$$s = \frac{m\sqrt{p_1\rho_1}\{974 + 3(T_1 - 32)\}}{\sqrt{2g\rho p_a}\{150 - T_f\}}\cos a. \quad \dots \quad (39)$$

Taking $\gamma = 10/9$

$$n = \left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma - 1}} = \cdot 4 \text{ nearly.}$$

Suppose, further, for example, that the boiler pressure is 160 lbs. per sq. in. absolute ; that the diameter of the water-steam throat is ·25 inch ; and assume the following values :—

$T_f = 60°F.$; $a = 13°$; $f = \cdot 5$;
$f_2 = \cdot 9$; $m = 3\ 6$, allowing for contraction.

Then $r = 42$; and diameter of steam nozzle

$$= \frac{\cdot 25}{\sqrt{\cdot 42}} = \cdot 38 \text{ inch.}$$

$$W = A_3\left\{\sqrt{2g\rho(p_1 - p_a)/f_2} - \frac{m}{r}\sqrt{p_1\rho_1}\right\} \quad \dots \quad (40)$$

$$= 3 \cdot 0 \text{ lbs. per second.}$$

For the exhaust steam injector we have merely to substitute p_a for p_1 where it is associated with n, and in (31) where it appears under the radical. Thus

$$r = \frac{2p_a(n + s)}{\frac{2 + f_1}{f_2}(p_1 - p_a) + p_a} \quad \dots \quad \dots \quad (41)$$

$$s = \frac{m\sqrt{\rho_a} \times 1028}{\sqrt{2g\rho}\{150° - T_f\}}; \quad \dots \quad \dots \quad (42)$$

and

$$W = A_3\{\sqrt{2g\rho(p_1 - p_a)/f_2} - \frac{m}{r}\sqrt{p_a\rho_a}\}. \dots \quad (43)$$

On Rankine's Formula for Earth Pressure.

By Dr A. C. Elliott.

In a short course of lectures on "Railway Practice," the author was recently called upon to deal with the mechanical principles involved in the design of retaining walls. What has come to be known as Rankine's method had to be explained, at all events, in its practical application. But the time at the author's disposal did not admit of the general consideration of the theory of stress by which Rankine in characteristic fashion leads up (in his *Applied Mechanics*) to the particular problem under discussion. The author had therefore to choose between omitting a demonstration or making a short cut, which at the same time should be of a rigorous nature.

The following process, which in the circumstances was the one employed, may be regarded, from a mathematical point of view, as a direct solution of the problem :—Given the angle between two conjugate stresses to determine their ratio when the maximum obliquity on *any* section has a given value. In the actual problem one conjugate stress is produced at any (a certain) point by the weight of the superincumbent column of earth ; the other conjugate stress is produced by the reaction of the wall ; and the maximum obliquity when the wall is exerting its least possible resistance is the angle of repose for the earthy material in question. The plane of rupture can of course be found, and it will be defined as that plane on which the obliquity of the stress has the given maximum value. (Fig. 56).

Let ABC represent a section of a triangular prism of earth whose length-axis is horizontal and parallel to the face of the wall ; and suppose moreover that the prism is of unit length. Also let $AB = a$ be vertical and parallel to the face of the wall ; and let BC be parallel to the upper surface of the earth. Let p_v be the intensity of the vertical thrust due to the weight of the earth ; and let p_h represent the conjugate thrust on AB. When, following Rankine, the friction of the wall is neglected the direction of p_h will be parallel to BC. Farther, let R be the total intensity of the stress on AC and α the obliquity ; β the angle which BC makes with the horizontal, and θ the angle BAC.

To find BC in terms of a we have

$$\frac{\mathrm{BC}}{a} = \frac{\sin\theta}{\cos(\theta+\beta)}.$$

Hence resolving vertically and using this value of BC

$$\mathrm{R}\sin(\theta+\alpha) = p_v a\frac{\sin\theta}{\cos(\theta+\beta)} - p_h a\sin\beta \; ; \quad \ldots \quad (1)$$

and resolving horizontally

$$\mathrm{R}\cos(\theta+\alpha) = p_h a\cos\beta. \; \ldots \quad \ldots \quad \ldots \quad (2)$$

$$\therefore \quad \tan(\theta+\alpha) = \frac{1}{\mathrm{S}}\,\frac{\sin\theta}{\cos\beta\cos(\theta+\beta)} - \tan\beta, \quad \ldots \quad \ldots \quad (3)$$

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putting $$p_h/p_v = \mathrm{S}.$$

Now for a given value of S to find the plane AC for which α is maximum, put $\frac{d\alpha}{d\theta}=0$. Writing α explicitly

$$\theta+\alpha=\tan^{-1}\left\{\frac{1}{\mathrm{S}}\frac{\sin\theta}{\cos\beta\cos(\theta+\beta)}-\tan\beta\right\}. \quad \ldots \quad (4)$$

$$\therefore \quad 1=\frac{1}{1+\left\{\frac{1}{\mathrm{S}}\frac{\sin\theta}{\cos\beta\cos(\theta+\beta)}-\tan\beta\right\}^2}\cdot\frac{1}{\mathrm{S}\cos\beta}$$
$$\times\frac{\cos(\theta+\beta)\cos\theta+\sin\theta\sin(\theta+\beta)}{\cos^2(\theta+\beta)};$$

or
$$1=\frac{1}{1+\left\{\frac{1}{\mathrm{S}}\frac{\sin\theta}{\cos\beta\cos(\theta+\beta)}-\tan\beta\right\}^2}\cdot\frac{1}{\mathrm{S}\cos^2(\theta+\beta)}. \quad \ldots \quad (5)$$

Equation (5) determines θ corresponding to α a maximum. When rupture is about to take place the maximum value of α is equal to the angle of repose, say ϕ. To find the value of S corresponding to this limiting state, we have therefore to eliminate θ between the two equations

$$\tan(\theta+\phi)=\frac{1}{\mathrm{S}}\frac{\sin\theta}{\cos\beta\cos(\theta+\beta)}-\tan\beta \quad \ldots \quad (6)$$

and

$$\mathrm{S}\cos^2(\theta+\beta)=\frac{1}{1+\left\{\frac{1}{\mathrm{S}}\frac{\sin\theta}{\cos\beta\cos(\theta+\beta)}-\tan\beta\right\}^2}. \quad \ldots \quad (7)$$

Using (6), (7) may be written

$$\mathrm{S}\cos^2(\theta+\beta)=\frac{1}{1+\tan^2(\theta+\phi)}=\cos^2(\theta+\phi); \quad \ldots \quad (8)$$

or
$$\sqrt{\mathrm{S}}\cos(\theta+\beta)=\cos(\theta+\phi). \quad \ldots \quad \ldots \quad (9)$$

Hence the two equations between which θ is to be eliminated may be written

$$\tan(\theta+\phi)=\frac{\sin\theta}{\sqrt{\mathrm{S}}\cos\beta\cos(\theta+\phi)}-\tan\beta, \quad \ldots \quad \ldots \quad (10)$$

and
$$\sqrt{\mathrm{S}}\cos(\theta+\beta)=\cos(\theta+\phi). \quad \ldots \quad \ldots \quad (11)$$

Writing in (10)

$$\sin\theta = \sin(\overline{\theta + \phi} - \phi),$$

expanding and transforming, there results

$$\tan(\theta+\phi)\left\{\frac{1}{\sqrt{S}}\,\frac{\cos\phi}{\cos\beta} - 1\right\} = \frac{1}{\sqrt{S}}\,\frac{\sin\phi}{\cos\beta} + \tan\beta. \quad \ldots \quad (12)$$

Next putting (11) in the form

$$\sqrt{S}\cos(\theta + \phi - \overline{\phi - \beta}) = \cos(\theta + \phi),$$

expanding and arranging, there results

$$\tan(\theta+\phi) = \frac{1}{\sin(\phi-\beta)}\left\{\frac{1}{\sqrt{S}} - \cos(\phi-\beta)\right\}. \quad \ldots \quad (13)$$

Combining (12) and (13)

$$\frac{1}{\sin(\phi-\beta)}\left\{\frac{1}{\sqrt{S}} - \cos(\phi-\beta)\right\}\left\{\frac{1}{\sqrt{S}}\,\frac{\cos\phi}{\cos\beta} - 1\right\} = \frac{1}{\sqrt{S}}\,\frac{\sin\phi}{\cos\beta} + \tan\beta. \quad \ldots \quad \ldots \quad (14)$$

Multiplying up and reducing we have finally

$$S - 2\sqrt{S}\,\frac{\cos\beta}{\cos\phi} + 1 = 0. \quad \ldots \quad \ldots \quad (15)$$

$$\therefore \quad \sqrt{S} = \frac{\cos\beta}{\cos\phi} \pm \sqrt{\frac{\cos^2\beta}{\cos^2\phi} - 1}. \quad \ldots \quad \ldots \quad (16)$$

$$\therefore \quad S = \frac{\cos\beta \mp \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta \pm \sqrt{\cos^2\beta - \cos^2\phi}}. \quad \ldots \quad \ldots \quad (17)$$

And interpreting the signs according to the principle of least resistance, we have when the yielding is about to take place by failure of the wall

$$S = \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} \quad \ldots \quad \ldots \quad (18)$$

The other set of signs represents the case of yielding by *crushing in of the wall.*

[See also Elliott, *Proc.*, R.S.E., Jan. 1887.]

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