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Foreword: special issue on coalgebraic logic

ERNST-ERICH DOBERKAT[†] and ALEXANDER KURZ[‡]

[†]*Chair for Software Technology, Technische Universität Dortmund, Dortmund, Germany*
Email: ernst-erich.doberkat@udo.edu
[‡]*Department of Computer Science, University of Leicester, Leicester, U.K.*
Email: kurz@mcs.le.ac.uk

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The second Dagstuhl seminar on coalgebraic logics took place from October 7–12, 2012, in the Leibniz Forschungszentrum Schloss Dagstuhl, following a successful earlier one in December 2009. From the 44 researchers who attended and the 30 talks presented, this collection highlights some of the progress that has been made in the field. We are grateful to Giuseppe Longo and his interest in a special issue in *Mathematical Structures in Computer Science*.

1. Coalgebraic logic

Coalgebras generalise important structures such as automata, transition systems, Kripke structures, Markov chains and many more. For each of the different types of coalgebra, there is a notion of behavioural equivalence or *bisimilarity* identifying states that cannot be observationally distinguished. An important role is played by *final coalgebras* which can be understood as the quotient by bisimilarity of the disjoint union of all coalgebras (of a given type). In a final coalgebra, one has the proof principle of *coinduction* saying that bisimilar states are equal. Final coalgebras play an important role in program semantics, both because they model *infinite data types* such as streams and because they provide domains for solving *recursive equations*.

Even though notions such as bisimulation and coinduction, independently invented in modal logic and the theory of concurrency, precede the advent of coalgebra as a research area, these notions do find their natural place in the theory of coalgebras. The fact that the type of a coalgebra is given by the category theoretic notion of a functor allows us to study a wide variety of different dynamic systems and their associated notion of bisimilarity from a uniform perspective. An important part in this direction of research has been played by Markov chains and other probabilistic transition systems.

The coalgebraic point of view also allows us to formally study and employ the category theoretic duality of algebras and coalgebras. This not only sheds light on induction vs coinduction, but also has applications, via Stone duality, to the development of modal logics for specifying the behaviour of systems. Next to the duality of algebra and coalgebra, it is also interesting to study structures that have both algebraic and coalgebraic components, such as programming languages the syntax of which is constructed inductively and hence algebraically, the structural operational semantics of which is given coinductively and hence coalgebraically, thus leading to the topic of bialgebras.

Another important insight brought on by coalgebras is that coalgebras, and in particular final coalgebras, provide universal domains for solving recursive equations. Roughly speaking, and this is closely related to Aczel's axiom AFA of non-well founded set theory, every guarded equation has a unique (uninterpreted) solution in the final coalgebra.

2. The contributions

The papers in this special issue contribute in various ways to the topics sketched above.

In *Well-founded coalgebras: Revisited* Jean-Baptiste Jeannin, Dexter Kozen, and Alexandra Silva extend the range of the notion of well-founded (or recursive) coalgebra. Recursive coalgebras generalise initial algebras and obey an induction principle which guarantees a unique solution to the recursive definition of many recursive algorithms. Moreover, even in the case of non-well founded coalgebras where uniqueness of the solution is not guaranteed, one often can obtain a least solution in a canonical way. The paper both develops the theory and gives new examples of how to apply (non)-well-founded coalgebras to programming.

In *Practical Coinduction*, Dexter Kozen and Alexandra Silva adress that coinduction as a proof technique is still not used by practitioners with the same ease as induction. The paper presents a coinduction proof principle and illustrates its use in examples from programming, emphasising that coinduction is not only about proving bisimularity.

Enhanced Coalgebraic Bisimulation by J. Rot, F. Bonchi, M. Bonsangue, D. Pous, J. Rutten and A. Silva presents a systematic study of bisimulation-up-to techniques. First introduced by Milner, these techniques are known to greatly reduce the difficulty of proving bisimilarity in many situations. Here, they are studied systematically from a category theoretic perspective using the notions of coalgebra and bialgebra.

In *Bisimilarity is not Borel*, P. Sanchez Terraf shows that bisimilarity between countable labelled transition systems is not Borel and thus cannot be characterised by a countable logic with measurable semantics. It is shown that it is Σ^1_1-complete, however. The proof methods come from descriptive set theory.

Smooth coalgebra: testing vector analysis by D. Pavlovic and B. Fauser shows how the vector calculus arises from coalgebras over manifolds. Dynamic systems are coalgebras $M \to T_*M$ where M is a manifold and T_*M its tangent bundle. Their main result shows how both the tangent and the cotangent vectors arise by testing of differentiable functions $M \to \mathbb{R}$ along curves $\mathbb{R} \to M$. This parallels the approach to coalgebraic logic in which logics arise from a testing relation between states and propositions.

Quasivarieties and varieties of ordered algebras by A. Kurz and J. Velebil is motivated by the formulation of coalgebraic logic that proceeds by extending Stone duality to (co)algebras. One of the advantage of this approach is that logics can be investigated uniformly using the methods of universal algebra. Being interested in extending this duality from the discrete to the ordered setting, the paper investigates some basic notions of categorical universal algebra in the ordered setting.

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