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Quadratic and Cubic Couplings of Oscillation Modes of Stars

T. Van Hoolst

Instituut voor Sterrenkunde, Katholieke Universiteit Leuven, Celestijnenlaan 200B, 3001 Heverlee, Belgium

1. Introduction

The strength of nonlinear interactions of oscillation modes of stars is determined by the amplitudes as well as by the eigenfunctions of the oscillation modes. The intrinsic couplings of modes through their eigenfunctions can be described by coupling coefficients. Here, we concentrate on quadratic and cubic coupling coefficients that describe the nonlinear coupling of modes with itself and are called self-coupling coefficients.

We considered radial and nonradial oscillation modes of polytropic models with degrees of central condensation that correspond to central condensations of main sequence stars to highly condensed evolved stars. We study the influence of the radial order and the degree of the oscillation mode on the self- coupling coefficients.

We adopt the isentropic approximation.

2. Coupling coefficients

When the nonlinear interactions of the stellar oscillation modes are taken into account up to the third order in the amplitudes, the time-behaviour of the amplitudes $a_k(t)$ of the linear, isentropic eigenfunction $\vec{\varphi}_k(\vec{r})$ of mode k is governed by the coupled-mode equation:

$$\frac{d^2 a_k}{dt^2} + \omega_k^2 a_k = Q_k^{nm} a_n a_m + C_k^{nmr} a_n a_m a_r$$

(see Van Hoolst & Smeyers 1993, Van Hoolst 1994), where Q_k^{nm} and C_k^{nmr} are quadratic and cubic coupling coefficients and are measures of the strengths of the nonlinear interactions of the modes. Coupling coefficients are defined as the ratios of integrals involving the eigenfunctions $\vec{\varphi}_n(\vec{r})$ of the modes n considered and the oscillatory moment of inertia of the mode k. The integrals of the quadratic and the cubic self-coupling coefficients contain the linear, isentropic eigenfunction $\vec{\varphi}_k(\vec{r})$ of mode k three and four times, respectively.

In the oscillation calculations, the generalised isentropic coefficient $\Gamma_1 = (\partial \ln \rho / \partial \ln P)_S = 5/3$. The linear, isentropic eigenfunctions are normalised by setting the radial part of the relative radial displacement equal to one.

We determined dimensionless self-coupling coefficients Q_k^{kk}/ω_k^2 and C_k^{kkk}/ω_k^2 for modes k as well as their isentropic frequencies ω_k and oscillatory moments of inertia.

3. Discussion

3.1. Radial order

With increasing radial order n, the frequencies and the absolute values of the dimensionless self-coupling coefficients increase whereas the moments of inertia decrease. Specifically, the quadratic self-coupling coefficient is found to increase almost linearly with radial order. An increase of coupling coefficients with increasing radial orders has also been noted by Van Hoolst (1994), and by, e.g., Takeuti et al. (1992) for quadratic coupling coefficients.

3.2. Spherical degree

The quadratic and cubic self-coupling coefficients Q_k^{kk} and C_k^{kkk} are proportional to the angular integral $\int |Y_k|^2 Y_k d\Omega$ and $\int |Y_k|^2 (Y_k)^2 d\Omega$, respectively. Here, Y_k is the spherical harmonic associated with the mode k, and $d\Omega = \sin\theta d\theta d\phi$ is the infinitesimal solid angle in angular coordinates θ and ϕ. The integrals over three spherical harmonics decrease and the integrals over four spherical harmonics increase with increasing degree.

The integral over the radial parts of the eigenfunction and the oscillatory moment of inertia also depend on the degree. We find that the dimensionless quadratic self-coupling coefficient decreases slightly with increasing degree l, and that the dimensionless cubic self-coupling coefficient decreases almost linearly with the degree.

3.3. Polytropic index

With increasing polytropic index n_e, or central mass-concentration, the frequencies increase and the moments of inertia decrease. The absolute values of the dimensionless self-coupling coefficients increase with increasing polytropic index for the lowest-order modes and decrease for the higher-order modes. Around polytropic index 4, this behaviour is sometimes reversed for low-order modes. Generally, the influence of n_e is not very large, so that it can be expected, at least as concerns the density profile, that coupling coefficients for various classes of pulsating stars are very similar to each other and do not differ much from the coupling coefficients for polytropic models.

Full details will be published elsewhere.

References

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