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Solutions of the nonlinear diffusion equation: existence, uniqueness, and estimation

John Howard Knight

In the first part of the thesis a problem for the nonlinear diffusion equation with diffusion coefficient a function of concentration, which is reduced by a similarity substitution to a boundary value problem for a nonlinear ordinary differential equation, is considered. Existence of a solution with certain upper and lower bounds is demonstrated for diffusion coefficient piecewise continuous and satisfying a local Lipschitz condition, and uniqueness is proved for non-increasing, continuous diffusion coefficient. An iterative method of Crank and Henry [1] for solving this problem is investigated and is proved to converge for non-decreasing diffusion coefficient, thus extending the existence result in this case. A perturbation method is used to derive a general series solution to the problem for a class of diffusion coefficients of power-law and exponential form.

More general problems are considered in the last two chapters of the thesis. It is shown that when the diffusion coefficient has the form

$$D(\theta) = a(1-b\theta)^{-2} , \quad a > 0 , \quad \theta \neq b^{-1} ,$$

where θ is the concentration, the diffusion equation in a semi-infinite region with general initial conditions and flux specified on the boundary can be transformed to a linear equation. New exact solutions are given to various problems of practical interest involving this nonlinear equation.

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An iterative method proposed by Parlange [2] to solve various problems for the nonlinear diffusion equation and related equations is investigated, and it is shown that the method of Parlange fails to converge. The problems are formulated as integral equations, and a new iterative method is described which gives accurate solutions with a minimum of iteration.

References

[1] J. Crank and M.E. Henry, "Diffusion in media with variable properties. I. The effect of a variable diffusion coefficient on the rates of absorption and desorption", *Trans. Faraday Soc.* **45** (1949), 636-650.

[2] Jean-Yves Parlange, "Theory of water-movement in soils: I. One-dimensional absorption", *Soil Sci.* **111** (1971), 134-137.

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