FIELD MEASUREMENT OF THE LIQUID-WATER CONTENT OF SNOW

By E. M. MORRIS

(Institute of Hydrology, Crowmarsh Gifford, Oxfordshire OX10 8BB, England)

ABSTRACT. Field trials show that the liquid-water content of snow can be determined simply and cheaply by a version of Bader's solution method.

RÉSUMÉ. *Mesures de terrain sur la teneur en eau liquide de la neige.* Des essais de terrain montrent que la teneur en eau liquide de la neige peut être déterminée simplement et bon marché par une adaptation de la méthode de la solution de Bader.

ZUSAMMENFASSUNG. *Feldmessung des Wassergehaltes von Schnee.* Feldversuche zeigen, dass der Wassergehalt von Schnee einfach und billig mit einer Version von Baders Lösungsmethode bestimmt werden kann

INTRODUCTION

Bader (1948, 1950) has suggested that the liquid-water content of a snow sample can be determined in the field by adding a solution of known concentration and measuring the equilibrium temperature of the mixture. Colbeck (1978) mentioned Bader's method in his survey of various techniques of measuring the water content of snow and has given an estimate of its potential accuracy. This note describes an improved version of Bader's method which has been used successfully in a wide variety of field conditions.

THEORY

Suppose that a sample of mass b of wet snow contains mass a_0 of ice and mass x of water, then

$$b = a_0 + x. \tag{1}$$

A volume of a solution containing mass c_0 of water and mass M of a solute at temperature t_1 is added to the sample in a Dewar of heat capacity H and initial temperature t_2. Phase equilibrium is established at a temperature t by melting a mass a_m of ice. If $a_m < a_0$, $|t|$ is the freezing-point depression of the solution, which has been diluted by the addition of the mass $a_m + x$ of water. The mass concentration A of a given solute, in grammes per 100 g of solution, which produces a given freezing-point depression can be found from standard chemical tables. Thus, if t is known, the mass of water c_1 in the solution at equilibrium can be calculated as

$$c_1 = 100 \frac{M}{A} - M. \tag{2}$$

Conservation of mass of H_2O gives

$$a_m = c_1 - c_0 - x. \tag{3}$$

Conservation of energy gives

$$La_m = \delta(c_0 t_1 - c_1 t) - \gamma t(a_0 - a_m) + H(t_2 - t), \tag{4}$$

where L is the latent heat of fusion of ice $(3.34 \times 10^2 \text{ J g}^{-1})$, δ the specific heat of the solution. and γ the specific heat of ice $(2.09 \text{ J g}^{-1} \text{ K}^{-1})$. The volumetric water content of the snow is

$$\theta = x/\rho_w V, \tag{5}$$

where V is the volume of the sample and ρ_w the density of water. The relative error in the water content is approximately

$$\left| \frac{\Delta \theta}{\theta} \right| < \left| \frac{\Delta x}{x} \right| + \left| \frac{\Delta V}{V} \right|, \tag{6}$$

175

where

$$|\Delta x| < \left| \frac{\delta}{L} \left\{ \left(\frac{\Delta c_{\mathrm{I}}}{c_{\mathrm{I}}} + \frac{\Delta t}{t} \right) c_{\mathrm{I}} t + \left(\frac{\Delta c_0}{c_0} + \frac{\Delta t_{\mathrm{I}}}{t_{\mathrm{I}}} \right) c_0 t_{\mathrm{I}} \right\} \right| +$$
$$+ \left| \frac{\gamma}{L} \left\{ \left(\frac{\Delta t}{t} + \frac{\Delta b + \Delta c_{\mathrm{I}} + \Delta c_0}{(b - c_{\mathrm{I}} - c_0)} \right) t (b - c_{\mathrm{I}} - c_0) \right\} \right| +$$
$$+ \left| \frac{1}{L} \left\{ \left(\frac{\Delta H}{H} + \frac{\Delta t + \Delta t_2}{(t + t_2)} \right) H(t + t_2) \right\} \right| + |\Delta c_{\mathrm{I}}| + |\Delta c_0|. \qquad (7)$$

If $A \ll 100$,

$$\left| \frac{\Delta c_{\mathrm{I}}}{c_{\mathrm{I}}} \right| < \left| \frac{\Delta A}{A} \right| + \left| \frac{\Delta M}{M} \right|, \qquad (8)$$

and

$$\left| \frac{\Delta c_0}{c_0} \right| \approx \left| \frac{\Delta M}{M} \right|. \qquad (9)$$

Bader (1950) tested the freezing-point depression method in conditions such that:

(i) the solution and Dewar were initially at 0°C, i.e.

$$t_{\mathrm{I}} = t_2 = 0;$$

(ii) the solution was sufficiently dilute for a linear relation between t and A to be specified;

(iii) the depression of the freezing point was small enough for the heat content $-Ht$ of the Dewar to be neglected.

However, in the author's opinion these conditions are too restrictive for a general field method. Even though snow is available as a coolant, it is not always possible to keep the initial temperatures of Dewar and solution to 0°C in the field, especially if measurements are being made at short intervals. The freezing point depression has to be at least 2–3 K, so that it can be measured to a reasonable precision, and this will normally mean that the heat content of the Dewar cannot be neglected. The method may also be more accurate if a fairly concentrated solution is used.

FIELD TESTS

The method was tested in the field with an apparatus consisting of a wide-mouthed 1 l Dewar fitted with a polystyrene stopper 5 cm thick. Temperatures were measured using a "Digitron" platinum resistance probe and pre-prepared 100 ml samples of NaCl solution were carried to the field site in light plastic bottles. The snow samples were taken with a 500 cm³ sampling tube and weighed using a spring balance. The relative error in the weight of wet snow was $|\Delta b/b| \approx 0.06$, including instrumental error in the spring balance and random error in the volume of the snow sample. The relative error in the measured heat capacity of the Dewar and its stopper was $|\Delta H/H| \approx 0.21$. This error could certainly be reduced if the design of the lid were improved. The absolute error in the temperature measurements was $\Delta t = \Delta t_{\mathrm{I}} = \Delta t_2 = \pm 0.1$ K. This is probably the minimum value that can be achieved at the moment in the field with lightweight battery-operated equipment. Since the brine solution was made up into samples of known weight and concentration in the laboratory, the errors in c_0 and H were very small; $|\Delta c_0/c_0| \approx |\Delta H/H| \approx 0.001$.

Given these fixed errors, the most effective brine concentration could be estimated by calculating the relative error in the water content, $|\Delta \theta/\theta|$, as a function of the mass of salt M added to the snow. Figure 1 shows this variation for $t_{\mathrm{I}} = t_2 = 0$°C, $b = 250$ g, $c_0 = 100$ g, and $H = 627$ J K^{-1}. The relative error decreases fairly rapidly for brine concentrations of up to about 5% by weight, but increasing the mass of salt beyond $M \approx 5$ g is not very effective in reducing $\Delta \theta$ for water contents in the expected range of $\theta \approx 5$–10%.
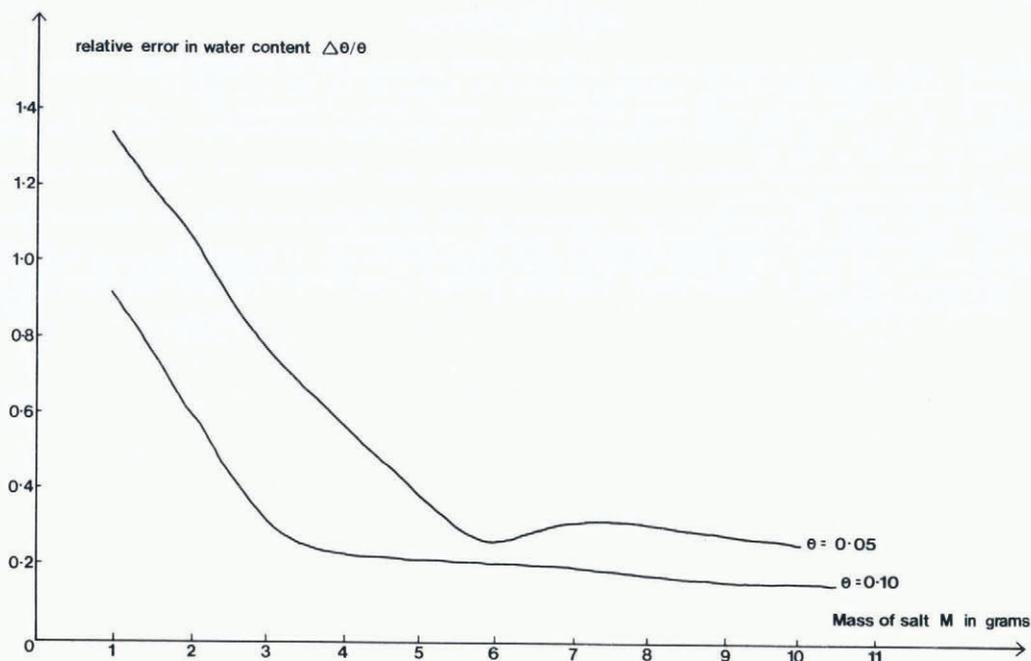
Fig. 1. Variation of the relative error in liquid-water content as a function of the mass of salt added to the snow.

It was found that using brine samples with $M = 5$ g, values of water content which were repeatable to within the estimated error $\Delta\theta \approx \pm 0.02$ could be obtained. This error is equivalent to about $\pm 4\%$ by weight and comparable to the 5% bands given by Wakahama (1968) and Wakahama and others (1968), who used a calorimeter to determine water content. Leaf (1966) reports an accuracy of $\Delta\theta = \pm 0.005$ using a freezing calorimeter but as Colbeck (1978) remarks this is unlikely to be a sensible estimate. In environments ranging from winter sub-arctic, with high winds and sub-zero temperatures, to summer high alpine, with intense solar radiation and warm air temperatures, the precision of the method could be maintained provided that

(i) the initial temperatures of the Dewar and brine were measured,
(ii) a relatively concentrated brine solution was used, and
(iii) the heat capacity of the Dewar and lid was included in the calculations.

The absolute accuracy of the method in the field could not be determined directly, since an alternative field apparatus for measuring water content was not available, but the measured values of $2-4\%$ for the liquid water content of a ripe pack rising to $9-10\%$ as the melt wave passed through agree with those reported by other authors. Measurements on frozen snow gave $\theta = 0.0 \pm 0.02$.

REFERENCES

Bader, H. 1948. Theory of non-calorimetric methods for the determination of the liquid water content of wet snow. *Schweizerische Mineralogische und Petrographische Mitteilungen*, Bd. 28, Ht. 2, p. 355–61.
Bader, H. 1950. Note on the liquid water content of wet snow. *Journal of Glaciology*, Vol. 1, No. 8, p. 466–67.
Colbeck, S. C. 1978. The difficulties of measuring the water saturation and porosity of snow. *Journal of Glaciology*, Vol. 20, No. 82, p. 189–201.
Leaf, C. F. 1966. Free water content of snowpack in subalpine areas. *Proceedings of the Western Snow Conference*, 34th annual meeting, p. 17–24.
Wakahama, G. 1968. Sekisetsu-nai ni okeru yūsetsu-sui no idō. III [Infiltration of melt water into snow cover. III]. *Teion-kagaku: Low Temperature Science*, Ser. A, [No.] 26, p. 77–86.
Wakahama, G., *and others*. 1968. Sekisetsu-nai ni okeru yūsetsu-sui no idō. II [Infiltration of melt water into snow cover. II]. [By] G. Wakahama, T. Nakamura, Y. Endō. *Teion-kagaku: Low Temperature Science*, Ser. A, [No.] 26, p. 53–75.