



Turbulent boundary layer response to uniform changes of the pressure force contribution

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We investigate a turbulent boundary layer (TBL) under uniform pressure force variations, focusing on understanding its response to local pressure force, local pressure force variation (local disequilibrating effect) and upstream history. The flow starts as a zero-pressure-gradient (ZPG) TBL, followed by a uniform increase in the ratio of pressure force to turbulent force in the outer region and concludes with a uniform decrease of the same magnitude. This last zone includes a subzone with a diminishing adverse pressure gradient (APG), followed by an increasing favourable pressure gradient (FPG). In both subzones, the impact remains the same: mean momentum gain and turbulence reduction. In the outer region, the mean flow responds to force balance changes with a considerable delay. The accumulated flow history leads to a FPG TBL at the domain's end with a momentum defect comparable to APG TBLs. Below $y^+ = 10$, the mean flow responds almost instantaneously to pressure force changes. In the overlap layer, velocity profiles deviate from the conventional logarithmic law of ZPG TBL. Outer-layer turbulence decays more slowly than it increases initially, the latter turbulence increase persisting even after the pressure force begins to decrease. As a result of the slow turbulence decay, the FPG TBL at the domain's end exhibits unusually high outer turbulence levels. Near the wall, turbulence responds with a delay to pressure force changes, partly due to the influence of large-scale turbulence. All these significant cumulative effects of continuous pressure force variation indicate that parameters based solely on local variables cannot fully describe the physics of non-equilibrium TBLs.

Key words: turbulent boundary layers

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1. Introduction

Most boundary layers found in industrial applications or in nature are complex because, in addition to being turbulent, they are affected by pressure gradient, wall curvature, wall roughness, heat transfer or a combination of these factors. As a result, one of the fundamental characteristics of these turbulent boundary layers (TBLs) is that they are non-equilibrium flows. This paper focuses on non-equilibrium effects caused by the pressure gradient. Its premise is that in order to better understand and to better model TBLs subjected to a pressure gradient, we have to understand, first and above all, how the pressure gradient unbalances the boundary layer.

To avoid any confusion, it is important to define what is meant here by ‘flow equilibrium’. Flow equilibrium refers to the state where all forces acting on the fluid maintain the same balance as the flow develops (Rotta 1953; Clauser 1956; Townsend 1956; Maciel, Rossignol & Lemay 2006). The properties of an equilibrium flow, for instance its thickness, velocity and forces’ magnitudes, can evolve in the streamwise direction but the balance of forces remains identical. The flow thus remains dynamically similar. It is important to note that in the context of turbulent flows, ‘equilibrium’ can also have a different meaning; it is also employed to express particular types of turbulent energy balance, as discussed by Spalart (2015).

In TBLs, the situation is more complex than in laminar flows and turbulent free shear flows due to the presence of two dynamically distinct wall-normal layers, with independent length scales: the inner and outer layers. The balance of forces differs significantly between these layers. In the inner layer, viscous forces are important while mean inertia forces are small, whereas the opposite is observed in the outer layer. This means that the inner layer can reach a state of equilibrium or near equilibrium, for instance corresponding to the law of the wall, whereas the outer layer remains in a state of disequilibrium. When we refer to ‘near equilibrium’, we mean that the balance of forces undergoes slow variation in the streamwise direction. The zero-pressure-gradient (ZPG) TBL serves as an example of a near-equilibrium TBL, where the force balance in the outer region experiences slow variations due to the increasing local Reynolds number. Unlike the sink-flow TBL, which is a single-layer flow (Townsend 1956; Rotta 1962), ZPG TBLs and favourable-pressure-gradient (FPG)/adverse-pressure-gradient (APG) ‘equilibrium’ TBLs are anticipated to achieve exact equilibrium only in the theoretical scenario of infinite Reynolds number, owing to their two-layer nature.

The concepts of self-similarity and equilibrium are interconnected. A flow or a region of a flow is self-similar if its statistical properties, or at least some of them, such as the mean velocity profile, depend solely on local flow variables (scales). If a flow region reaches a self-similar state, then it can only depend on its upstream history through its length and velocity scales. It has therefore become an equilibrium flow. A review of these ideas can be found in Maciel *et al.* (2006) and Devenport & Lowe (2022). Equilibrium boundary layers are seldom encountered in the real world and are challenging to achieve experimentally and numerically. There exist only a few experimental and numerical realisations of equilibrium APG and FPG TBLs with varying degrees of success (see Devenport & Lowe 2022). The interest of equilibrium TBLs lies in the fact that they represent theoretically simpler forms of TBLs, as they can be described by local parameters alone.

The Rotta–Clauser pressure gradient parameter, β_{RC} , is a commonly used pressure gradient parameter in the literature:

$$\beta_{RC} = \frac{\delta^*}{\rho u_\tau^2} \frac{dp_e}{dx}, \quad (1.1)$$

TBL response to uniform changes of the pressure force

where ρ is density, δ^* is the displacement thickness, u_τ is friction velocity and p_e is the mean pressure at the edge of the boundary layer. It can be obtained from the momentum integral equation as the ratio of the total streamwise pressure force to the wall shear force acting on the whole boundary layer momentum defect. Therefore, it is not an inner or outer pressure gradient parameter but rather a global one that encompasses the entire boundary layer. Although this paper utilises pressure gradient parameters specific to the inner and outer layers, β_{RC} will also prove useful in analysing non-equilibrium effects in the current paper.

In general PG TBLs, the overall local dynamic state of the inner region can be expressed with the pressure gradient parameter β_i (Mellor 1966), which represents the ratio of pressure force and apparent turbulent force (gradient of Reynolds shear stress)

$$\beta_i = \Delta p^+ = \frac{\nu}{\rho u_\tau^3} \frac{dp_w}{dx}, \quad (1.2)$$

where ν is viscosity and p_w is the mean pressure at the wall. In the literature, β_i is also denoted Δp^+ or p^+ and considered as the local pressure gradient in wall units. Equilibrium in the near-wall region implies that β_i remains constant.

Equilibrium in the outer region, including the overlap layer, is more difficult to achieve because of the inertia forces. For this reason, the various existing similarity analyses (Townsend 1956; Rotta 1962; Mellor & Gibson 1966; Castillo & George 2001; Maciel *et al.* 2006) find necessary conditions for self-similarity, and thus equilibrium, expressed with outer layer parameters. The main (driving) condition is that the outer pressure gradient parameter, in any chosen form as long as it represents a ratio of forces, must remain constant. By using five non-equilibrium and one equilibrium APG TBL databases, Maciel *et al.* (2018) showed that the ratio of forces in the outer layer can be accurately followed with three parameters when these parameters are expressed with Zagarola–Smits scales as the outer scales: $U_o = U_{ZS} = (\delta^*/\delta)U_e$ and $L_o = \delta$, where δ is the boundary layer thickness and U_e is the velocity at the edge of the boundary layer. Such a correspondence was not achieved with the friction velocity or the outer pressure gradient velocity scale, $U_{po}^2 = (\delta^*/\rho)(dp_e/dx)$ whatever the length scales chosen. The outer pressure gradient parameter is therefore expressed with Zagarola–Smits scales in the present study:

$$\beta_{ZS} = \frac{\delta}{\rho U_{ZS}^2} \frac{dp_e}{dx}, \quad (1.3)$$

and it precisely follows the ratio of pressure force to turbulent force in the outer region of the six APG TBLs in the study of Maciel *et al.* (2018) and in the APG TBL region of the current flow. The present study suggests that the correspondence is not as accurate for FPG TBLs, at least for the present one which is recuperating from strong downstream APG effects, but β_{ZS} still follows the variation of the force ratio in the FPG region of the flow.

From the previous discussion, it becomes clear that a TBL which is in a non-equilibrium state in the outer region because of the pressure force is basically a TBL for which the pressure gradient parameter β_{ZS} varies. It is important to stress that it is not the pressure gradient in itself that causes a non-equilibrium state, but rather the streamwise evolution of the pressure gradient parameter. In other words, $d\beta_{ZS}/dx$ is a measure of the local disequilibrating effect of the pressure force whereas β_{ZS} is a measure of the local direct effect of the pressure force. This study illustrates this point further by showing that the sign of the pressure gradient does not always reflect the type of disequilibrating effects. It will be seen that, as counterintuitive as it may appear, a positive pressure gradient ($dp/dx > 0$;

APG) can lead to the boundary layer filling up with momentum if $d\beta_{ZS}/dx < 0$. We will call the disequilibrating effects of $d\beta_{ZS}/dx < 0$ as momentum-gaining conditions since the qualifier favourable refers in the literature to $dp/dx < 0$.

In addition to not fully understanding such local pressure gradient effects on the evolution of a TBL, we are unable to make a clear distinction between them and Reynolds number and upstream history effects. In the case of mild APG effects, Sanmiguel Vila *et al.* (2020a), Pozuelo *et al.* (2022) and Deshpande *et al.* (2023) provide insights on the differences between APG and Reynolds number effects. If the imposed pressure gradient effect is strong (large β_{ZS} and/or $d\beta_{ZS}/dx$) and the Reynolds number is high, Reynolds number effects can probably be neglected. The question of upstream history effects is more complex since they cannot be neglected and they can hardly be represented by a local parameter. Upstream history effects are, by definition, cumulative effects that can only be taken into account with the integration of the governing equations. They have been studied for different types of non-equilibrium PG TBLs, including cases where the transition from APG to FPG, or *vice versa*, is considered. Three recent studies have focused on comparing APG TBLs with different upstream histories at locations where both β_{RC} and the friction Reynolds number Re_τ match (Bobke *et al.* 2017; Vinuesa *et al.* 2017; Tanarro, Vinuesa & Schlatter 2020). The findings revealed that upstream non-equilibrium APG conditions affect the outer region more significantly than the inner region. The latter is found to be more dependent on the local value of β_{RC} . However, these studies assume that upstream history effects are the same for both the inner and outer regions by using only β_{RC} as the pressure gradient parameter, but in reality, they differ.

Although the boundary layer does not respond in the same manner to continuous gradual changes in the force balance and to rapid or step changes of flow conditions, the latter cases can nonetheless help us understand the response of the boundary layer to non-equilibrium conditions. From the equations of motion, it can be deduced that the mean shear rate $\partial U/\partial y$ in the outer region cannot respond immediately to a step change in the pressure gradient since viscous diffusion is negligible there (Smits & Wood 1985). Consequently, turbulence in the outer layer cannot also respond immediately to a step change in the pressure gradient. Based on scaling arguments, Devenport & Lowe (2022) suggest that it takes tens of boundary layer thicknesses for both the mean flow and turbulence to adjust to pressure gradient changes. This is consistent with the boundary layer recovery distances observed by Volino (2020) downstream of a FPG-to-ZPG transition.

The extended recovery in the outer region is also observed in channel flows with a step change from rib- or cube-roughened surface to a smooth wall (Ismail, Zaki & Durbin 2018) and in pipe flows subjected to various strong localised perturbations (Van Buren *et al.* 2020; Ding & Smits 2021; Ding *et al.* 2021). In their studies on pipe flows, Smit's research group found that turbulence recovery in the far-field was oscillatory, resembling a second-order dynamical system, overshooting the eventual fully developed condition. This oscillatory behaviour is due to the asynchronous recovery between the mean velocity and the Reynolds shear stress.

In contrast to the situation in the outer layer, turbulence in the inner layer adjusts itself very quickly. For instance, in experiments with rods placed inside ZPG TBLs (Clauser 1956; Marusic *et al.* 2015), the inner layer recovers after a few boundary layer thicknesses, whereas the outer layer takes tens or even hundreds of boundary layer thicknesses to recover. A similar situation occurs in the TBL recovery from separation due to a step, where the outer region is strongly energised by the perturbation (Vaquero, Renard & Deck 2022). Cases of a very fast transition from FPG to APG (Tsuji & Morikawa 1976;

Parthasarathy & Saxton-Fox (2023) also show that the near-wall region adapts more rapidly than the outer region. In such cases, a new internal layer grows inside the APG TBL, starting from the wall, in reaction to the change, whereas the upper part of the TBL responds much more slowly. Parthasarathy & Saxton-Fox (2023) found that this internal layer establishes its own inner and outer regions. The triggering of an internal layer is more commonly found in TBLs with abrupt changes of wall conditions: surface curvature, roughness, wall heat flux (Smits & Wood 1985). In the case of rough-wall TBLs with step changes in roughness, it has been shown that the outer-layer recovery is nonlinear and depends on the sign of the step change (Smits & Wood 1985). The return to an equilibrium state in the outer region is more rapid when turbulence is augmented (smooth-to-rough step change) than when turbulence is reduced (rough-to-smooth step change). Turbulence appears to take more time to decay than to build up in the outer region of TBLs.

The response of turbulence in the outer region is known to lag even in cases of continuous changes of the pressure force. For example, Devenport & Lowe (2022) remark that at the streamwise location where $\beta_{RC} \approx 2$, the Reynolds shear stresses in the cases of an increasing pressure force effect ($d\beta_{RC}/dx > 0$) of Nagano, Tagawa & Tsuji (1993) and Spalart & Watmuff (1993) are about half of those of the equilibrium case of Bobke *et al.* (2017).

To better understand the direct local pressure force effect, the local disequilibrating effect of the pressure force and the upstream history effect, we have designed a non-equilibrium flow with an almost linear increase followed by an almost linear decrease of the pressure gradient parameter β_{ZS} , as shown in figure 2(a). The local pressure gradient effects start therefore as momentum-losing conditions, of constant strength, followed by momentum-gaining conditions of the same strength. The momentum-gaining conditions ($d\beta_{ZS}/dx < 0$) are maintained on a long length to also include a zone of FPG ($dp/dx < 0$) at the end of the domain. Note that the middle region of the flow is a zone of APG ($dp/dx > 0$) that is momentum-gaining for the flow ($d\beta_{ZS}/dx < 0$). In other words, the boundary layer is filling up with momentum even if the external flow is still decelerating. Although this may appear counterintuitive as stated previously, it is consistent with an analysis of the mean momentum equation, boundary layer computations of the current flow and the results presented in the paper. This is a unique feature that has not been specifically studied, although it is also encountered in the TBLs alternating from mild APG to mild FPG of Tsuji & Morikawa (1976) or Fritsch *et al.* (2022a).

The current TBL is experiencing moderate APG disequilibrium (indicated by a moderate value of $d\beta_{ZS}/dx > 0$). However, due to the long length of the imposed pressure force increase, it attains a significant velocity defect ($H = 2.87$). This places it between quasi-equilibrium APG TBLs with large velocity defects (Skåre & Krogstad 1994; Kitsios *et al.* 2017) and our recent APG TBL, which is in a state of strong disequilibrium (Maciel *et al.* 2018; Gungor, Maciel & Gungor 2022). This feature, combined with the previous one, makes it an excellent intermediate test case to determine what distinguishes equilibrium and disequilibrium APG TBLs in strong pressure gradient conditions. Likewise, the slow deceleration of the TBL in a long domain helps us determine under what conditions the classical law of the wall starts to lose its validity.

Because of the switch from momentum-losing to momentum-gaining pressure conditions, it is possible to study the delayed response of the mean flow and turbulence in both the inner and outer layers. In addition, the final segment of the domain represents an FPG TBL at high Reynolds number with a distinctly unique flow history.

The present work aims to better understand and distinguish the three different effects of the pressure force, local direct, local disequilibrating and upstream variation, on TBLs.

2. The database

2.1. The numerical details

The novel flow case is generated using a direct numerical simulation (DNS) code that employs a fractional step method to solve the three-dimensional incompressible Navier–Stokes equations with primitive variable formulation within a three-dimensional rectangular volume. The methodology for this DNS code is based on Kim & Moin (1985). In terms of computational specifics, the grid is structured and staggered. Spatial discretisation entails a fourth-order compact finite-difference scheme for convective and viscous terms, as well as a standard second-order discretisation for the pressure term in both the streamwise and wall-normal directions. The spanwise direction uses spectral expansion, de-aliased via the 2/3 rule, for discretisation (Lele 1992). Temporal advancement is accomplished through a semi-implicit three-step Runge–Kutta method. For more comprehensive insights into the DNS code and further particulars, refer to Simens *et al.* (2009), Borrell, Sillero & Jiménez (2013) and Sillero (2014).

The DNS computational set-up consists of two simultaneous simulation domains, depicted in figure 1 and elaborated on further in Borrell *et al.* (2013). The auxiliary DNS, possessing coarser resolution, aims to deliver realistic turbulent inflow data for the primary DNS. It involves a ZPG TBL, where inlet conditions are derived from a recycling plane within the domain. Data from this recycling plane (π_r in figure 1) are rescaled using a modified version of method of Lund, Wu & Squires (1998) and Simens *et al.* (2009). The positioning of the recycling plane with respect to the inlet is determined to ensure sufficient distance for flow decorrelation from inlet effects (Simens *et al.* 2009). A transfer plane (π_t in figure 1) from the auxiliary simulation is designated as the source of temporal data for inflow conditions in the main simulation. For the transfer plane, the data from the auxiliary domain are extrapolated in the freestream because the wall-normal height of both domains are different from each other. The recycling and transfer planes are located at approximately $x = 46\delta_{0,a}$ and $124\delta_{0,a}$, where $\delta_{0,a}$ is δ at the inlet of the auxiliary domain. The adoption of two domains is needed to provide inflow conditions for the main DNS at a lower computational cost.

Regarding the other boundary conditions, the lower surface represents a flat plate with a no-slip (zero velocity) and impermeability condition. As for the upper boundary, wall-normal velocity is applied via suction and blowing, generating favourable/adverse pressure gradients. In addition, the free-slip conditions are enforced for the wall-parallel components. At the exit of the domain, convective boundary conditions are employed. In addition, the auxiliary domain was initialised with a flow realisation of a ZPG TBL.

The computational domains of the main and auxiliary cases are rectangular volumes. The domain length and grid properties are given in table 1 for both cases. Here, the average boundary layer thickness (δ_{av}) is calculated within the useful range of the main domain. The useful range is $36\delta_{av}$ long. For domain sizes, the wall-normal length of the domain is chosen as approximately 3 times δ at the exit and spanwise length is chosen based on two-point correlations. The Reynolds number based on momentum thickness (Re_θ) of the main case ranges from 1941 to 12970.

Spatial resolutions, Δx^+ and Δz^+ , range from 1.56–9.84, and 1.18–7.37, respectively, with the coarsest resolutions near the inlet and finest resolutions where the flow exhibits a significant velocity defect. Furthermore, Δy^+ at the wall does not exceed 0.49, and the maximum Δy^+ across the boundary layer remains below 11.06. The time step size, Δt^+ , is maintained below 0.17 throughout the domain.

The data for the statistics were collected by averaging over time and the spanwise direction. The duration of data collection was 11.5 and 7.7 flow-through times based on

TBL response to uniform changes of the pressure force

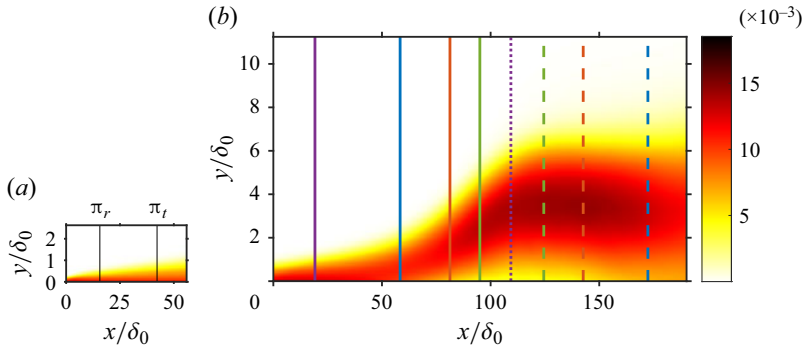


Figure 1. Spatial evolution of the turbulent kinetic energy, normalised with $U_{e,0}$ and δ_0 , in the auxiliary (a) and main (b) domain as a function of x/δ_0 and y/δ_0 where δ_0 is the boundary layer thickness at the inlet of the main domain. Here π_r and π_t indicate the location of the recycling and transfer planes, which are represented by the black vertical lines. The vertical lines on the main domain indicate the streamwise stations selected for the detailed analysis. The x -axis is not scaled proportionally to the y -axis. Only the lower half of the domain is shown for both domains.

Domain	$(L_x, L_y, L_z)/\delta_{av}$	$(L_x, L_y, L_z)/\delta_e$	N_x, N_y, N_z	Re_θ	Re_τ
Auxiliary	12.0, 1.2, 6.3	46.7, 4.2, 16.5	2161, 316, 2700	620–2380	260–810
Main	40.2, 4.7, 6.3	26.1, 3, 4.1	12 801, 770, 2700	1941–12 970	654–2489

Table 1. Domain properties of the main and auxiliary DNS. Here δ_{av} is the average thickness of the boundary layer of the main domain in the useful range and δ_e is the thickness of the boundary layer at the exit of each domain.

the edge velocity, U_e , at the inlet and exit, respectively. The eddy-turnover time, based on u_τ , is highest at the inlet, 85.9. It decreases to 2.2 at $x/\delta_{av} \approx 20$, which is very close to SP5. However, in TBLs with a large-velocity defect, u_τ is no longer a characteristic scale for large-scale structures. Therefore, we also computed the eddy-turnover time using U_{ZS} , which gives an eddy-turnover time of 323 at the inlet and a minimum value of 21. The simulations were performed on the Marconi KNL and Marconi100 clusters at Cineca, as well as on the Niagara cluster of the Digital Research Alliance of Canada. The cost of the simulation for collecting statistics is approximately 10 million core-hours on Niagara of Digital Alliance of Canada.

3. Results

Before discussing the results, the edge of the boundary layer needs to be defined clearly. The definition of the boundary layer edge and edge velocity in the case of pressure gradient TBLs and TBLs over complex geometries is challenging due to variations in freestream velocity. Griffin, Fu & Moin (2021) provide a review of eight methods of computing the boundary layer thickness in such cases. We have tested several methods, and a summary of the main results are presented in the [Appendix](#). The method based on the local reconstruction of the inviscid-flow velocity profile by Griffin *et al.* (2021) is the most theoretically rigorous and empirically consistent. Unfortunately, we cannot use it in the present study because it requires mean static pressure data, which we lack for the near-equilibrium TBL database of Kitsios *et al.* (2017) used for comparison. Instead, we chose the recent method of Wei & Knopp (2023), because it yielded the most consistent

results across all tested databases, although, like most methods, it lacks a theoretical basis. In this method, the boundary layer thickness, represented as δ , is defined as the location near the freestream where the Reynolds shear stress reaches 5 % of its maximum. The edge velocity, denoted as U_e , is defined as the mean streamwise velocity at that location.

3.1. The mean flow and force balance

To first understand the local and upstream effects of the pressure force on the mean flow, we investigate global flow parameters, mean momentum equation budgets and mean velocity profiles separately for the outer and inner layers. The streamwise mean momentum equation where each term can be viewed as a force acting on the flow is given in

$$0 = \left(-U \frac{\partial U}{\partial x} - V \frac{\partial U}{\partial y} \right) - \frac{1}{\rho} \frac{dp_e}{dx} - \frac{\partial \langle uv \rangle}{\partial y} + \nu \frac{\partial^2 U}{\partial y^2}, \quad (3.1)$$

where U and V are the mean velocities in the streamwise and wall-normal directions, u and v are the fluctuation velocities, ν is the viscosity and $\langle \cdot \rangle$ indicates ensemble averaging. The terms on the right-hand side of the equation represent the inertia force, the pressure force, the turbulent force (the gradients of the normal stresses are not considered because they are negligible) and the viscous force, respectively.

3.1.1. Outer layer

The global parameters of the flow and the ratio of forces in the inner and outer layers are presented in [figure 2](#) as a function of x/δ_{av} . [Figure 2\(a\)](#) presents the outer-layer pressure gradient parameter β_{ZS} and the ratio of pressure force at the edge of the boundary layer ($F_{p,o}$) to the maximum of turbulent force in the outer layer ($F_{t0,max}$), where the outer layer is defined as the region above 0.1δ . [Maciel et al. \(2018\)](#) demonstrated that β_{ZS} follows the ratio of pressure to turbulence force in the outer region using six APG TBLs. This is confirmed in the present flow as β_{ZS} and the ratio of forces follow each other very closely in the region where the pressure gradient is positive (adverse). However, β_{ZS} and the force balance do not correspond in the FPG region, although the trend is still the same for both. This could be due, at least partly, to the delay in turbulence response to the flow changes. As presented in [§ 3.2](#), Reynolds shear stresses remain very high in the outer region of the whole momentum-gaining zone, even higher than downstream. Their expected decay only starts at $x/\delta_{av} \approx 28$. This could explain why the absolute value of the ratio of pressure force to turbulence force is low.

As was already presented in the introduction, in the first part of the domain, until approximately $x/\delta_{av} = 12$, β_{ZS} increases at a fairly constant rate as it was aimed for in the design of this flow. Afterwards, it decreases at a similar absolute rate until the end of the domain. Such a streamwise evolution of β_{ZS} implies a non-equilibrium boundary layer with a constant increase in importance of the pressure force with respect to the other forces followed by a constant decrease as it can be seen with $F_{p,o}/F_{t0,max}$. Until approximately $x/\delta_{av} \approx 21.5$ where the shape factor reaches its maximum ([figure 2d](#)), the flow is similar to some strong non-equilibrium APG TBL cases in the literature ([Hickel & Adams 2008](#); [Gungor et al. 2016](#); [Hosseini et al. 2016](#); [Gungor et al. 2022](#)). In this region, the boundary layer constantly loses momentum (H increases) because the pressure force's importance increases upstream for a long extent, as reflected by $F_{p,o}/F_{t0,max}$ and β_{ZS} . Afterwards, the flow is still under the effect of an APG until $x/\delta_{av} = 28$. However, the flow gains momentum in this region because the pressure force's importance decreases (β_{ZS} decreases), which is a particular behaviour not studied previously, as discussed in

TBL response to uniform changes of the pressure force

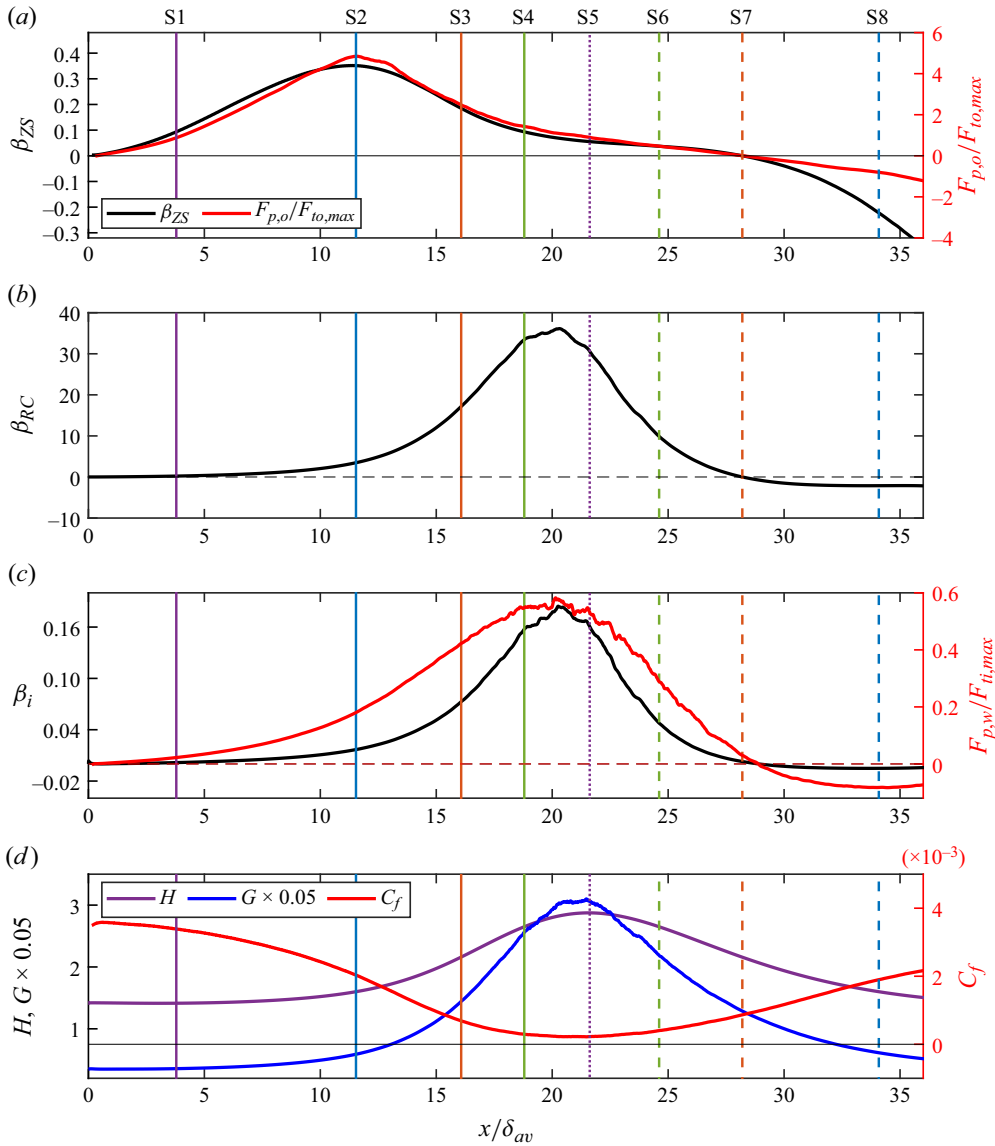


Figure 2. Streamwise development of the main parameters of the APG TBL. The vertical lines indicate the streamwise stations selected for the detailed analysis.

the introduction. The rest of the domain is an FPG TBL where the pressure gradient is maintained with a negative $d\beta_{ZS}/dx$ similar to that upstream.

Although we present the remaining parameters in figure 2 later, for now, we focus on discussing the mean momentum budgets to provide a more comprehensive description of the streamwise evolution of the force balance. Figure 3 presents the outer-scaled mean momentum budgets for various streamwise positions. These positions are identified with vertical lines in figure 2. Details of these positions, including the acceleration parameter ($K = (\nu U^2)/(dU_e/dx)$), are given in table 2 and the corresponding mean velocity profiles are shown in figure 4. In figure 3, the x -axis ranges have been selected to ensure that the reference turbulent force value ($F_{to,max}$) is approximately located at the same position in all subfigures.

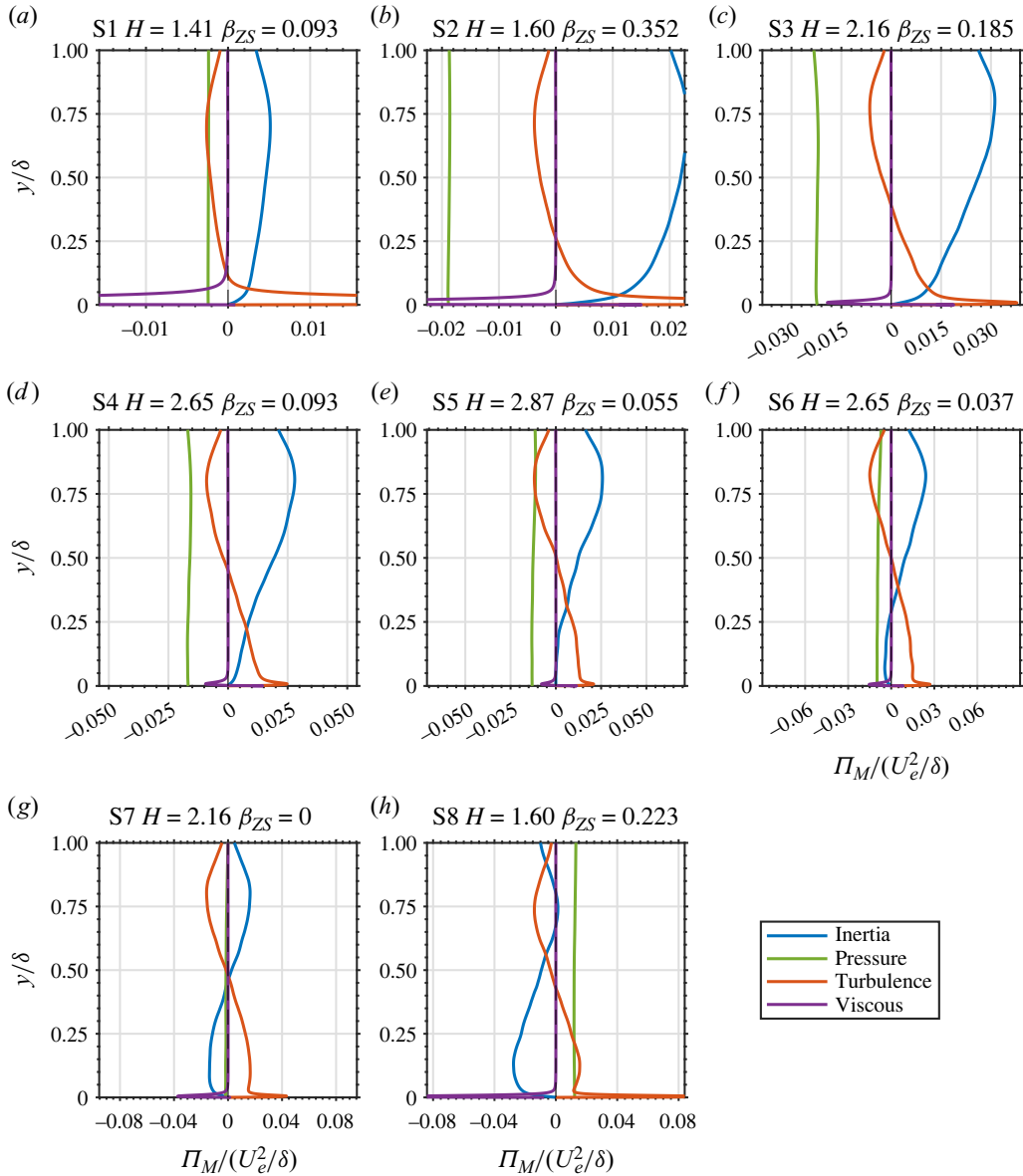


Figure 3. The mean momentum budget profiles of the eight streamwise positions as functions of y/δ . The budget terms are normalised with U_e and δ . The x-axis ranges are chosen so that the reference turbulent force value $F_{t0,max}$ is approximately at the same location in all subfigures.

The momentum budgets reveal the non-equilibrium nature of the flow since the balance of forces changes from one station to another. The budgets of the first two stations (S1 and S2) show explicitly that the pressure force increases in importance with respect to the turbulent force in the outer region. Further downstream, the pressure force's relative magnitude with respect to the other forces begins to decrease as can be seen with the momentum budgets starting from S3. This decrease continues until the pressure force becomes zero at S7. Afterwards, the pressure gradient becomes negative (FPG zone) until the end of the domain.

Position	x/δ_{av}	H	C_f	Re_τ	Re_θ	β_{ZS}	β_i	β_{RC}	$K \times 10^{-6}$
S1	3.8	1.41	0.0034	809	2486	0.093	0.0016	0.22	0.18
S2	11.5	1.60	0.0020	1012	4931	0.352	0.0168	3.47	0.53
S3	16.1	2.16	0.0007	865	7961	0.185	0.0729	17.21	0.54
S4	18.8	2.65	0.0003	702	9635	0.093	0.1575	33.49	0.29
S5	21.6	2.87	0.0002	709	10952	0.055	0.1607	30.59	0.18
S6	24.6	2.65	0.0004	1006	12099	0.037	0.0476	9.83	0.11
S7	28.2	2.16	0.0009	1489	12827	0.000	0.0028	0.00	-0.01
S8	34.1	1.60	0.0019	2181	12278	-0.223	-0.0051	-2.11	-0.12
ZPG	N/A	1.38	0.0030	1334	3904	0.000	0.0000	0.00	—
EQ1	N/A	1.57	0.0025	811	3358	0.104	0.0056	1.04	—
EQ2	N/A	2.57	0.0004	698	8584	0.090	0.1058	34.24	—

Table 2. The main parameters of selected streamwise positions in the present case along with the ZPG TBL of Sillero, Jiménez & Moser (2013) and the near-equilibrium APG TBL cases of Kitsios *et al.* (2017).

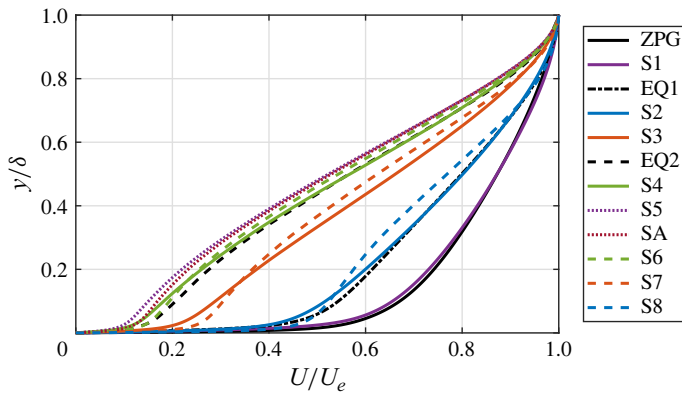


Figure 4. The mean velocity profiles for nine streamwise positions of the present case along with the ZPG TBL and near-equilibrium cases EQ1 and EQ2 as a function of y/δ . The profiles are normalised with U_e .

In the momentum budgets of S1 to S6, the inertia term indicates that the fluid is losing momentum in the streamwise direction ($dU/dx < 0$), except near the wall for S6. This is confirmed by the mean velocity profiles in figure 4, but it is important to note that these are normalised profiles U/U_e as a function of y/δ that are representative of U variation in approximate streamline directions, not x . This explains why the fluid has gained momentum from S5 to S6 in the normalised representation of figure 4 although figure 3 indicates that dU/dx is still negative at S6 in most of the outer region.

Indeed, while it was constantly losing momentum upstream, the fluid starts to gain momentum around S5 because of the decrease of the pressure force’s relative magnitude ($d\beta_{ZS}/dx < 0$) that began slightly after S2. The fact that momentum gain only starts at S5, and not right after S2, shows that the mean flow responds with a delay to changes in the force balance because of inertia. As explained in the introduction, it is not the sign of the pressure gradient that indicates momentum gain or loss (disequilibrium), but the streamwise evolution of the relative magnitude of the pressure force, reflected by $d\beta_{ZS}/dx$ in the outer layer and $d\beta_i/dx$ in the inner layer. The present flow was specifically designed to produce a zone, from S5 to S7, where the fluid gains momentum in the boundary layer

(in the normalised representation U/U_e as a function of y/δ of figure 4) even if the pressure gradient is still positive (APG).

The momentum changes observed in figure 4 can also be followed with the distribution of the shape factor, H , as presented in figure 2(d), with increasing H indicating momentum loss and *vice versa*. The velocity defect is small at S1 ($H = 1.41$), increases to become large and maximal at S5 ($H = 2.87$) and then monotonously decreases downstream. The shapes of the profiles in the momentum-losing (streamwise positions S2 to S4) and momentum-gaining zones (S6 to S8) are different from each other at matching or very close shape factors as illustrated in figure 4. This happens due to the different upstream flow history in the two zones. The main difference is that the profiles in the momentum-gaining region are fuller in the inner layer due to the faster response of the flow in that layer.

To examine how the momentum gain starts in the boundary layer, we consider two streamwise positions, S5 and the position denoted as SA, where the shape factor is 2.78, located just slightly downstream of S5. As it can be seen from figure 4, the inner layer gains momentum at SA but the defect in the outer layer remains almost the same. The inner layer therefore starts gaining momentum before the outer layer even if the momentum-gaining effect of the pressure force initiates earlier in the outer layer ($d\beta_{zs}/dx$ becomes negative before $d\beta_i/dx$). This implies that the inner layer reacts to the pressure gradient much more rapidly than the outer layer. This is consistent with the fact that, in the inner layer, inertia forces are small and turbulence adjusts more quickly.

As mentioned in the introduction, β_{RC} is not an outer pressure gradient parameter, but a global pressure gradient parameter for the whole boundary layer. It is presented in figure 2(b) and a comparison of $F_{p,o}/F_{to,max}$ and β_{RC} shows that β_{RC} indeed is not an outer region pressure gradient parameter for large-defect TBLs. It follows in fact more closely, but not exactly, the ratio of pressure to turbulent forces in the inner region given by the inner-layer pressure gradient parameter, β_i (figure 2c).

To enhance our understanding of how the present flow responds to disequilibrium conditions, figure 4 illustrates mean velocity profiles from the two near-equilibrium APG TBLs of Kitsios *et al.* (2017). These near-equilibrium flows correspond to cases of small velocity defect ($H = 1.57$), denoted herein as EQ1, and large velocity defect ($H = 2.57$), denoted as EQ2. The main parameters of these flows are given in table 2. Figure 4 illustrates that S2 and EQ1, sharing similar H values, exhibit comparable velocity, but the near-equilibrium profile is fuller in the inner region. However, the near-equilibrium TBL attains this state with a globally smaller local pressure force effect (smaller β_{RC}) and a smaller effect in the outer region (smaller β_{zs}). This suggests that the current non-equilibrium flow does not attain the anticipated equilibrium state corresponding to a given local pressure force effect ($\beta_{zs} = 0.352$ and $\beta_i = 0.0168$). In other words, if the flow were to respond immediately to the pressure force, S2 would exhibit a larger velocity defect than EQ1, given that $\beta_{zs} = 0.352$ at S2 compared with $\beta_{zs} = 0.104$ at EQ1.

Mellor & Gibson (1966) showed that for equilibrium boundary layers (at infinite Reynolds number) there is a unique relationship between the Rotta–Clauser defect shape factor G and β_{RC} , where $G = (U_e/u_\tau)(1 - 1/H)$. The relationship was obtained numerically from their similarity equation computations. A plot of G as a function of β_{RC} effectively illustrates, in a single graph, the departure from equilibrium of a TBL. Such plots have been used recently, for instance, by Knopp *et al.* (2021), Fritsch *et al.* (2022b) and Volino & Schultz (2023). We also use here G as a function of β_{RC} instead of H as a function of β_{zs} for three reasons: G is less sensitive to Reynolds number effects than H , the pressure gradient parameter that is compatible with G from the similarity analysis is β_{RC} (not β_{zs}), plots of G as a function of β_{RC} are commonly used.

TBL response to uniform changes of the pressure force

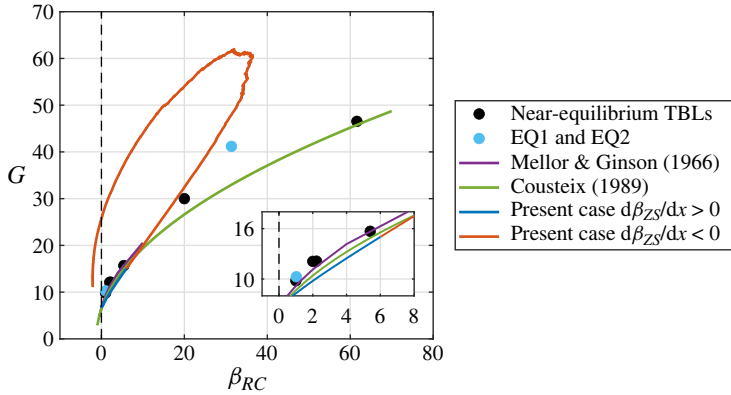


Figure 5. The defect shape factor as a function of β_{RC} for the present DNS (blue and orange are for the regions where $d\beta_{ZS}/dx > 0$ and $d\beta_{ZS}/dx < 0$, respectively), near-equilibrium cases from the literature (Bradshaw 1967; East & Sawyer 1980; Skåre & Krogstad 1994; Bobke *et al.* 2017; Kitsios *et al.* 2017; Sanmiguel Vila *et al.* 2020b), computations of Mellor & Gibson (1966) and correlation of Cousteix (1989).

Figure 5 presents such a plot with the current flow case, together with three types of equilibrium or near-equilibrium data to appreciate its departure from equilibrium. The latter are the equilibrium TBL data of Mellor & Gibson (1966) from their similarity analysis, an empirical correlation from Cousteix (1989) based on near-equilibrium TBL data from the 1968 Stanford conference (Kline, Coles & Hirst 1969) that allows a comparison at high values of β_{RC} , and six cases of near-equilibrium APG TBLs from the literature (Bradshaw 1967; East & Sawyer 1980; Skåre & Krogstad 1994; Kitsios *et al.* 2016; Bobke *et al.* 2017; Kitsios *et al.* 2017; Sanmiguel Vila *et al.* 2020b). The Mellor and Gibson equilibrium data are probably reliable; however, a turbulence closure model had to be employed for its computation. The near-equilibrium flow case at $\beta_{RC} = 34.2$ of Kitsios *et al.* (2017) seems to deviate from the trend given by both the case of East & Sawyer (1980) at $\beta_{RC} = 61.6$ and Cousteix's empirical correlation, but it is impossible to know which dataset might be more biased. The departure from equilibrium of the present flow case can still be appreciated despite these discrepancies.

Indeed, the response lag of the mean flow (mean velocity defect) can be seen throughout the streamwise evolution of the present case. At the start, the defect shape factor G is very slightly lower than the equilibrium case for a given value of β_{RC} , indicating that the momentum-losing effect of the pressure force is not immediate. At $\beta_{RC} \approx 6$, the pressure force starts decreasing in importance in the outer region which should lead to momentum gain. However, the momentum defect continues to increase substantially due to the influence of the upstream momentum-losing effect in the outer region and the increased impact of the pressure force in the inner region. The momentum defect even becomes superior to the equilibrium one at identical β_{RC} . Figure 5 illustrates that the Rotta–Clauser parameter β_{RC} effectively reflects the momentum-losing effect of the pressure gradient across the entire boundary layer. The defect eventually starts decreasing, but it always remains higher than the equilibrium case. In the FPG zone at the end ($\beta_{RC} < 0$), the defect remains significantly higher than that in equilibrium FPG TBLs.

3.1.2. The inner layer

As described in the introduction, β_i represents the ratio of pressure force to turbulent force in the inner layer. Figure 2(c) displays both β_i and the ratio of pressure force to the

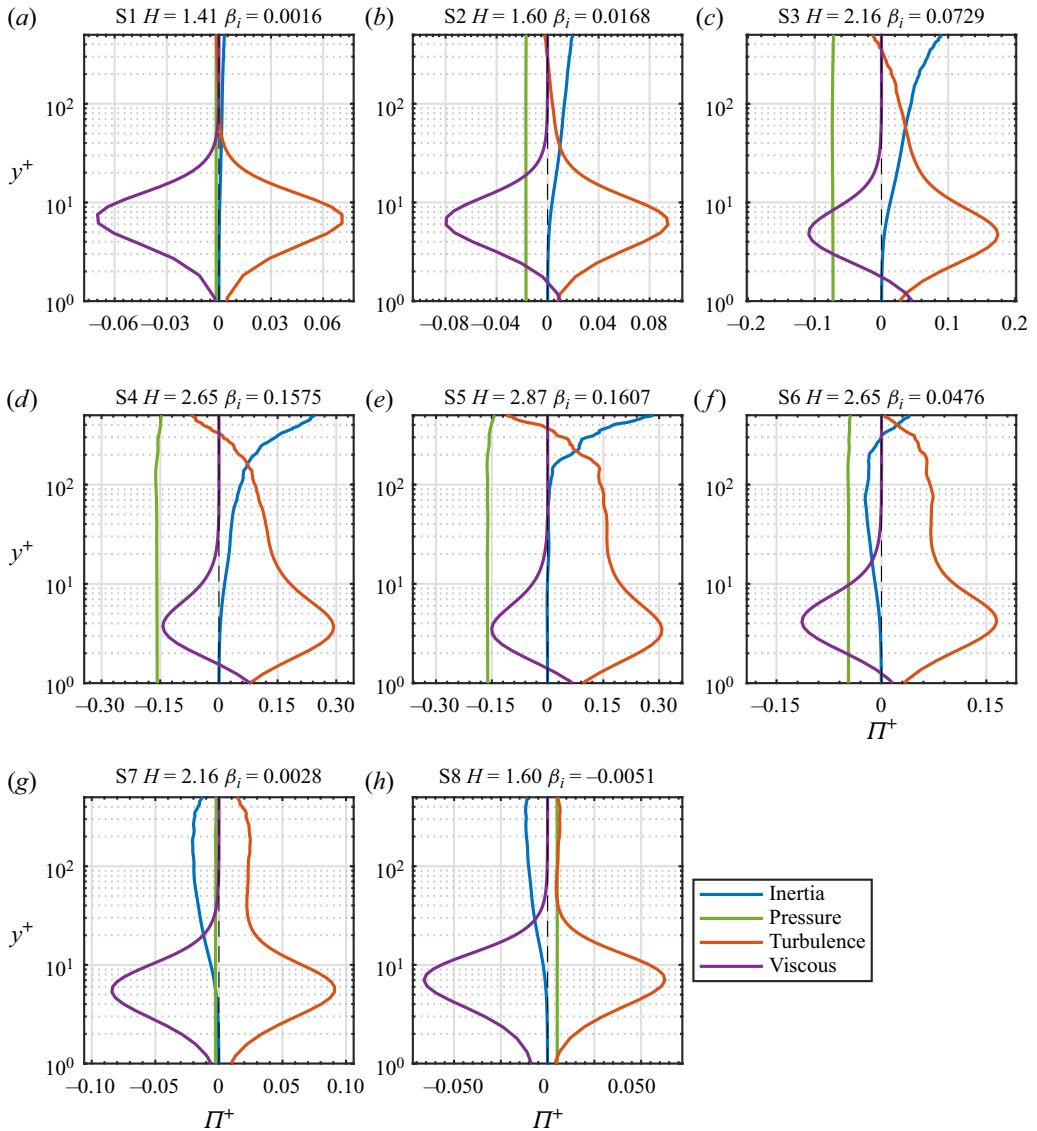


Figure 6. The mean momentum budget profiles of the eight streamwise positions as functions of y^+ . The budget terms are normalised with friction-viscous scales. The x -axis ranges are chosen so that $F_{ii,max}$ is approximately at the same location in all subfigures.

maximum turbulent force in the inner layer, where $F_{p,w}$ is the pressure force at the wall. Although the distributions of force balance and β_i do not follow each other as close as β_{ZS} and the force balance in the outer layer, the trend is the same. Here β_i shows the changes in force balance well and its behaviour is consistent with the behaviour of C_f as given in figure 2(d). Figure 6 displays the force balance at the eight streamwise positions for the inner layer as a function of y^+ . The β_i distribution indicates that the pressure force's relative importance (as a momentum-losing force) in the inner layer increases until approximately $21\delta_{av}$ (between S4 and S5). The force balance from S1 to S4 also demonstrates this as the pressure force's relative importance in the inner layer with respect

TBL response to uniform changes of the pressure force

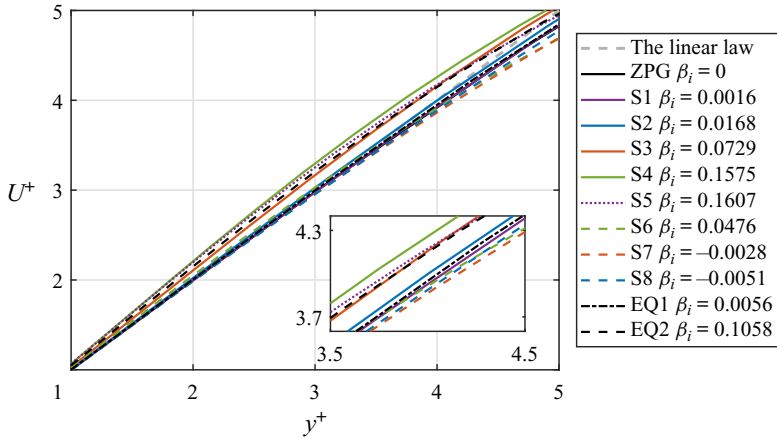


Figure 7. The friction-viscous scaled mean velocity profile of the eight streamwise positions, the near-equilibrium cases and the ZPG TBL case in the viscous sublayer as a function of y^+ . The black line (ZPG TBL) is hidden behind the purple line (S1) as they coincide within the viscous sublayer.

to the other forces increases and later decreases from S5 to S7, and becomes a positive force (FPG) at S8.

This change in force balance affects the mean velocity significantly in the inner layer. Figure 7 displays the mean velocity as a function of y^+ for the eight streamwise positions along with the ZPG TBL case in the viscous sublayer. Note that the ZPG TBL curve, in black, is hidden behind the purple curve of S1 as both curves coincide in the viscous sublayer. As the pressure force's relative importance increases (β_i increases), U^+ increases and deviates from the linear law. This increase is expected as it reflects the change from a linear law to a quadratic law at the wall when the pressure force increases in importance with respect to the wall shear stress force (Patel 1973; Skote & Henningson 2002). This behaviour of U^+ in the viscous sublayer of APG TBLs was already reported before by Gungor *et al.* (2016). The deviation from the ZPG TBL curve reaches its maximum near the position where β_i is the highest. After β_i reaches its maximum, U^+ constantly decreases, which means the pressure force's relative importance begins decreasing. Figure 7 shows that the mean velocity reacts almost instantly to the relaxation of the pressure force in the viscous sublayer. This immediate response of the mean flow to the decrease of the pressure force's relative importance suggests that the effect of flow history is small in the viscous sublayer. This is consistent with the fact that inertia forces, which are linked to the flow history's effect, are small in the viscous sublayer.

There is still a deviation from the ZPG TBL profile in S6 to S8 which are located in the region where $d\beta_i/dx$ is negative, albeit the deviation is mild. The decrease of pressure force's importance from S6 to S8 leads to a decrease in U^+ in the viscous sublayer. In the last position, the pressure force is positive (FPG) which makes S8 different from all other positions. It is important to highlight that the flow would have not become a ZPG TBL if we had kept maintaining $d\beta_{zs}/dx < 0$ in a longer domain. The boundary layer would have eventually relaminarised.

Now, we discuss the overlap layer. It is important to state that there is no consensus regarding the nature of the overlap layer and the presence of the logarithmic law in APG TBLs in the literature, particularly when considering large-defect APG TBLs and those in disequilibrium. Furthermore, the overlap layer may be small or non-existent due to the current flow being at a moderate Reynolds number. To investigate the potential presence of

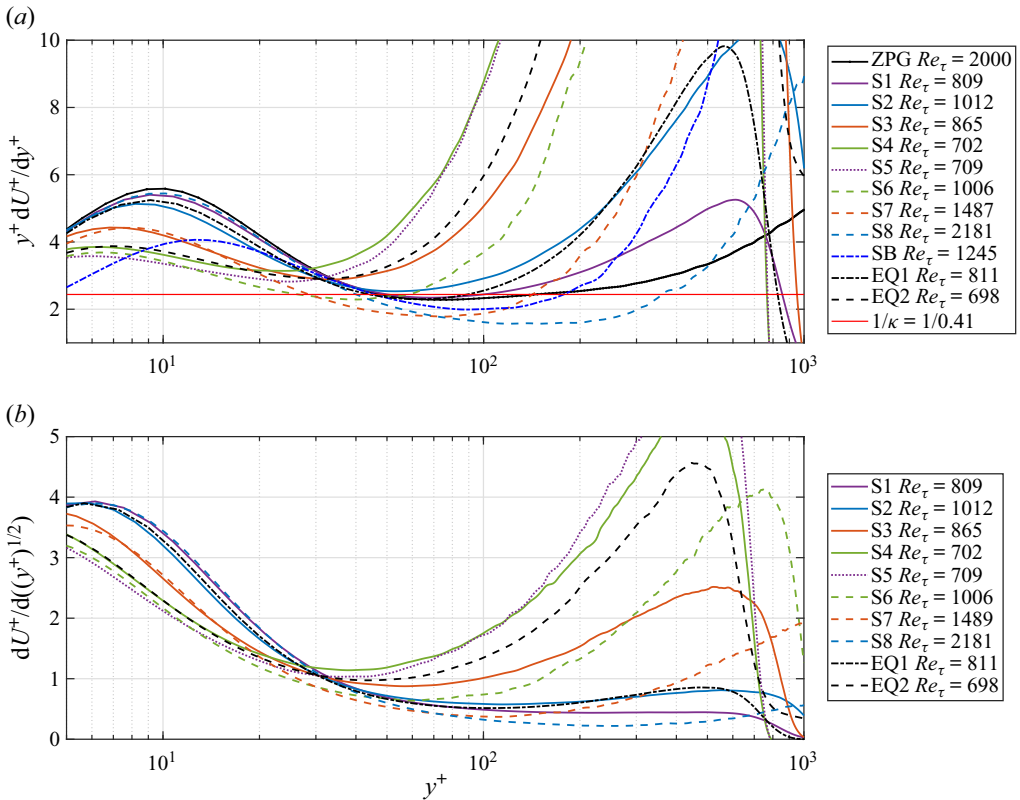


Figure 8. Diagnostic plots for the logarithmic law (a) and the half-power law (b) as a function of y^+ . Straight red line in the top plot indicates the traditional Kármán constant value $1/\kappa = 1/0.41$.

a logarithmic law, [figure 8\(a\)](#) displays the diagnostic plot associated with such a law. The canonical logarithmic law does not hold, even in the small-defect case S1, as well as in the equilibrium cases EQ1 and EQ2. Nonetheless, the trends with respect to the logarithmic behaviour can be analysed. [Figures 8 and 9\(c,d\)](#) show that the flow deviates from the logarithmic law with the traditional values of κ and B , 0.41 and 5, respectively, even at S1 and S2 which are small-defect cases. [Figure 9\(a,b\)](#) illustrate that the consistently positive β_i from S1 to S7 results in a drop of U^+ below the log law. Such an effect has been documented in numerous studies of non-equilibrium APG TBLs (Gungor *et al.* 2016). What is perhaps more surprising is its presence in the near-equilibrium cases depicted in [figure 9\(a\)](#). This suggests that the phenomenon is, in part, a direct effect of the pressure gradient, rather than solely its disequilibrating nature (as indicated by an increasing β_i). In the case of three equilibrium APG TBLs with mild APGs, Lee (2017) also observed a decreasing trend of U^+ below the log law with increasing β_{RC} , though these cases were at very low Reynolds numbers. Past experimental studies of near-equilibrium TBLs tended to confirm a log law behaviour (Clauser 1954; East & Sawyer 1980; Skåre & Krogstad 1994; Elsberry *et al.* 2000), even for cases with very large velocity defects, but since the wall shear stress was not measured directly, it is impossible to know if these flows follow the classical log law. Note that the large-defect near-equilibrium case EQ2 in [figure 8\(a\)](#) does not show a log law behaviour, which could be due to the moderate Reynolds number of the flow.

TBL response to uniform changes of the pressure force

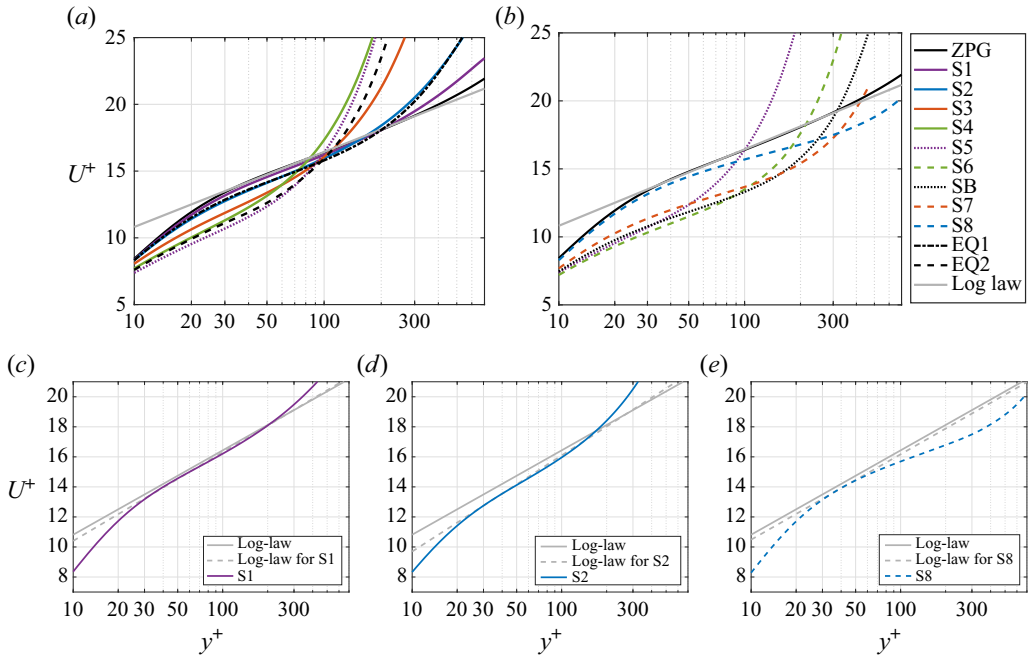


Figure 9. (a,b) Mean velocity profile of various streamwise positions, the ZPG TBL case and equilibrium cases in the overlap layer as a function of y^+ . The streamwise positions from S1 to S5 are given on the left and the streamwise positions from S5 to S8 on the right for the sake of clarity. (c–e) Mean velocity profiles of three streamwise positions, S1 (c), S2 (d) and S8 (e) along with the logarithmic law using the traditional constants and values obtained with the correlations of Nickels (2004), which were used in Knopp *et al.* (2021).

Knopp *et al.* (2021) found similar deviations from the classical log law for an APG TBL with values of H and β_i close to those of S2 but with Re_θ an order of magnitude higher, above 20 000. But as they pointed out, the logarithmic law may exist with different values of κ and B . Figure 9(c,d) show that the mean velocity at S1 and S2 approaches the logarithmic law obtained with the correlations of Nickels (2004) based on β_i , which were used in Knopp *et al.* (2021) for APG TBLs. The wall-normal extent of the approach towards a logarithmic law is thin in part because of the low Reynolds number. However, it is thinner in S2 than in S1 even though the Reynolds number is higher. This indicates that as the pressure force strength increases, the logarithmic layer becomes thinner, eroding from above, as was found by Alving & Fernholz (1995), Knopp *et al.* (2021) and Knopp (2022).

To illustrate the history effects on the mean flow in the overlap layer, we can consider another streamwise position (named SB) where β_i has the same value 0.0167 as S2 but with $H = 2.40$ (between S6 and S7). The mean velocity profile of SB is shown in figure 9(b), and its log-law diagnostic curve in figure 8(a). The main differences between this position and S2 are the flow history and the sign of $d\beta_i/dx$ (positive in S2 and negative in SB). At SB, the flow does not follow the classical log law at all and it behaves completely differently from S2 due to upstream history effects. As another illustration that the non-equilibrium nature of the TBL transforms the overlap layer, β_i is nearly zero at S7 but the mean velocity profile in the overlap region is very different from the ZPG TBL, where β_i is nearly zero too. The behaviour of this non-equilibrium TBL clearly shows that it is not only the local balance of forces that matters, as reflected by β_i , but also the upstream history and the local type of disequilibrating effect of the pressure force ($d\beta_i/dx$).

Because of these effects, the correlations of Nickels (2004) and Knopp *et al.* (2021) do not work for the other stations from S3 to S8. Figure 9(e) shows the mean velocity profile and the logarithmic law for S8. The mean velocity profile seems to approach the modified logarithmic law between $y^+ = 30$ and 50 but it cannot be considered as such because the profile is not logarithmic there as can be seen from figure 8(a). The profile at S8 might actually approach a log law between $y^+ = 100$ and 200. It should also be noted that the flow at S8 is an FPG TBL so if the decrease of β_{ZS} (and β_i and β_{RC}) would have been maintained in a longer domain, which means stronger FPG effects, the velocity profile would have eventually risen above the log law (Patel & Head 1968; Warnack & Fernholz 1998; Dixit & Ramesh 2008). Such a rise above the log law was found in the transition from an APG TBL to an FPG TBL studied by Tsuji & Morikawa (1976)

We also examine the presence of the half-power law which is expected to exist for large-defect TBLs (Stratford 1959; Perry, Bell & Joubert 1966; Coleman *et al.* 2017; Coleman, Rumsey & Spalart 2018; Knopp *et al.* 2021). Figure 8(b) shows the diagnostic plot for the half-power law. It seems there is an approach towards the half-power law in large-defect cases S4, S5 and S6. The region where this approach occurs is located above the region where the mean flow approaches the logarithmic law as discussed previously. Therefore, the wall-normal location of the half-power region is consistent with the literature (Perry 1966; Knopp *et al.* 2021). As with the logarithmic layer, the half-power region is thin.

3.2. Turbulence

3.2.1. Outer layer

The local and upstream pressure force effects on turbulence are now examined. Figure 10 presents outer-scaled Reynolds stresses normalised with U_e . As was reported numerous times for APG TBLs, outer-layer turbulence becomes dominant and inner-layer turbulence loses its importance as the velocity defect increases in the momentum-losing zone (Gungor *et al.* 2016; Maciel *et al.* 2018). Moreover, the Reynolds stress levels in the outer layer increase with increasing velocity defect when they are normalised with U_e . However, the continuing rise of the Reynolds stresses while the flow is under turbulence-reducing conditions ($d\beta_{ZS}/dx < 0$), starting at S3, is indicative of the delayed response of turbulence to changes in the pressure force. This delayed response of turbulence is even more pronounced than that observed in the mean flow. As the velocity defect decreases in the outer layer, the mean shear also decreases (see figure 4) and turbulence is expected to decay. But figure 10 shows that outer turbulence keeps increasing all the way down to S7 due to the delay in response of turbulence to the changes in the pressure gradient and mean velocity. The delay in response is more pronounced for normal stresses $\langle v^2 \rangle$ and $\langle w^2 \rangle$, which continue to rise further downstream than $\langle u^2 \rangle$. This is reasonable since their energy depends on the redistribution from $\langle u^2 \rangle$ through pressure strain. The Reynolds shear stress follows the same trend as $\langle v^2 \rangle$ and $\langle w^2 \rangle$, with levels at S8 still higher than those of S5. It is important to remember that in an equilibrium FPG TBL or an FPG TBL monotonously evolving from a ZPG TBL, the Reynolds stresses in the outer region are lower than those of the ZPG TBL when scaled with U_e (Harun *et al.* 2013; Volino 2020). The Reynolds stress levels at S8 are therefore extremely high for an FPG TBL and this is due to the persistent effect of the upstream flow history.

We now compare the near-equilibrium flow cases EQ1 and EQ2 with the present flow to isolate the effect of flow history. We first focus on the small-defect cases: S2 and EQ1. Similar to our approach in analysing the mean flow, we choose S2 because both cases

TBL response to uniform changes of the pressure force

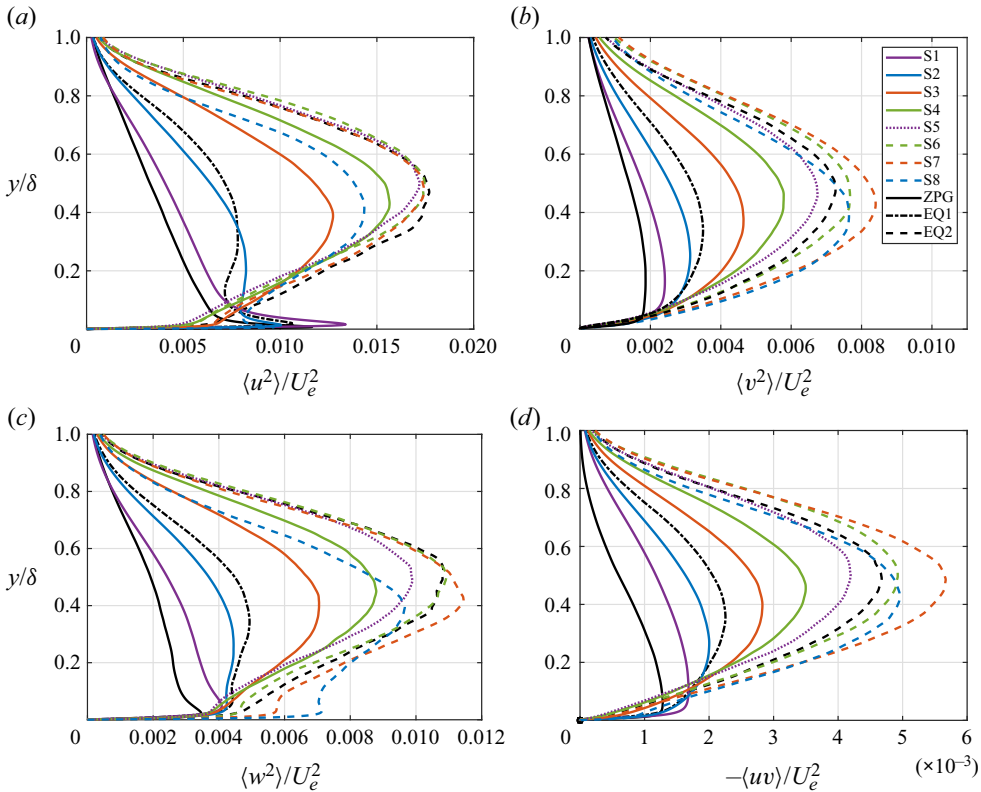


Figure 10. Reynolds stress profiles normalised with the outer scales as functions of y/δ for the eight streamwise positions, the ZPG TBL case and the near-equilibrium cases of EQ1 and EQ2.

exhibit a comparable velocity defect. The mean velocity profile, and consequently the mean shear profile, are similar in both cases. However, the local pressure force effect at S2 is stronger, as indicated by the values of β_{ZS} and β_{RC} in table 2. In addition, S2 is at a higher Reynolds number. Despite the similarities in the mean velocity profiles, the Reynolds stress profiles differ. There exists a disparity in the distribution of intensity among the Reynolds stress components. Specifically, the transverse Reynolds stresses ($\langle v^2 \rangle$ and $\langle w^2 \rangle$) exhibit higher intensity in the case of EQ1. Furthermore, the outer maxima of all Reynolds stress components are situated farther away from the wall in EQ1. In the current flow at S2, the pressure force has reached its local level through a ramp-up process. Consequently, the flow lacks the time needed to generate as much turbulence in the middle of the outer region compared with the near-equilibrium TBL. Moreover, there is insufficient time for the flow to redistribute as much energy to the transverse Reynolds stresses.

Now, we compare the large-defect flow cases, S4 and EQ2, which exhibit similar mean velocity profiles. In this case, the disparity in Reynolds stress levels is even more pronounced. The Reynolds stress levels at S4 are considerably lower than those in the near-equilibrium case. However, unlike the small defect cases, the difference in the wall-normal positions of the outer maxima is less significant, although they still remain lower in the current non-equilibrium flow. The redistribution of energy to the transverse Reynolds stresses continues to be less important in the non-equilibrium flow.

The delayed response of turbulence can be observed more comprehensively through the streamwise evolution of the outer maxima of the Reynolds stresses and turbulent kinetic

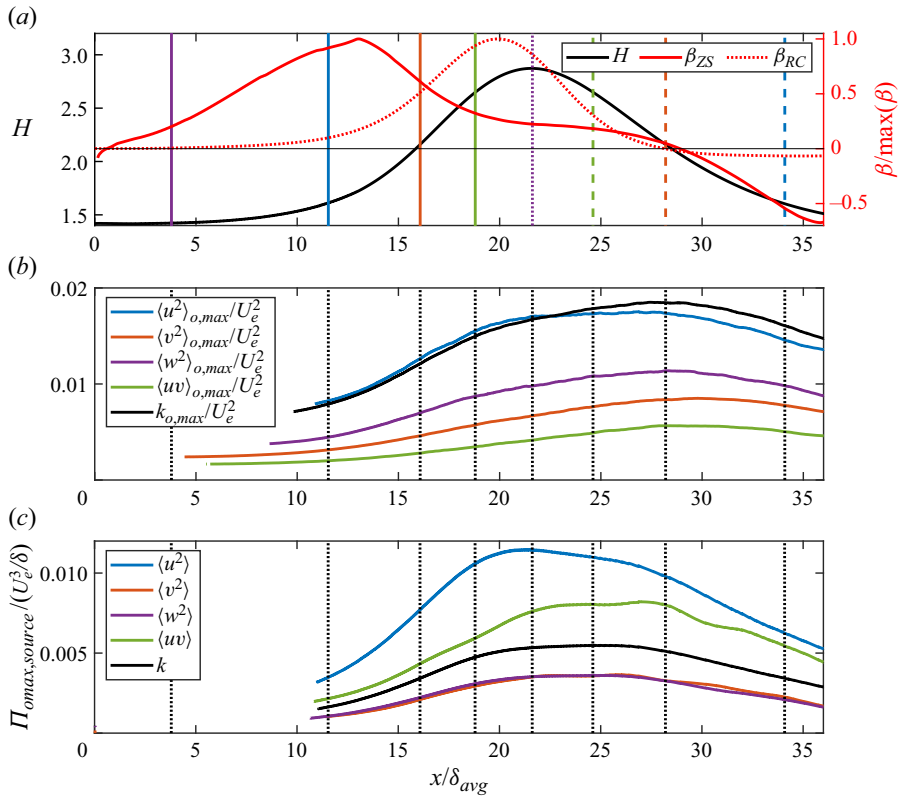


Figure 11. The spatial development of H , $\beta_{ZS}/\max(\beta_{ZS})$ and $\beta_{RC}/\max(\beta_{RC})$ (a), the maxima of turbulent kinetic energy (k) and the Reynolds stresses (b) and pressure-strain for $\langle v^2 \rangle$ and $\langle w^2 \rangle$ components (c) in the outer layer as a function of x/δ_{av} . Data only shown when an outer maximum is present. The levels are normalised with U_e and δ .

energy, k , as depicted in figure 11(b). For ease of comparison with the pressure force effect and the mean flow response, figure 11(a) reproduces the streamwise distributions of β_{ZS} , β_{RC} , and H , also available in figure 2. The starting points of the curves in figure 11(b) correspond to the positions where the outer maxima of the Reynolds stresses begin to manifest. We have already seen that the mean flow responds with a delay, as evident from the H distribution in figure 11(a), which is shifted downstream by approximately 10δ compared with the distribution of β_{ZS} . As mentioned earlier, the delay is even more significant for the Reynolds stresses in the outer region and it varies among the different Reynolds stress components. The maxima of $\langle u^2 \rangle$ and k peak at $x/\delta_{av} \approx 27$ and 28, respectively, which is roughly 14δ downstream of the maximum of β_{ZS} . The delay is more pronounced for $\langle v^2 \rangle$ and even more so for $\langle w^2 \rangle$, again underscoring the delay in energy redistribution.

Outer turbulence therefore continues to increase for a considerable distance even after the pressure force begins to diminish. Furthermore, figure 11(b) illustrates that the subsequent decay of turbulence is slower than the initial upstream turbulence growth, even if the increase and decrease in pressure force are of the same amplitude (comparable to $|\mathrm{d}\beta_{ZS}/\mathrm{d}x|$). This slow turbulence decay cannot be solely attributed to the prolonged survival of energetic turbulence structures convected from upstream, as this cannot explain the observed turbulence buildup where the pressure force is decreasing. A tentative

TBL response to uniform changes of the pressure force

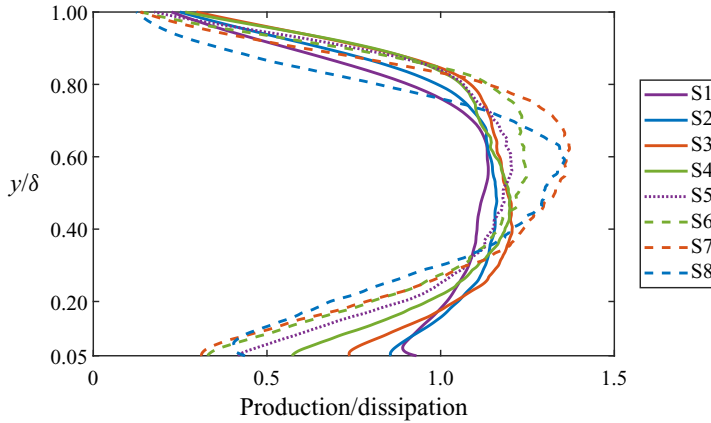


Figure 12. The ratio of production to dissipation of turbulent kinetic energy as a function of y/δ .

explanation can be offered by analysing the source and sink terms of k and Reynolds stresses. Figure 11(c) illustrates the streamwise evolution of the maxima of the source terms of k and the Reynolds stresses. In the case of k and $\langle u^2 \rangle$, the source term is the corresponding production term whereas the main contribution comes from the pressure-strain (redistribution) term for $\langle v^2 \rangle$ and $\langle w^2 \rangle$. In addition, the $\langle v^2 \rangle$ production is considered for $\langle v^2 \rangle$, albeit its contribution is small. It is observed in this figure that the production of k and $\langle u^2 \rangle$ has peaked around S5. One might expect a subsequent decrease of k ; however, as noted earlier, k continues to increase up to $x/\delta_{av} \approx 28$ (figure 11b). This growth is attributed to an increase of the ratio of production to dissipation of k in the middle of the boundary layer, from S5 to S7, as illustrated in figure 12. Indeed, figure 12 demonstrates that the ratio of production to dissipation increases after S5 in the approximate wall-normal range $y/\delta = 0.3$ to 0.8 . Given that this wall-normal range corresponds to the zone where production is maximal, the rise in the ratio of production to dissipation results in the continued increase of k between S5 and S7 even if production has decreased in absolute terms.

As a concluding exploration of outer region turbulence, we examine the budget of turbulent kinetic energy to gain a deeper understanding of turbulence behaviour in the current non-equilibrium flow. The transport equation for the turbulent kinetic energy is given as follows:

$$0 = - \left(U \frac{\partial k}{\partial x} + V \frac{\partial k}{\partial y} \right) - \left(\frac{\partial U}{\partial x} \langle u^2 \rangle + \frac{\partial U}{\partial y} \langle uv \rangle + \frac{\partial V}{\partial x} \langle uv \rangle + \frac{\partial V}{\partial y} \langle v^2 \rangle \right) - \epsilon - \frac{1}{2} \left(\frac{\partial \langle uk \rangle}{\partial x} + \frac{\partial \langle vk \rangle}{\partial y} \right) + \nu \left(\frac{\partial^2 k}{\partial x^2} + \frac{\partial^2 k}{\partial y^2} \right) - \frac{1}{\rho} \left(\frac{\partial \langle pu \rangle}{\partial x} + \frac{\partial \langle pv \rangle}{\partial y} \right). \quad (3.2)$$

The terms are (in order) mean convection, production, dissipation, turbulent transport, viscous diffusion and pressure transport. The budgets are illustrated in figure 13. For the sake of simplicity, for the current flow, we focus only on four representative stations that depict distinct flow situations: the small-defect case S2, the largest defect cases S4 and S5, and the small-defect case in the FPG region S8 which shares the same shape factor as S2. In each plot, a near-equilibrium case (ZPG, EQ1 or EQ2) is included as a reference for comparison.

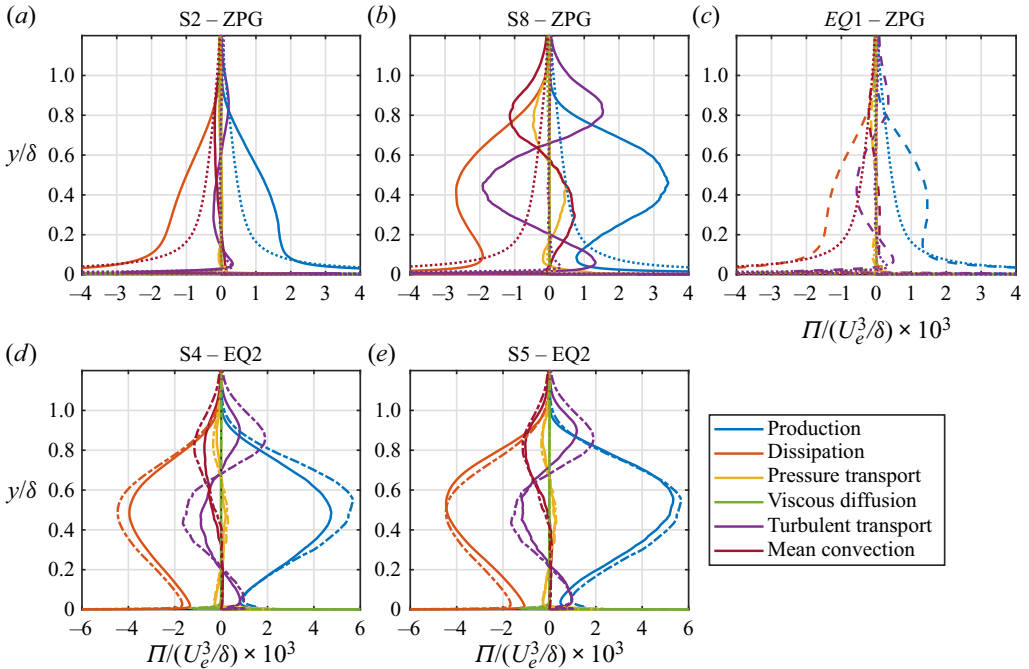


Figure 13. The turbulent kinetic energy budget for S2, S8, EQ1, ZPG, S4, S5 and EQ2 as a function of y/δ . The terms are normalised with outer scales (U_e and δ). The straight, dashed, dashed-dotted and dotted lines are for the present case, EQ1, EQ2 and ZPG, respectively.

In the small-defect case S2, the turbulent kinetic energy budget, shown in figure 13, resembles that of canonical wall flows as the comparison with the ZPG TBL indicates. However, there is an accumulation of production and dissipation in the outer region. This characteristic is typical of small-defect APG TBLs that either originated as ZPG TBLs (Gungor *et al.* 2022) or are in near-equilibrium state (Kitsios *et al.* 2016; Bobke *et al.* 2017; Kitsios *et al.* 2017). Nevertheless, when comparing the budget of S2 with that of the near-equilibrium case EQ1, which shares the same shape factor but has smaller pressure gradient parameters β_i , β_{ZS} and β_{RC} , the near-equilibrium case exhibits a local maximum of production around $y/\delta = 0.35$, a feature absent in S2. The absence of a production maximum in S2 serves as another manifestation of the delay in the turbulence response of the current flow. Turbulent transport is also much stronger in the near-equilibrium case.

The two lower plots compare the large-defect cases S4 and S5 with EQ2. In all these large-defect cases, there is a noticeable presence of production, dissipation and turbulent transport in the middle of the boundary layer, consistent with the heightened turbulent kinetic energy observed in that region. The value of β_{ZS} at S4 and EQ2 is almost the same, but the budget terms at S4 have not reached the levels observed in EQ2, once again indicating a delay in turbulence response. S5 is the case with the largest mean velocity defect. Even so, the budget terms are not more significant than those for EQ2.

The FPG TBL case S8 is compared with the ZPG TBL in the top-middle plot. Despite S8 being an FPG TBL, the outer maxima of the budget terms persist in the middle of the boundary layer due to the delayed response of turbulence, and their levels are much stronger than those in the ZPG TBL. An equilibrium FPG TBL with the same β_{ZS} as S8 would exhibit a budget similar to the ZPG TBL, albeit with even lower levels in the outer region due to reduced mean shear. Although production and dissipation have decreased

TBL response to uniform changes of the pressure force

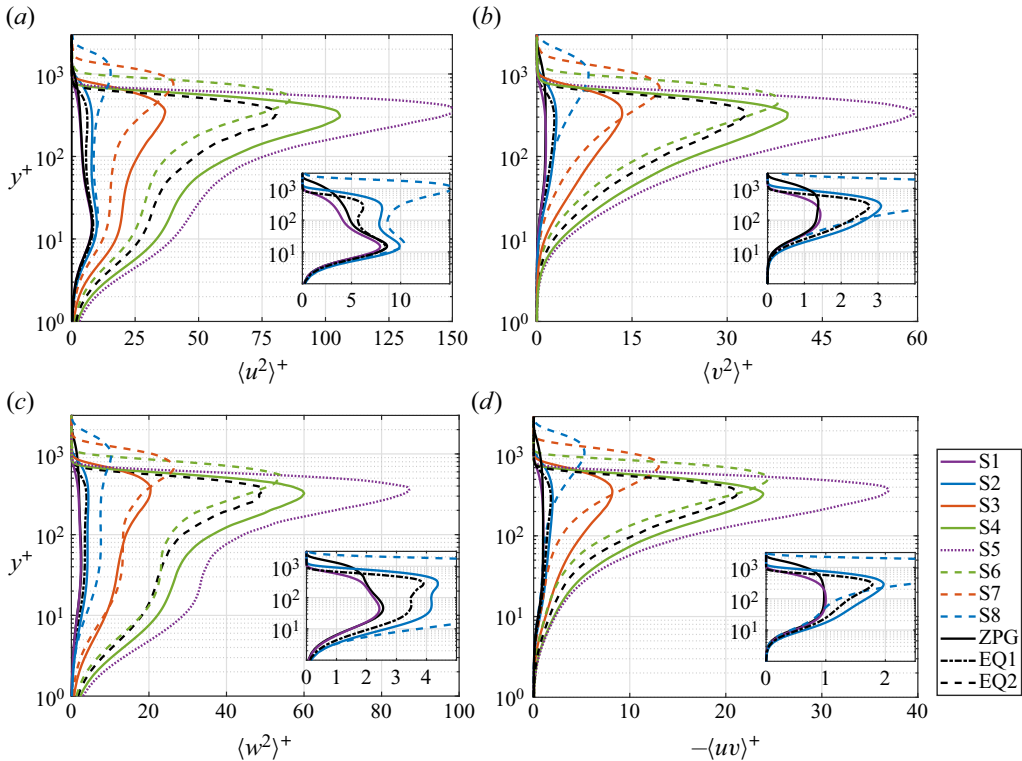


Figure 14. Reynolds stress profiles normalised with friction-viscous scales as functions of y^+ for the eight streamwise positions, the ZPG TBL case and the near-equilibrium cases EQ1 and EQ2.

since S5, they remain elevated. Surprisingly, in contrast, turbulent transport at S8 is higher than at S5. Figure 10 illustrates that the Reynolds stress profiles at S8 are wider than those at S5 due to these sustained high levels of turbulent transport.

3.2.2. Inner layer

Figure 14 illustrates Reynolds stress profiles as functions of y^+ . When these profiles are normalised with friction-viscous scales, it is observed that the Reynolds stresses increase everywhere as β_i and the velocity defect in the momentum-losing zone (until S5). This phenomenon has been well-documented in previous studies on APG TBLs (Gungor *et al.* 2016; Maciel *et al.* 2018). The increase in Reynolds stress levels in such boundary layers occurs because the friction velocity is inadequate as a scaling parameter for Reynolds stresses when dealing with large-velocity-defect TBLs (Maciel *et al.* 2018). As the value of β_i decreases, starting from S5 and continuing onwards, the amplitude of Reynolds stresses also decreases.

In the near-wall region where inertia effects are negligible, below $y^+ = 10$, if turbulence were to respond instantly to the pressure force, the Reynolds stress profiles should be similar at identical β_i . However, stations S4 and S5, with close values of β_i , demonstrate that this is not the case. The Reynolds stress levels are higher at S5 than S4 in the near-wall region. Another indication of this discrepancy in response to the pressure force is the fact that the Reynolds stress profiles at S8 are higher than those in the ZPG TBL. Since β_i is negative at S8 (FPG TBL), the Reynolds stress levels should be lower than those of

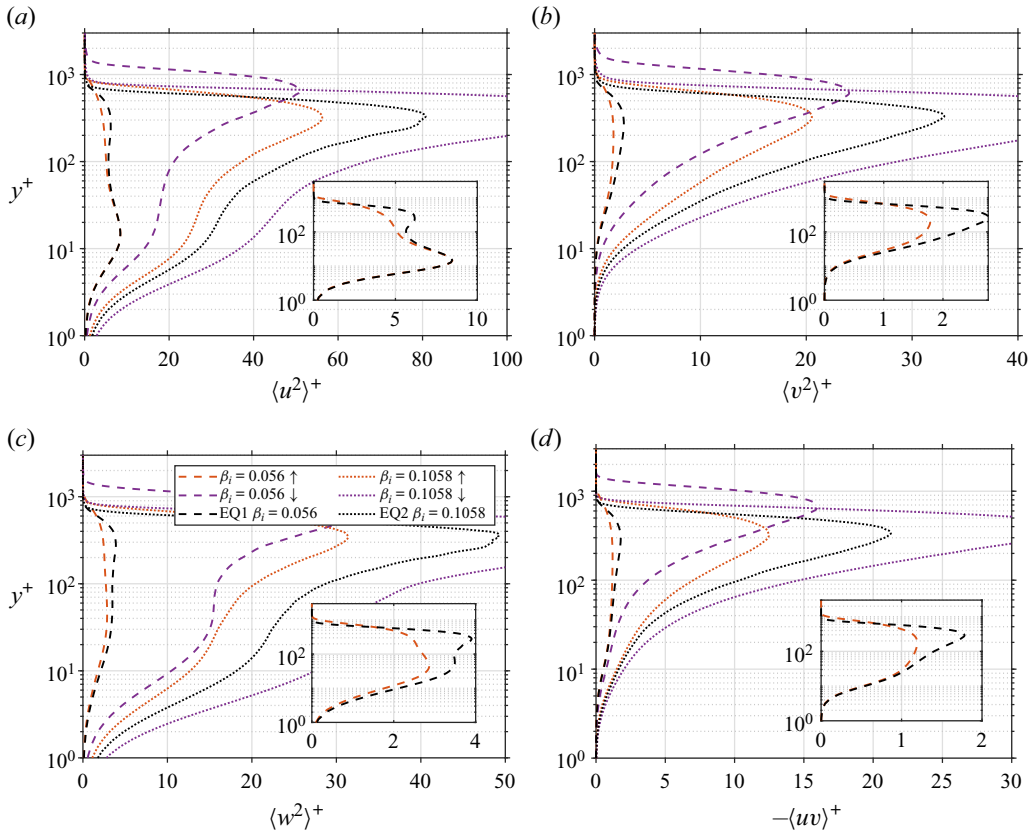


Figure 15. Reynolds stress profiles normalised with friction-viscous scales as a function of y^+ for the streamwise positions of the present case along with EQ1 and EQ2 at matching β_i . The arrow indicates whether β_i is increasing or decreasing.

the ZPG TBL in an equilibrium situation. These two observations suggest a delay in the response of turbulence. However, this delay may not be solely local; it could also be an indirect effect. As discussed later, part of the reason lies in the influence of large-scale turbulence on the near-wall region. The delayed response of the large-scale structures affects the near-wall region.

As for a comparison with the near-equilibrium cases, figure 15 illustrates the Reynolds stress profiles of the present flow, EQ1 and EQ2 at matching values of β_i . For the current flow, two profiles are shown at each β_i value: one closer to the beginning of the domain, where the APG turbulence-promoting effect is increasing ($d\beta_i/dx > 0$) and depicted in orange, and another further downstream where the APG effect is decreasing ($d\beta_i/dx < 0$) and shown in purple. For the small-defect cases, a comparison between EQ1 and the first position in the current flow (dashed orange) reveals, in the enlarged-view plots of figure 15, that the Reynolds stresses in the inner region match up to approximately $y^+ = 30, 10$ and 20 for $\langle u^2 \rangle^+, \langle v^2 \rangle^+$ and $\langle uv \rangle^+$, respectively. For $\langle w^2 \rangle^+$, the profiles start diverging very near the wall and no explanation has been found for such distinctive behaviour. Overall, in the present flow, the Reynolds stresses in the inner region have not yet attained the levels observed in the near-equilibrium case EQ1. Nevertheless, these history effects remain relatively minor when compared with all other stations located further downstream.

TBL response to uniform changes of the pressure force

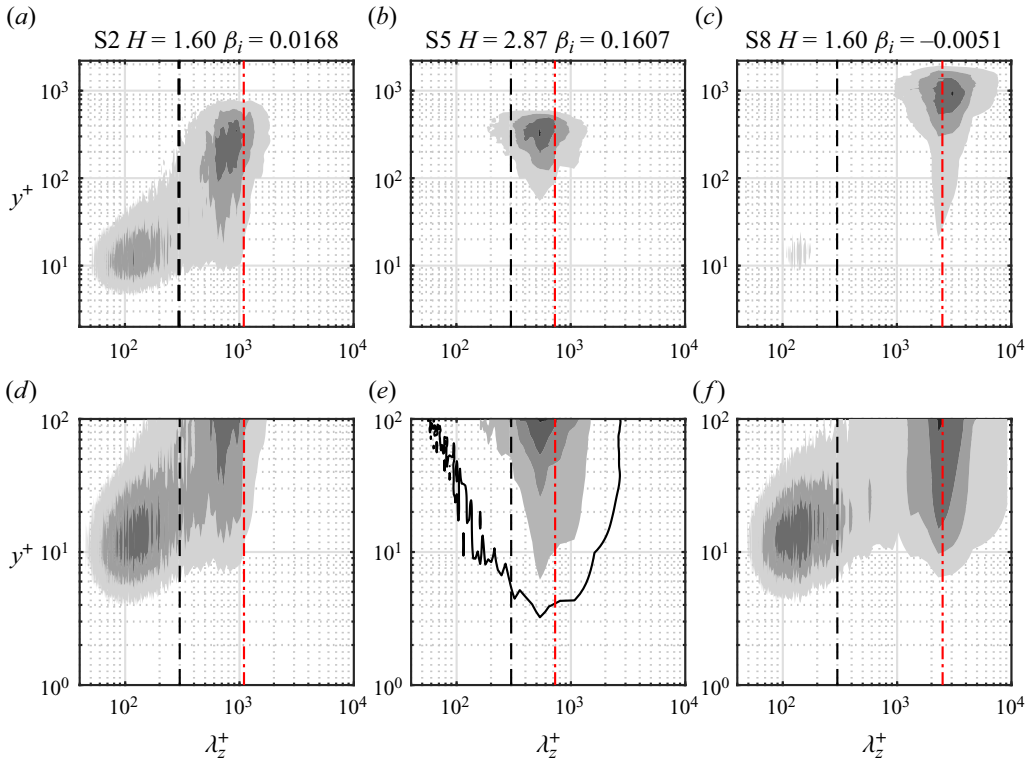


Figure 16. The spectral distribution of $\langle u^2 \rangle$ as a function of λ_z^+ and y^+ for S2, S5 and S8. The spectra are plotted for the whole wall-normal range (a–c) and for the region up to $y^+ = 100$ (d–f). The flooded-contour levels are 0.3, 0.5, 0.7 and 0.95 of the maxima of each spectra. The black-line contour is 0.1 of the maxima. The dashed black lines indicate the cutoff filter $\lambda_z^+ = 300$ and the red dash-dotted lines indicate $\lambda_z^+ = \delta^+$.

Indeed, at the second small-defect station (dashed orange), much further downstream, the deviations from the near-equilibrium case are significant everywhere, even very near the wall. As for the large-defect cases, the Reynolds stresses in the inner region at the first position in the current flow (dotted orange) have not reached the levels of the near equilibrium case EQ2. Meanwhile, at the second station (dotted purple), they have exceeded those levels. This underscores again the substantial delay in turbulence response to changes in the pressure force.

Turning our attention to $\langle u^2 \rangle^+$, figure 14(a) demonstrates that the near-wall peak, observed at $y^+ = 15$ in the ZPG TBL, becomes less sharp and eventually disappears as the local APG pressure force effect increases. This phenomenon, evident in both non-equilibrium and near-equilibrium flows, is attributed to the significant increase in $\langle u^2 \rangle^+$ above the near-wall region. The vanishing of the near-wall peak, coupled with the rise of $\langle u^2 \rangle^+$, has been documented previously (Nagano, Tsuji & Houra 1998; Drózdź, Elsner & Drobnik 2015; Gungor *et al.* 2016). This suggests that the small-scale near-wall turbulence activity might be obscured by the footprints of the energetic large-scale structures located above the near-wall region. To investigate this, we analyse the spectral content of $\langle u^2 \rangle$ and $\langle uv \rangle$, along with wall-normal profiles of these Reynolds stresses decomposed into small and large scales.

Figures 16 and 17 depict the premultiplied $\langle u^2 \rangle$ and $\langle uv \rangle$ one-dimensional spanwise spectra, respectively, as functions of λ_z^+ and y^+ for S2, S5 and S8. The black dashed lines

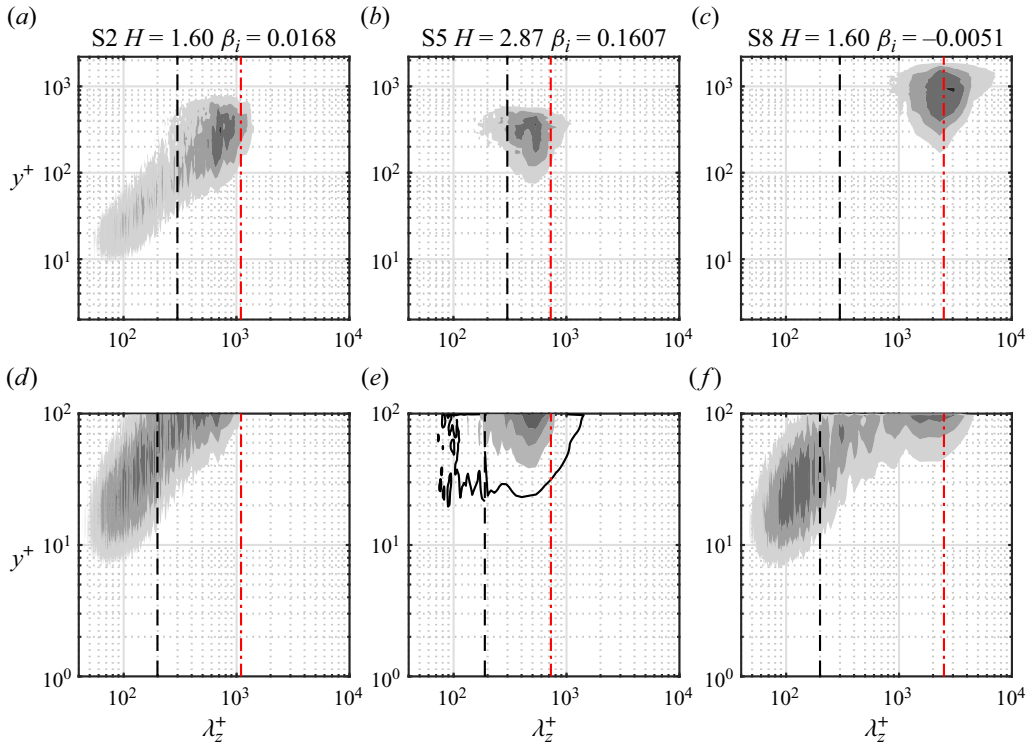


Figure 17. The spectral distribution of $\langle uv \rangle$ as a function of λ_z^+ and y^+ for S2, S5 and S8. The spectra are plotted for the whole wall-normal range (a–c) and for the region up to $y^+ = 100$ (d–f). The flooded-contour levels are 0.3, 0.5, 0.7 and 0.95 of the maxima of each spectra. The black-line contour is 0.1 of the maxima. The dashed black lines indicate the cutoff filter $\lambda_z^+ = 300$ and the red dash-dotted lines indicate $\lambda_z^+ = \delta^+$.

in these figures delineate the wavenumber sharp filter cutoff that will later be employed to decompose small and large scales. The bottom row presents the same spectra as the top row but only up to $y^+ = 100$ and with chosen contour values to facilitate clearer visualisation of the energy content in the inner region.

The spectral distribution of $\langle u^2 \rangle$ at S2 exhibits an inner peak, indicative of the near-wall streaks. These streaks are narrow structures with a λ_z^+ of approximately 120, a characteristic extensively discussed in the literature (Smith & Metzler 1983). As the velocity defect increases from S2 to S5, the outer-layer turbulence becomes dominant, and the inner peak vanishes from the spectra. This is consistent with the literature for large-defect TBLs (Kitsios *et al.* 2017; Lee 2017; Gungor *et al.* 2022). Despite the absence of the inner peak, it remains unclear whether the streaks have vanished or if they are obscured by the presence of more intense large-scale structures (Gungor 2023; Gungor, Maciel & Gungor 2024).

Following the reversal to momentum-gaining conditions induced by the pressure gradient effect ($d\beta_{ZS}/dx < 0$ and $d\beta_i/dx < 0$), the inner peak re-emerges at S8 with a similar λ_z^+ , but it is weak compared with the outer peak. This finding is significant, demonstrating that inner-layer turbulence is becoming like that of canonical flows, as expected for an FPG TBL with a moderate β_i value, despite the presence of history effects. It is important to emphasise once again that the flow is not reverting to a ZPG TBL state, as $d\beta_{ZS}/dx < 0$ is maintained. If the pressure force variation were imposed in a longer domain, the flow would eventually laminarise.

In the inner region, $\langle u^2 \rangle$ is also associated with wide structures at all stations. These large-scale structures have a λ_z on the order of δ , as in the outer region. At S5, these structures carry the majority of $\langle u^2 \rangle$ in the inner layer. Furthermore, at S8, the relative contribution of the large scales in the inner region is higher than at S2 due to the presence of very large and energetic structures convected from upstream. This history effect also results in the structures being wider, with respect to δ , at S8 than S2. Gungor *et al.* (2024) showed that these large-scale structures primarily belong to the outer layer.

The spectral distributions of $\langle uv \rangle$ in figure 17 exhibit similar results to those of $\langle u^2 \rangle$. The inner peak is present in the small defect cases S2 and S8, although it is notably weaker at S8 when compared with the outer peak. The spectral distribution of $\langle uv \rangle$ at S2 is consistent with those found in the literature (Lee 2017; Gungor *et al.* 2022). In the large-defect case S5, the inner peak has vanished, and $\langle uv \rangle$ is influenced significantly by large-scale structures in the inner region, although small-scale structures still contribute. Overall, the effect of the large-scale structures in the inner layer is less important for $\langle uv \rangle$ than for $\langle u^2 \rangle$, a trend also noted by Gungor *et al.* (2024).

To investigate small-scale properties in the inner layer, we employ a filtering process to isolate the contributions of small scales by separating them from the large scales. To achieve this, we decompose the Reynolds stresses using a wavenumber sharp filter cutoff in the spectral domain, as depicted in figures 16 and 17 with vertical black dashed lines. We set the cutoff wavelength at $\lambda_z^+ = 300$, chosen for its effective differentiation between small and large scales in the inner region. This value is also used in the recent study by Deshpande *et al.* (2023). We have tested that the reconstructed Reynolds stress distributions are relatively insensitive, but only qualitatively (see the discussion in the following), to the specific filter cutoff value.

Figure 18 presents the Reynolds stress profiles of small scales ($\langle u^2 \rangle_{SS}^+$ and $\langle uv \rangle_{SS}^+$) and large scales ($\langle u^2 \rangle_{LS}^+$ and $\langle uv \rangle_{LS}^+$) as functions of y^+ . The full Reynolds stress profiles are presented at the top of the figure to facilitate comparison. In contrast to the full $\langle u^2 \rangle^+$ profiles, the small-scale $\langle u^2 \rangle_{SS}^+$ profiles at S3 and S7 reveal an inner peak. Furthermore, the inner peak has become sharper at S1, S2 and S8. These results indicate that large-scale structures indeed influence the Reynolds stress distributions in the inner layer, leading to the obscuration of small-scale contributions and the near-wall cycle of turbulence production.

Deshpande *et al.* (2023) found that the inner peak of $\langle u^2 \rangle_{SS}^+$ collapses well across ZPG TBLs and TBLs subjected to very mild APG ($\beta_{RC} < 1.7$), across various Reynolds numbers. The reported peak value of $\langle u^2 \rangle_{SS}^+$ is 5.5 at $y^+ = 15$. Recall that we employ the same filtering procedure. In the current flow, the inner peak of $\langle u^2 \rangle_{SS}^+$ closely resembles that of Deshpande *et al.* (2023) only at the first station, S1, with a small velocity defect and minimal history effects. As the velocity defect increases, the $\langle u^2 \rangle_{SS}^+$ inner profile also increases, eventually leading to the disappearance of the peak. Positions with substantial velocity defects, specifically S4 to S6, do not exhibit inner peaks in the profiles, except for a hump observed at S6. The disappearance of the near-wall peak suggests that the near-wall cycle could be significantly attenuated or even absent in TBLs with large velocity defects.

Nevertheless, interpreting the decomposed Reynolds stresses requires caution. First, the filtering procedure based on one-dimensional spectra has inherent limitations. A more refined separation between small and large scales could be achieved by employing a filtering approach based on two-dimensional streamwise–spanwise spectra, as demonstrated by Lee & Moser (2019). Similar to Deshpande *et al.* (2023), Lee & Moser (2019) reported a Reynolds number independence of the $\langle u^2 \rangle_{SS}^+$ profile up to $y/\delta = 0.2$ in channel flow cases. Yet, in their case, the peak value of $\langle u^2 \rangle_{SS}^+$ exceeded 7. Lastly, it is

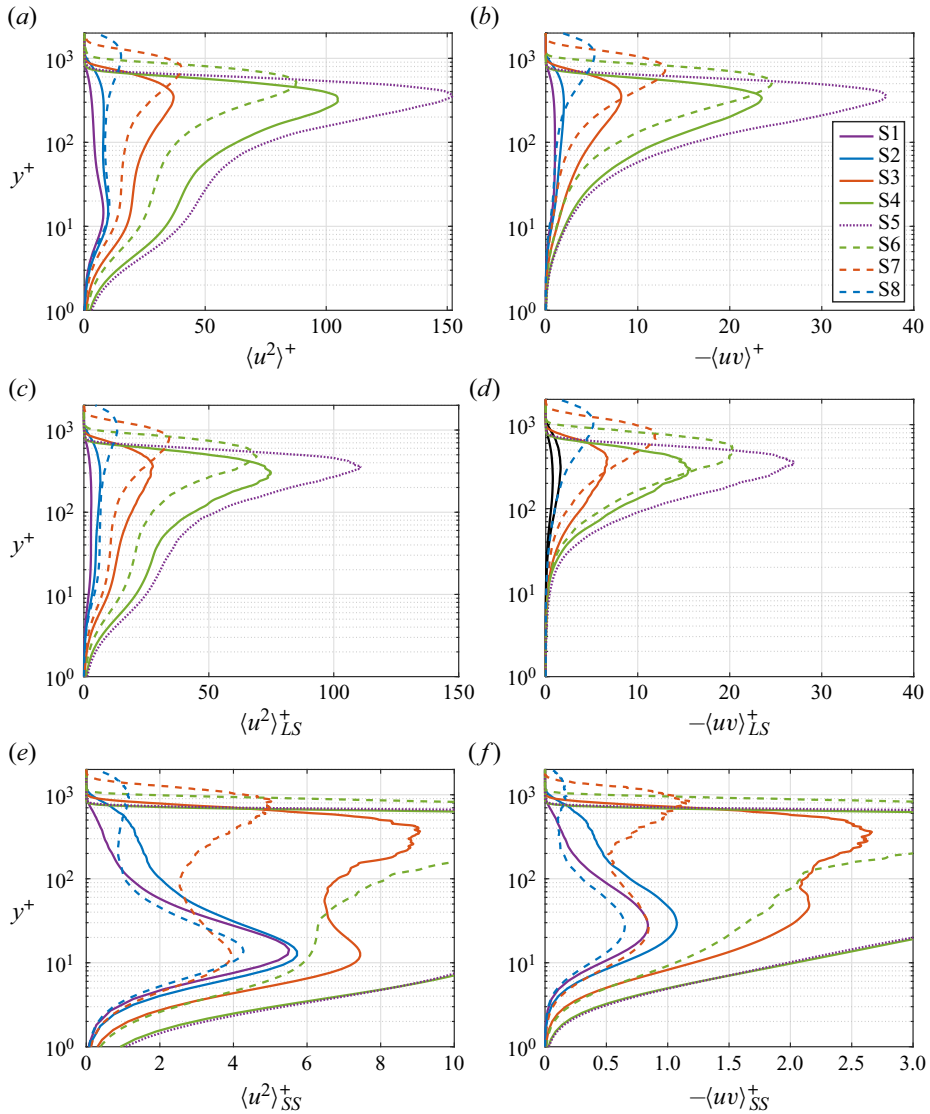


Figure 18. Reynolds stress profiles as a function of y^+ for the full signal (a,b), large-scale structures (c,d) and small-scale structures (e,f).

plausible that the Reynolds number in our study might not be sufficiently high to achieve a clear-cut scale separation of scales.

The inner peak of $\langle u^2 \rangle_{SS}^+$ at S7 is rather surprising because it is below that at S1, the latter closely following the ZPG TBL and small defect behaviour presented in Deshpande *et al.* (2023). At S7, the pressure gradient is null. One could therefore expect the inner peak to be similar to the ZPG TBL one (and, therefore, also to the S1 peak) or even higher due to the upstream trend. The lower-level peak might suggest that the near-wall cycle has not yet fully recovered from the upstream pressure gradient effects, if it was indeed dampened or destroyed by them. It should be noted that, in contrast to the situation of $\langle u^2 \rangle_{SS}^+$, the inner peak of the small-scale Reynolds shear stress $-\langle uv \rangle_{SS}^+$ at S7 is similar to that observed at S1.

TBL response to uniform changes of the pressure force

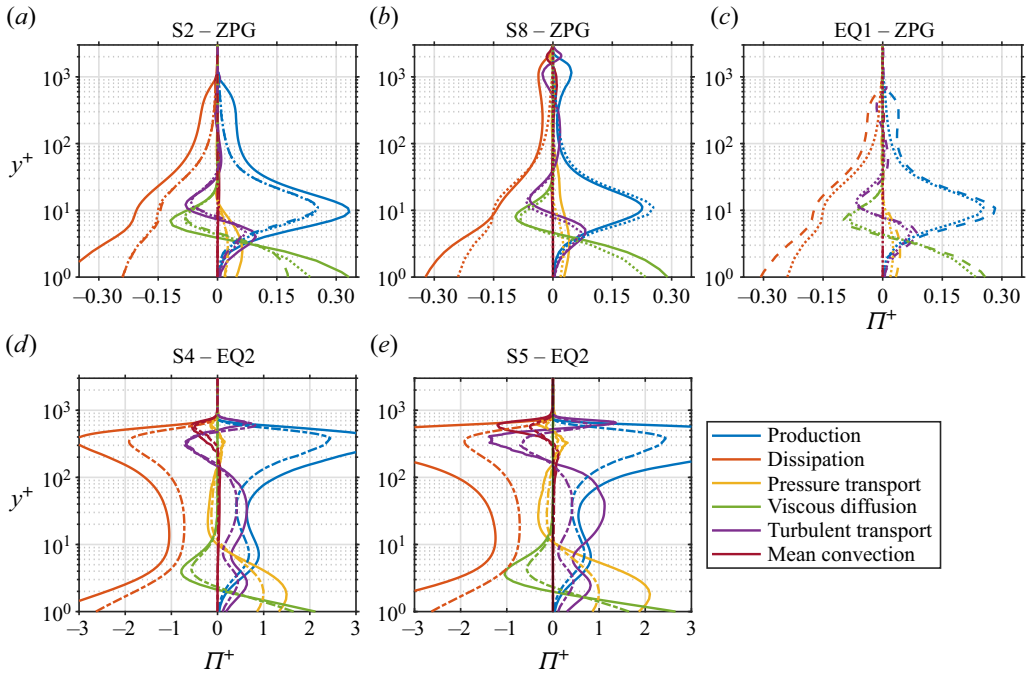


Figure 19. The turbulent kinetic energy budget for S2, S8, EQ1, ZPG, S4, S5 and EQ2 as a function of y^+ . The terms are normalised with friction-viscous scales. The straight, dashed, dashed-dotted and dotted lines are for the present case, EQ1, EQ2 and ZPG, respectively.

Regarding the inner peak of $\langle u^2 \rangle_{SS}^+$ at S8, it is expected that it is lower than that at S1. FPG TBLs that are in near-equilibrium or are monotonically developing from a ZPG TBL have lower $\langle u^2 \rangle^+$ levels in the inner region (complete profile) compared with ZPG TBLs (Bourassa & Thomas 2009; Harun *et al.* 2013; Volino 2020), even if the difference is not as pronounced as in the outer region.

As for the small-scale Reynolds shear stress $-\langle uv \rangle_{SS}^+$, it behaves similarly to $\langle u^2 \rangle_{SS}^+$. The notable difference, as mentioned previously, is that the inner peak at S7 is similar to that at S1, as would be expected if there were no history effects. The inner peak of $-\langle uv \rangle_{SS}^+$ is associated with the near-wall production of $\langle u^2 \rangle_{SS}^+$.

The profiles of large-scale Reynolds stresses demonstrate that large scales indeed contribute significantly to the inner region, and this contribution increases with the mean velocity defect (not with β_i). As expected from the spectral distributions, $-\langle uv \rangle_{LS}^+$ is small below $y^+ = 10$ whereas $\langle u^2 \rangle_{LS}^+$ is significant in that region, comparable to $\langle u^2 \rangle_{SS}^+$. The large-scale structures reaching the very-near-wall region cannot have a large wall-normal velocity. They are long streaky u -structures rather than sweeps and ejections.

To gain a deeper understanding of inner layer turbulence behaviour, we analyse the turbulent kinetic energy budget at the streamwise positions S2, S8, S4 and S5 along with ZPG, EQ1 and EQ2. Figure 19 illustrates the budget terms, normalised with friction-viscous scales, as functions of y^+ , with a similar format as the outer-scaled budgets in figure 13. The balance of the budget terms in the inner layer at S2 closely resembles that of canonical wall flows; however, the levels of the terms are approximately 20–30% higher at S2. Another distinction is the higher level of production around

$y^+ = 100$ relative to its peak value at $y^+ = 10$ in the APG case, aligning with the behaviour of the Reynolds stresses depicted in figure 14. These similarities and differences have been observed in small-defect APG TBLs, either originating as ZPG TBLs (Gungor *et al.* 2022) or in near-equilibrium states (Bobke *et al.* 2017; Kitsios *et al.* 2017).

Moving from S2 to S4 and S5, the inner wall-normal distributions of the various energy transfer mechanisms become notably different. For instance, production still exhibits an inner maximum, but it is no longer dominant compared with the other terms due to high levels of turbulence being transported from the outer layer. The budget terms have higher values than those for EQ2, as can be expected given that β_i is higher at S4 and S5. At S8, the inner budget has returned to one typical of canonical wall flows, with levels comparable to those of the ZPG TBL and smaller than at S2. However, turbulent transport remains strong above $y^+ = 40$ due to significant outer turbulence resulting from history effects. These history effects likely explain why the Reynolds normal stresses in the inner layer are much higher at S8 than at S2 (figure 14) despite the lower energy transfer terms.

4. Conclusion

The current study has investigated the response of the TBL to uniform changes in the pressure force contribution to the force balance. The primary objective was to enhance our understanding and differentiation of the three types of effects resulting from the pressure force: the direct impact of the local pressure force, the influence of a local pressure force change (local disequilibrating effect) and the impact of the upstream pressure force evolution. The contribution of the pressure force to the force balance can be characterised by a pressure gradient parameter. However, due to differences in this contribution between the inner and outer regions (assuming two layers without consensus on this matter for pressure gradient boundary layers), two pressure gradient parameters become necessary. In this study, we have chosen the pressure gradient parameters β_i and β_{ZS} for the inner and outer regions, respectively, as we show that they follow reasonably well the ratio of pressure force to turbulent force in each region.

To achieve the aforementioned objective, we devised a flow with a specific streamwise distribution of the pressure force contribution in the outer region, by choosing a distinctive distribution of β_{ZS} . The emphasis is placed on the outer region due to its more intricate and significant response compared with the near-wall region. Consequently, the pressure force contribution in the inner region becomes a result of this selection and, to a lesser extent, the Reynolds number of the flow. We designed and computed, using direct numerical simulation, a non-equilibrium flow characterised by a nearly linear increase followed by a nearly linear decrease of β_{ZS} . In the first zone, the uniformly increasing APG impact leads to mean momentum loss in the boundary layer and an increase in turbulence when normalising with U_e or u_τ . In the second zone, the constant $d\beta_{ZS}/dx < 0$ results in two subzones: a first one of uniformly decreasing APG impact and a final one of uniformly increasing FPG impact. Despite this differentiation, the global effect in both subzones is the same: mean momentum gain in the boundary layer and a decrease in turbulence when normalising with U_e or u_τ . Notably, in the first subzone with $d\beta_{ZS}/dx < 0$, the flow exhibits a deliberately sought-after singular region where the boundary layer gains momentum while the external flow is still decelerating due to the APG.

As anticipated, the mean flow's response to changes in the force balance differs significantly between the outer and inner layers. In the outer region, the results reveal a substantial delay in the mean flow response for both cases of momentum-losing ($d\beta_{ZS}/dx > 0$) and momentum-gaining ($d\beta_{ZS}/dx < 0$) pressure force effects. The order

of magnitude of the delay is 10 boundary layer thicknesses. Comparisons with the two near-equilibrium APG TBLs of Kitsios *et al.* (2017) indicate that in the first momentum-losing zone, the delay results in an APG TBL with a smaller velocity defect compared to a near-equilibrium APG TBL at equivalent β_{ZS} or β_{RC} . This is a manifestation of the delayed response to a local pressure force change. In the subsequent momentum-gaining zone, the prolonged delay leads to an FPG TBL at the end of the domain with a significant momentum defect in the outer region, comparable to that of APG TBLs with a moderate defect, and a hollowed-out mean velocity profile. These are clear effects of the upstream flow history, specifically the presence of a long momentum-losing zone upstream.

In the momentum-gaining zone, the increase in momentum initiates near the wall due to the rapid response of the inner region to pressure force changes. Very close to the wall, in the viscous sublayer, the mean flow responds almost instantaneously because inertia effects are negligible in that region. Inertia effects become noticeable above approximately $y^+ = 10$. Moving further away from the wall, the conventional logarithmic law of the ZPG TBL breaks down, even for cases with small defects in the current flow and in the near-equilibrium APG TBL of Kitsios *et al.* (2017). The local pressure force, therefore, has a direct impact on the logarithmic law. It is important to note that we cannot draw definitive conclusions about the logarithmic law due to the moderate Reynolds numbers of these three flows. However, the results of Knopp *et al.* (2021) at a Reynolds number an order of magnitude higher appear to confirm them. A positive β_i results in a drop of U^+ below the log law and a change in the profile shape. This direct local effect of the pressure gradient can be accounted for in the initial part of the flow with the modifications of the logarithmic law proposed by Nickels (2004) and observed by Knopp *et al.* (2021), using the local value of β_i . However, these modifications fail further downstream due to the non-equilibrium nature of the flow or the large values of β_i . The overlap layer is indeed greatly affected by upstream history and local changes in the pressure force. Streamwise locations with the same value of β_i exhibit significantly different inner-scaled mean velocity profiles. One striking example occurs at a position where the pressure gradient is zero, yet the profile deviates considerably from that of the ZPG TBL.

Turbulence responds with a delay to the mean flow changes in both the inner and outer regions, with the delay being much more prominent in the outer region. In the current flow, the pressure force contribution in the outer region was intentionally increased and then decreased with the same absolute amplitude ($|\mathrm{d}\beta_{ZS}/\mathrm{d}x|$). The outcomes indicate that outer turbulence takes significantly more time to decay in the second region than it takes to build up in the initial region. In fact, turbulence continues to intensify for a considerable length even after the pressure force begins to diminish, and its subsequent decay is very gradual. This slow decay of turbulence is not solely attributed to the persistence of intense turbulence structures convected from upstream. The results demonstrate that it also occurs because of the increase of the excess of turbulence production over its dissipation in the momentum-gaining zone of the flow. The Reynolds stress profiles also become fuller due to sustained high levels of turbulent transport. As a result of the slow decay of turbulence, the FPG TBL located at the end of the domain exhibits unusually high levels of outer turbulence, significantly surpassing those in the ZPG TBL. The comparison of Reynolds stresses in the outer region with those of the near-equilibrium APG TBLs by Kitsios *et al.* (2017) also indicates a delay in the redistribution of energy from $\langle u^2 \rangle$ to $\langle v^2 \rangle$ and $\langle w^2 \rangle$ in the current flow. Furthermore, the outer maxima of all Reynolds stress components do not shift as far away from the wall as observed in the near-equilibrium cases.

Throughout the entire inner region, turbulence exhibits a delayed response to changes in the pressure force, even in the near-wall region below $y^+ = 10$, where mean inertia

effects are negligible. Part of the reason for this delay can be attributed to the influence of large-scale turbulence on the near-wall region. The one-dimensional spanwise spectra of the Reynolds stresses reveal that large-scale structures with spanwise wavelengths on the order of δ significantly contribute to the Reynolds stresses in the inner region. This is particularly evident in the final momentum-gaining part of the flow, owing to the presence of energetic structures convected from upstream. Upon filtering out the large-scale contribution to the Reynolds stresses, the behaviour of the small-scale Reynolds stresses aligns more consistently with the local value of β_i .

As documented in previous studies of both non-equilibrium and near-equilibrium large-defect APG TBLs, $\langle u^2 \rangle^+$ increases in the near-wall region as the local APG effect intensifies, similar to the behaviour in the rest of the boundary layer. Simultaneously, the near-wall peak of $\langle u^2 \rangle^+$ becomes less sharp and eventually disappears in the large-velocity-defect zone of the flow. With the filtering out of the large-scale contributions, the inner peak of small-scale $\langle u^2 \rangle^+$ in the initial small-defect zone of the flow resembles that of the ZPG TBL. The peak is even present in large-defect cases with H around 2. However, it eventually disappears for larger defect cases, suggesting that the near-wall cycle could be significantly attenuated or even absent in TBLs with very large velocity defects.

The current study contributes to our understanding of how the boundary layer responds to the three types of pressure force effects. Isolating the direct impact of the local pressure force from the other two effects is relatively straightforward. However, distinguishing between the local effect of a pressure force change (local disequilibrating effect) and the impact of the complete upstream pressure force history is challenging. This differentiation was achieved only in a few response situations in the present study. A more comprehensive distinction would necessitate various flow cases at high Reynolds numbers, encompassing a wide range of carefully selected pressure gradient parameter distributions. To complicate matters further, even if we provide convincing evidence for the appropriateness of the selected inner and outer pressure gradient parameters, this selection is still rightfully debatable.

Not surprisingly, the significant cumulative effects of continuous pressure force variation suggest that characteristic parameters based only on local variables cannot fully capture the physics of non-equilibrium boundary layers. This limitation applies even to the streamwise derivative of the pressure gradient parameter ($d\beta/dx$), which characterises the local disequilibrating effect of pressure force changes. Similarly, our preliminary investigation into existing response length scales, derived for localised step changes in pressure force, reveals their inadequacy in addressing continuous and gradual pressure force changes. Our research into finding appropriate parameters and response length scales for non-equilibrium TBLs is ongoing.

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Declaration of interests. The authors report no conflict of interest.

Data availability statement. Statistical data for the current flow case are available at: <https://yanmaciel.gmc.ulaval.ca/databases/> and <https://web.itu.edu.tr/gungoray/databases/>.

TBL response to uniform changes of the pressure force

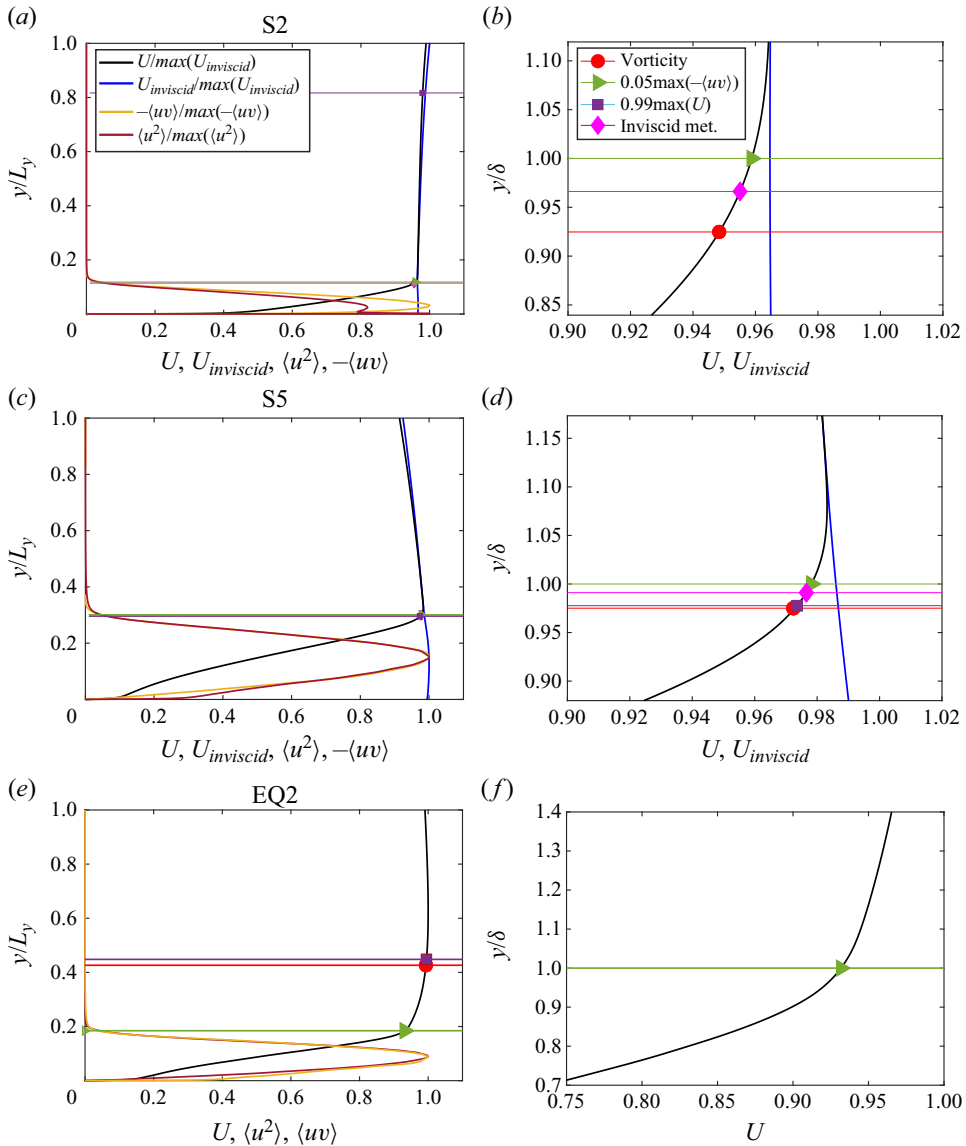


Figure 20. The mean velocity, inviscid velocity, $\langle uv \rangle$ and $\langle u^2 \rangle$ profiles as a function of y/L_y (a,c,e) and y/δ (b,d,f) where δ is computed using the boundary layer thickness definition in the paper. Here L_y indicates the wall-normal height of the domain.

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Appendix. The boundary layer thickness and edge velocity definitions

As discussed in the main text, defining the boundary layer thickness and edge velocity presents challenges for TBLs subjected to a pressure gradient or over complex geometries

where the freestream velocity varies. Here, we examine four methods outlined in the literature: the traditional δ_{99} method, a vorticity-based approach proposed by Spalart & Watmuff (1993), the method proposed by Wei & Knopp (2023) and the technique introduced by Griffin *et al.* (2021). A comprehensive review of these methods, along with others, is available in Griffin *et al.* (2021).

Figure 20 illustrates the comparison of the four methods for the present case and EQ2 of Kitsios *et al.* (2017). We do not present the results of the method by Griffin *et al.* (2021) for EQ2 because we do not have the mean pressure data. Nevertheless, the results indicate that the method is consistently reliable, aligning with the findings of Griffin *et al.* (2021). It is also the most theoretically rigorous. The method of Wei & Knopp (2023) is also reliable. The results for the other two methods exhibit significant discrepancies. The traditional δ_{99} method, which is commonly employed in the literature, performs poorly at S2 and for EQ2. Although the vorticity method yields satisfactory results for the present case, it does not work well for EQ2. It locates the boundary layer edge too high in the freestream. The method proposed by Wei & Knopp (2023) remains consistent with the approach of Griffin *et al.* (2021) across all cases and works well for EQ2. Consequently, since we cannot use the method of Griffin *et al.* (2021) for EQ1 and EQ2 because we do not have the mean pressure data, we have opted for this method for the present study. The edge velocity is considered to be the velocity at the position of the boundary layer edge determined using the aforementioned method.

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