53\frac{41}{143}$ min., B 10 hr. $39\frac{63}{143}$ min.; and A 10 hr. $39\frac{63}{143}$ min., B 7 hr. $53\frac{41}{143}$ min. On the other hand, however, solutions of the type A 1 hr. $5\frac{5}{11}$ min., B 1 hr. $5\frac{5}{11}$ min., when all four hands point in the same direction, only occur once. We therefore have in all 77 (*i.e.*, $66+11$) different combinations.

Problems can be made involving any relation between the directions of the hands in the two clocks.

Another type of clock problem is indicated by

Find the first time after 1 o'clock when the minute hand of a clock bisects the angle between the hour and seconds hands.

J. JACKSON.

**Multiplication and Division of Vulgar Fractions.**—The difficulty in teaching Multiplication and Division of Fractions consists solely in the fact that the ideas formerly associated with these operations when whole numbers were concerned no longer fit the case when we come to deal with fractions, *e.g.*, $9 \times 7$ can be intelligibly interpreted as meaning the number of articles in 9 bundles, each containing 7 articles. In this respect, multiplication is simply contracted addition. When we come to a case like $\frac{3}{5} \times \frac{2}{7}$, we cannot frame a question on the lines of the former. But if we present the problem in a concrete form we are more likely to be successful in teaching young pupils the method of multiplying and dividing vulgar fractions.

8 tons at 7/- per ton cost $(8 \times 7)$/-.

So we may say that £$(8 \times \frac{3}{10})$ represents the cost of 8 tons at £$\frac{3}{10}$ per ton.

But £$\frac{3}{10}$ is 3 florins.

Therefore 8 tons at £$\frac{3}{10}$ per ton cost $(8 \times 3)$ florins, *i.e.*, £$\frac{8\times 3}{10}$.

Therefore we now have that £$(8 \times \frac{3}{10}) =$ £$\frac{8\times 3}{10}$.

As before we may say that

$\pounds(2 \times \frac{3}{4})$ represents the cost of 2 tons at $\pounds\frac{3}{4}$ per ton.

But 2 tons at 3 crowns per ton cost $(2 \times 3)$ crowns, *i.e.*, $\pounds\frac{2 \times 3}{4}$.

Therefore we arrive at the conclusion that $\pounds(2 \times \frac{3}{4}) = \pounds\frac{2 \times 3}{4}$.

Coming now to a case like $\frac{3}{5} \times \frac{7}{4}$, we may say that $\pounds(\frac{3}{5} \times \frac{7}{4})$ represents the cost of $\frac{3}{5}$ of a ton at $\pounds\frac{7}{4}$ per ton.

This is the same as the cost of 12 cwts. at 35/- per ton which gives us an answer of 21/- or $\pounds\frac{21}{20}$.

We conclude therefore that $\pounds(\frac{3}{5} \times \frac{7}{4}) = \pounds\frac{21}{20}$.

If we now tabulate the results thus :—

(1) $\pounds(8 \times \frac{3}{10}) = \pounds\frac{8 \times 3}{10} = \pounds\frac{24}{10}$ ;

(2) $\pounds(2 \times \frac{3}{4}) = \pounds\frac{2 \times 3}{4} = \pounds\frac{6}{4}$ ;

(3) $\pounds(\frac{3}{5} \times \frac{7}{4}) = \pounds\frac{3 \times 7}{5 \times 4} = \pounds\frac{21}{20}$ ;

the method of arriving at the answer is immediately obvious. A few easy examples on the lines of (3), where the answer can be got without reference to fractions, will show the pupil that the method gives the correct solution.

A somewhat similar process enables us to get over the difficulty in the case of Division.

If 8 tons cost £16, we know that 1 ton costs $\pounds(16 \div 8)$.

On the analogy of this we say that

(1) $\pounds(5 \div \frac{1}{4})$ represents the price of 1 ton if $\frac{1}{4}$ ton cost £5.

But if $\frac{1}{4}$ ton cost £5, 1 ton costs $\pounds(5 \times 4)$.

$\therefore \pounds(5 \div \frac{1}{4}) = \pounds(5 \times 4)$.

(2) Similarly $\pounds(5 \div \frac{3}{4})$ represents the cost of 1 ton if $\frac{3}{4}$ ton cost £5.

In this case one ton costs one-third of what it cost in case (1), *i.e.*, $\pounds\frac{5 \times 4}{3}$.

$\therefore \pounds(5 \div \frac{3}{4}) = \pounds\frac{5 \times 4}{3}$.

In these two examples the pupil can see that we may arrive at the answer by inverting the divisor and proceeding as in multiplication.

To show that this method leads to the correct solution in any case, we may consider £$(\frac{3}{5} \div \frac{3}{10})$.

By our plan this is the same as £$(\frac{3}{5} \times \frac{10}{3}) =$ £$\frac{30}{15} =$ £2.

But £$(\frac{3}{5} \div \frac{3}{10})$ represents the cost of 1 ton if $\frac{3}{10}$ ton cost £$\frac{3}{5}$. The answer to this can be found by considering that if 6 cwts. cost 12/-, 1 ton costs £2.

The principle advocated here is to find first of all what the answer should be, and then to see how the fractions are to be manipulated to arrive at that answer.

G. PHILIP.

**A First Lesson in Algebra.**—A big stumbling-block in the way of the child beginning Algebra is the transition from the conception of definite numbers in Arithmetic to that of indefinite quantities in Algebra, and the performance on these of the fundamental operations with facility and certainty. It is a truism that our algebraic teaching must grow out of our arithmetical: in its initial stages it may with advantage be based on some such method as the following. From a box of counters, or little wooden cubes, or in fact any handy articles, ask several pupils—better still, all the pupils, if the class be of convenient size—to pick out respectively five counters, eight counters, and so on. Their possessions can be exhibited on the black-board thus:—"A has *five* counters," "B has *eight* counters," etc. The counters being replaced, now ask others to take out *some* counters each. So long as the boys hold these in their closed hands no one can tell what quantity of counters any boy has; all that can be said is that each boy has *some* counters. Hence on the black-board is written:—"L has *some* counters," "M has *some* counters," and so on for each boy. Now the pupils have been accustomed in their arithmetical work to use various shorthand symbols in order to avoid writing out in full frequently occurring words or phrases. Thus, instead of the word "one" we use 1; the expression "multiplied by" is represented by ×, and so on. We therefore rub out the words "five" and "eight" that we wrote at first. and write in place of them their symbols "5" and "8."