PROSPECTS FOR OBSERVATIONS OF RELATIVISTIC EFFECTS IN THE SOLAR SYSTEM

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ABSTRACT. The solar system is the traditional laboratory for testing theories of gravitation. The results of all tests are consistent with the predictions of general relativity. The differences between these predictions and those of Newton's theory of gravitation have been confirmed with uncertainties as small as one part in a thousand. To enhance significantly the accuracy of such tests, one must investigate novel techniques. In this paper we concentrate on an experiment that promises a dramatic improvement in a classical test of general relativity -- the deflection of light by solar gravity. The goal is to measure the post-post-Newtonian contribution of nearly 11 microarcseconds to this deflection. The technique we propose is based on use of an astrometric optical interferometer, POINTS, which could be operated from the bay of the Space Shuttle, mounted on the proposed Space Station, or supported by an independent spacecraft. POINTS should be able to measure the separation of stars about 90° apart with an uncertainty of only a few microarcseconds.

1. INTRODUCTION

Tests of general relativity clearly establish it as a significant improvement over previous descriptions of gravity. Below, we mention some of these tests and discuss briefly some planned future ones. In considering means of conducting deeper tests of general relativity, we conclude that an attractive candidate is a second-order deflection experiment conducted by means of an astrometric optical interferometer in earth orbit. The proposed interferometer, POINTS, is introduced in Section 3 and recent progress in its development is discussed. In particular, we find that the central question in the design of such an instrument is systematic error. We propose to address this question at three levels: use of stable construction materials and control of their environment; use of metrology; and correction of biases in the post-analysis of the data.

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2. PRESENT AND PLANNED TESTS

The Viking relativity experiment has confirmed the time-delay prediction of general relativity to within the standard error of 0.1% (Reasenberg et al, 1979). Other tests, all consistent with general relativity, have standard errors of about 1%. For example, the deflection experiment has been performed with an uncertainty of 0.9% by using radio interferometry at S-band and X-band (Fomalont and Sramek, 1977). An improved radio-deflection test, using signals at S-band, X-band, and K-band, is expected to yield an uncertainty of 0.1% (B.E. Corey and I.I. Shapiro, private communication, 1985), with little prospect of substantial further improvement within the decade.

There is one proposed NASA mission which could contribute substantially to our testing of general relativity: a gyroscope experiment, which would investigate velocity-dependent gravitational potentials (see, for example, Anderson et al., 1982). This experiment, over 20 years in preparation, is based on very impressive advances in several areas of technology, and will involve a set of gyroscopes in orbit about the earth in a "drag free" satellite system.

The Mars Observer, a NASA spacecraft planned for launch in 1990, is expected to arrive at Mars in August 1991. This mission, which is to be focussed on the study of Mars, has an intended duration at Mars of about 700 days. The spacecraft will be placed in a low (350 km altitude), circular, sun synchronous, nearly polar orbit which is well suited for "geophysical" investigations. Ranging observations of this spacecraft at X-band with an auxiliary S-band downlink could provide accurate earth-Mars "normal points" (NPs). However, the low orbit will be heavily perturbed by the highly irregular gravity field of Mars, making the NPs relatively difficult to determine accurately. This difficulty notwithstanding, these NPs, if available, would allow a reduction by a factor of about three of the standard error in the estimate of $\dot{G}$, the (hypothesized) rate of change of the "constant" of gravitation as measured in atomic units.

Should we continue testing general relativity principally by advancing the accuracy of classical tests by small factors at great costs? Or should we attempt experiments which are, at least in some sense, novel? The gyroscope experiment, which investigates a heretofore untested aspect of the theory, responds to the second alternative. An experiment which demonstrates the existence of gravitational waves is highly significant. The work of Taylor and Weisberg (1984), analyzing the arrival times of pulses from PSR 1913+16, has produced very strong evidence for the existence of gravitational waves. Their direct detection by an apparatus on earth or in space would be an additional significant step. Other astrophysical systems which display strong-field behavior would be attractive candidates for study. However, the reliable modeling of these systems is generally difficult. Thus, they tend to be studied more as astrophysical objects than as means for testing general relativity.

An additional possibility is to conduct one of the classical experiments to sufficient precision that it measures a relativistic effect to post-post-Newtonian order. To do such an experiment in the solar system would seem to require that some aspect of the experiment be near the sun, whose gravitational potential at its limb is 2×10^{-6} ($G = c = 1$). The NASA STARPROBE mission would have brought a spacecraft to within 4 solar radii of the sun, but would not have yielded a second-order test and, in any event, is no longer being considered actively. To conduct such a "second-order" test before the millennium, one must probe near the sun with photons. Thus, either a time-delay or light-deflection experiment is the natural candidate. Our evaluation of the complexity of each of these approaches has led us to conclude that the more promising is a deflection experiment. The remainder of this paper will be devoted to that possibility.

3. A POST-POST-NEWTONIAN GRAVITATIONAL EXPERIMENT IN SPACE

It has been shown by Epstein and Shapiro (1980) that the second-order (i.e., post-post-Newtonian) contribution of the sun's gravitational field to deflection is 11 microarcseconds (μas) at the limb and falls quadratically with impact parameter. The effect of the solar corona is sufficiently severe that no reasonable extrapolation of current technology will make possible a microwave deflection experiment at this level of accuracy. Therefore, we consider optical measurements. Fluctuations of the atmospheric refraction are so great that no ground-based instrument can be expected to be sufficient for measuring this effect and we must consider a space-based instrument. What type of instrument should we use?

3.1. Choice of Instrument

For any instrument, the so-called diffraction limit is of order λ/D where λ is the observing wavelength and D is the linear scale of the instrument perpendicular to the line of sight. For $\lambda/D = 5$ μas and $\lambda = 0.5$ μm, we have $D \sim 20$ km. Thus, for any practical-sized astrometric instrument, one must plan to use statistics to split the fringe or Airy disk. The analysis below shows that for instruments of comparable size, an interferometer requires orders of magnitude less integration time than an astrometric telescope to make a measurement of the same accuracy for the same target.

We assume an astrometric instrument collects light and causes it to pass through two exit ports with intensities I_1 and I_2 such that $\Delta \equiv (I_1 - I_2)/2$ changes as the instrument is rotated by δ with respect to the source direction. Then

$$\sigma(\delta) = I_0 / \left[2\sqrt{N} (\partial\Delta / \partial\delta) \right] \tag{1}$$

where $I_0 = I_1 + I_2$ and N is the number of detected photons (Reasenberg, 1986). We can apply this formula to an astrometric interferometer having two telescopes:

$$\sigma_I = \lambda/\left[2\pi L\sqrt{N}\right] , \tag{2}$$

where L is the baseline length.

Similarly, Equation (1) can be applied to a single-telescope instrument. Although more efficient detection is possible using a filter matched to the shape of the Airy spot, we assume a knife-edge detector since all practical instruments make use of this approach. We find that

$$\sigma_T \geq 3\pi\lambda/\left[16D\sqrt{N}\right] , \tag{3}$$

where D is the telescope diameter and the equality holds if the knife edge passes through the center of the Airy disk. We can use Equations (2) and (3) to compare a telescope to an interferometer under the assumption that the number of detected photons is equal in the two cases:

$$\sigma_T/\sigma_I = 3\pi^2 L/8D \tag{4}$$

If the instruments are to be of comparable size, then we might take the focal length of the telescope to be equal to the baseline of the interferometer. The resulting ratio of σ's is about 50, thus favoring the interferometer. (Use of a telescope with a folded optical path, i.e., a secondary mirror, is inappropriate for high-accuracy astrometric work; Gatewood, private communication, 1984.)

3.2. POINTS

It appears that use of an optical interferometer in space is the best approach for carrying out a second-order test of general relativity. Such an interferometer has been dubbed POINTS (Precision Optical INTerferometry in Space). Its design has been evolving with the goal of achieving measurements with microarcsecond precision. Reasenberg (1984) has described a "strawman" design for POINTS and discussed the basic questions of sensitivity and pointing requirements. We discuss some of these, and related issues, below.

POINTS is an articulated dual astrometric interferometer with two 2 m baselines and four 25 cm telescopes. The baselines are separated by an angle $\varphi = \varphi_0 + \Delta$ where φ_0 is approximately 90° and Δ is the articulation angle: $|\Delta| < \Delta_m$, where Δ_m, the articulation range, will probably be between 0.5° and 5°. Making φ_0 = 90° maximizes the number of reference stars for a given target star and articulation range. The articulation of the two interferometers allows the observed stars to lie within a few arcseconds of the optical axes of their respective interferometers, thus essentially eliminating off-axis distortions and the attendant biases.

The instrument design is centered around the critical question of systematic error. A displacement of one of the end mirrors toward the target by 0.5Å would produce a systematic error equal to the nominal measurement uncertainty of 5 μas. A comparable effect would result from a similar displacement of each of several internal optical components. Disturbance of the instrument can be expected from the earth's gravity gradient, a changing thermal environment, and the on-board mechanical devices necessary to maintain and adjust the spacecraft's orientation. Thus, for example, even if the spacecraft were to spend no time in shadow and were moderately well insulated, a re-orientation would result in the internal metering rods changing in temperature by the order of 1K. For the most thermally stable materials, this change would result in a change of length of at least 10^3 Å.

3.2.1. Internal Metrology. How does one assure that the precision inherent in the interferometric approach can be translated into accuracy? The high-precision star position measurement is made with respect to the optical axis of the interferometer, which is determined by the positions of the optical elements used to transfer the starlight. The acceptable error level in the knowledge of each of the positions of these elements is of order 0.1Å. Thus, given the expected high level of disturbance, accuracy comparable to precision seems possible only with a system of internal metrology, based on a series of laser interferometers.

What does it mean to know the position of a 25 cm mirror to an uncertainty of 0.1Å? Even under the most benign conditions, time-dependent mirror surface distortions will be many angstroms. In the present case, the quantity that we need to know is the average change in the starlight delay induced by all motions and distortions of an optical element. Such an average is determined most effectively by illuminating fully the optical element surface with the metrology light. A scheme for performing this full-aperture metrology is described by Reasenberg (1984). An added dividend of this approach is that it results in a pair of fiducial points located in front of each interferometer. These fiducial points, which lie on lines parallel to (or offset by a small fixed angle from) the interferometer baselines, can be used to determine φ, the angle between the two interferometers' optical axes.

A small assembly of mirrors, beamsplitters, detectors, etc. could be joined together as a solid "fiducial block" and located at each fiducial point. Since these fiducial blocks would provide the mechanical connections among the laser interferometers, a change of fiducial-block size or shape would cause a corresponding change in the bias of the metrology system. Therefore, the fiducial blocks would be kept in the most thermally stable environments of the instrument.

3.2.2. Beamwalk. The primary mirrors would serve as the entrance pupils of the optical instrument. As it rotates, the illuminated portion of each of the other optical elements would change but the portions of the beamsplitter illuminated by the two stars would move together. This "beamwalk" would not be followed by the full-aperture metrology illumination and would thus give rise to an instrument bias that would depend both on the angular displacement of the source from the instrument axis and on the irregularities in the optical elements. Fortunately, this bias is expected to be only of order 10 μas. Under the assumption that this bias would be stable on a time scale of one day, we have shown that if it is represented as a polynomial in δ, the coefficients can be estimated from routine observations with little increase in $\sigma(\theta)$.

3.2.3. Measurement Strategy. Since POINTS is a global instrument, it is natural to consider a global reference frame. (See Kovalevsky [1984] for a discussion of astrometric instruments in space and definitions of types of astrometry.) This approach is in sharp contrast to the use of local reference frames defined by the small fields of view of classical astrometric instruments. Even those classical instruments designed to perform "wide field" astrometry are, in the present perspective, limited to small angle, differential measurements. Thus, for POINTS, the reference frame will be defined by a grid of reference stars; the 90° baseline separation in POINTS will provide a maximum choice of grid stars to serve as references for each new "target" star.

Consider the following scenario: A set of S grid stars is selected so as to be distributed with moderate uniformity over the celestial sphere. The interferometer is used to observe each pair of stars within its articulation range Δ_m. The number of observations K will be proportional to $S^2\Delta_m$ and we define $M \equiv (K/S) \propto S\Delta_m$, the number of observations per star. We have performed series of Monte Carlo covariance studies (Chandler and Reasenberg, unpublished, 1981, 1984) based on this scenario. We considered the a posteriori uncertainty in the angular separation of each pair of stars whether directly observed or not. We found that, independent of Δ_m, the distribution of these uncertainties becomes essentially unimodal for M greater than a critical value between 3 and 4, depending on the particular set of random stars.

The Monte Carlo covariance studies have shown that when M is about five, the a posteriori estimated separation between about half of the pairs of stars (including those not measured directly) has an uncertainty $\bar{\sigma}(\theta)$ less than the measurement uncertainty $\sigma(\theta)$; for less than 5% of the pairs, $\bar{\sigma}(\theta) > 3\sigma(\theta)$. The stars of such a grid would be used as the reference stars for the majority of the scientific applications of POINTS.

A different strategy may be better for the light-deflection study. In covariance studies by Epstein and Shapiro (private communication, 1980) and by Chandler and Shapiro (private communication, 1983), specific stars in a small region of the ecliptic were designated as occultation targets and suitable reference stars were assumed. Corrections to the solar ephemeris were included along with star positions and four other parameters representing the firstand second-order deflection due to the solar mass, the deflection due to the sun's equatorial bulge, and the deflection due to the Lense-Thirring effect. Because these effects have significantly different signatures and symmetries, the corresponding estimated coefficients were not highly correlated.

In a long-duration mission with multiple objectives including the deflection experiment, the schedule of observations would combine the reference grid of stars with bright targets along the ecliptic specifically selected for the relativity test. Although we plan a covariance study of such a situation, we believe that the essential results are contained in the studies already completed.

3.2.4. Bias Correction. The metrology system described above has several problems in addition to beamwalk. Changes in wavelength of the metrology light source, in the thermal control system for the fiducial blocks, and in the fiducial blocks *per se* may change the metrology system bias. Short period changes in the fiducial block temperature, such as would be expected to result from the routine reorientation of the instrument with respect to the sun, could also change the bias. (A modest amount of insulation and heat capacity would provide a thermal time constant of a day.) Thus, we need to have a means of estimating the instrument bias on a time scale of a few hours.

Fortunately, such a bias estimate is naturally obtained by means of 360 deg closure, and special sequences of observations are not required for this purpose. Our sensitivity studies have shown that in the least-squares analysis to obtain the individual stellar coordinates (including proper motion and parallax), it is possible to estimate simultaneously several instrument bias parameters per day without significantly degrading the estimates of the stellar coordinates. In one study in which we assumed 60 observations per day, we found that with four bias parameters per day, $\overline{\sigma}(\theta)$ was increased by 10%; with 15 bias parameters per day, $\overline{\sigma}(\theta)$ was increased by a factor of 2.5 and the system was approaching instability.

4. CONCLUSION

The program of solar-system tests has confirmed various predictions of general relativity at the level of 1% to 0.1%. The study of the binary pulsar has provided strong evidence for the existence of gravitational waves and the proposed gyroscope experiment would investigate velocity-dependent gravitational potentials. To provide a significant new test, we are investigating use of a space-borne astrometric optical interferometer to measure the "second-order" contributions of solar gravity to the deflection of light.

POINTS, a small astrometric instrument capable of making tens of measurements per day, each with an uncertainty of a few microarcseconds, would enable such a second-order test of general relativity to be carried out. Although more complex than an

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astrometric telescope, a POINTS-type interferometer of comparable size has a much larger "scientific throughput" and offers the advantages associated with global astrometry.

The design of the instrument reflects a central concern for systematic error which is addressed at three levels: (1) use of stable materials and thermal control; (2) use of laser-interferometer metrology; and (3) estimation and correction of biases through data analysis. Both in its global nature and in its use of post-analysis to detect and correct instrument biases, POINTS is the interferometric counterpart to HIPPARCOS. The spacecraft has modest pointing requirements and would be able to operate from a mechanically noisy base such as the cargo bay of the Space Shuttle.

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DISCUSSION

Grishchuk : will it be necessary to take into accound the shape of the stars ?

Reasenberg : yes, it will be necessary.

Bertotti : will you be able to observe quasars ?

Reasenberg : POINTS will be able to observe any object up to magnitude 15. So dozens of quasars will be observable.

Nobili : will it be possible to use POINTS to observe laser beacons on the Earth.

Reasenberg : yes.

Cannon : what is the effect of the rotation of the star ?

Reasenberg : there is no such effect.

Tikhonov : JPL and CNES are planning to place an active ranging responder on the soviet Phobos lander planned for 1990 or so in order to test gravitational waves and $\dot{G}$. What do you think of this project ?

Reasenberg : there has been very great difficulties to construct such a responder, so the project seems to be abandoned.

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