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ARTICLE

Dynamic inefficiency by promoting relative consumption

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Abstract

I develop a dynamic model of consumption variety in status goods by introducing a realistic aspect that is new in the existing literature—that a good will not carry status appeal unless it is advertised. As advertisements will divert resources from new product research, growth in new products will be reduced. However, status-good advertisements also enhance distinctiveness of a good and increase a firm's profit. This will motivate more researches. With the two effects offsetting each other, the original market bias in a standard product-development model—insufficient research due to a general knowledge spillover—cannot be overcome. While introducing advertising into models of this kind does not reverse the original welfare implication of suboptimal growth, this makes available a new and better intervention option—taxing advertisements. This tax is superior to consumption tax, the conventional solution to inefficient status competition, as consumption tax is found to be ineffective in the present model. It is also superior to research subsidies, the conventional solution to suboptimal growth, as subsidies must be financed and is not a self-sufficient policy.

Keywords: Endogenous Growth; Relative Consumption; Status Effects; Advertising; Taxation Policy

JEL classifications: D91; M37; O31; O40; O41; Z00

1. Introduction

The primary welfare implication of status-seeking is now well known among economists. Since social status is assessed relative to others, each one must run to keep in the same (relative) place. But even if each runs faster, as others may have also done so, again one can keep only in the same (relative) place, and all extra efforts devoted to getting a higher relative position are simply wasted. When consumption is a major means to show off one's status, over-consumption is resulted. To correct this market bias, tax on consumption has been proposed [see, for example, Frank (1989)].

Nevertheless, this primary model of status-seeking is too simple and ignores many important elements. For instance, why will a consumer good be taken as a status symbol? Is it a result of some costly actions? If yes, who initiates these actions, and under what considerations? These are important questions but are rarely explored in the literature. In this paper, I argue that when these questions are examined seriously, a new dynamic inefficiency problem of status-seeking will appear.

As is well known since Frank's seminal paper (1985), only goods that are easily visible, or conspicuous consumption in Veblen's (1899) terminology, will be taken as a status symbol. In a way, the inherent characteristics of goods determine which goods are visible. Frank's own work (1985)

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has taken this particular aspect into account in his model. More interestingly, goods may become more observable and conspicuous when firms devote resources to promote or advertise their goods as something of status appeal. Promotion is costly. Do firms have motivations to take such actions? There are two reasons why they have. If promotions can change a consumer's mind, prompting them to take the promoted good as a status symbol, the demand for the good will be increased. Nonetheless, if all firms sell homogenous goods, this effect will also benefit nonpromoting firms, and so a firm will lack sufficient motivations to take the costly action. In contrast, in a market of heterogeneous goods, firms will have the incentives to promote their goods. Given this, the second reason for promotion appears to be even more important—promotions can create distinctiveness of a product, reducing the price elasticity of each good. Taken together, firms will expect that promotions can generate a higher profit for them, at least due to the elasticity-reducing effect.

Is such a promotion activity welfare reducing because it promotes the welfare-reducing activity of status-seeking? This depends on whether promotion offsets or reinforces some preexisting distortions in markets. In a dynamic setting, product variety is not fixed. It grows over time but, to invent new goods, costly research must be conducted. Innovations have positive spillover effects so that subsequent inventors can learn from their predecessors and invent new goods at a lower cost. In the relevant literature, under-investment in research is a typical result due to this effect. Promotion can enhance profit in a market of heterogenous goods, and this makes research on new goods more attractive. Thus, this positive welfare effect can offset some of the negative effects resulting from status-seeking. To assess the overall effect of promoting relative consumption, a more specific model, with the features outlined above, is needed.

The model I use is the standard product development model pioneered by Grossman and Helpman (1991, Chapter 3; GH henceforth), which explains the growth of new goods by inventions. Inventions, in turn, are supported by firms' profits through selling products. Two new elements, status and promotion, are added to GH: Each heterogeneous good may be taken by consumers for relative comparison if the good is visible. To make a good visible, firms must promote the good at a cost. With promotions, the price elasticity of a good is reduced, and the monopoly power of each firm is enhanced, resulting in a higher price of each good and a higher profit for each firm. While higher profits in the products motivate more researches in inventing new goods, promotion will also divert the resources available for research. Taking the two opposite effects together, more promotion may not result in a higher growth rate of new goods.

In my model, I will show that the growth rate of new goods in the steady state is indeed lower than that in social optimum. A major reason is that invention involves knowledge spillover (as introduced above). Therefore, a typical result of suboptimal growth will be generated by the presence of positive externality. As mentioned above, promotion brings about both growth-enhancing effect (higher profit) and growth-reducing effect (divertion of resources). With the two effects offsetting each other, the net effect, if any, is not sufficient to overcome the market bias of under-investment in research inherent in the GH model.

To correct this market bias, GH have identified research subsidy as a remedy. This directly corrects the knowledge spillover effect. In the present model, more policy instruments can be identified, including a tax on status goods and a tax on advertisements or promotional activities.

Contrary to the usual results in the status economics literature, I find that taxing status goods is useless. This tax reduces the consumers' willingness to pay for the good, reducing both outputs and profits and thus impeding the incentive for invention. Nonetheless, output contraction also releases more resources (labor in the present model) for invention, which also lowers the wage, or the cost of innovation. The two effects offset each other in the model.

For the tax on advertisements, I find that it is useful. Although a tax on advertisements also affects profits and resources in the same directions as in status-good tax, the magnitudes and patterns are different. Advertising is related to a firm's profit by creating distinctiveness of its product, or by reducing elasticity. This effect is firm-specific. It will not disappear even though other firms do the same thing. A tax on advertisements damages this firm-specific effect, and so will generate a greater negative impact on each firm's profit. In comparison, a status-good tax reduces not only the level of demand for each good but also the intensity of competition between firms—output reduction by one firm will increase the demand for the (imperfect) substitutes made by another firm. As such, status-good tax will only have a more moderate effect on firms' profits.

Reducing either promotion or manufacturing activity will release more resources for invention. Then, reducing which activity can release more resources? As explained above, taxing advertisements will produce a more pronounced effect than taxing goods. Therefore, taxing advertisements can increase innovation while taxing status goods has no such effect.

That taxing advertisements can enhance growth is an interesting finding[1] and brings about new issues for discussion.

In the original GH model, it is found that only a research subsidy can enhance growth. Now I find an additional instrument—a tax on advertisements. It is also a superior instrument when the problem of financing a research subsidy is taken into account. GH have never mentioned how research subsidies are to be financed. In their model, there is no static inefficiency but only dynamic inefficiency problem. If a subsidy is used to address the dynamic problem, one should assess if the static allocation would be affected by the way funds are collected for the subsidy. Thus, dynamic and static efficiency cannot be treated completely separately.

Meanwhile, a tax on advertisements generates revenue and can possibly lift equilibrium growth rate to its optimal level under certain conditions. Nevertheless, I found that when an advertising tax is imposed at a rate where optimal growth is attained, the aggregate output level is smaller than its optimal level. A question naturally arises: Will the revenue obtained from taxing advertisements be sufficient to be used to fill up the output shortfall? If so, the policy scheme is self-sufficient. If not, extra problems will be generated. I will show that the scheme is self-sufficient (Proposition 3) under certain assumptions, which are not particularly strong.

1.1. Literature Review

In the existing literature, there are many papers studying the relation between growth and relative-consumption effect (status-seeking), for example, Rauscher (1997), Carroll et al (2000), Alvarez-Cuadrado et al. (2004), Fisher and Hof (2000), and Doi and Mino (2008). So far, however, promotion or advertising is not an element in these papers.

Notably, there is a strand of literature [such as Futagami and Shibata (1998); Fisher and Hof (2005); and Tournemaine and Tsoukis (2008)] which explores the relation between growth and relative wealth, not relative consumption. The study of relative wealth is certainly important as wealth (sometimes specified as capital goods), instead of consumption, is arguably a more direct contributor to output growth. Nevertheless, the present paper is concerned with the growth of the variety of consumer goods, not the aggregate consumption level or simply output growth. The issue of advertising is also more appropriate in this context (as advertising is normally used to promote consumption), not in the context of wealth, or relative wealth.[2]

The relation between advertising and status goods has surprisingly received only few attentions although the tie between the two seems to be quite obvious. Several studies do exist, such as Krähmer (2006) and Pepall and Reiff (2016), but none of them is concerned with growth.

The link between advertising and status is apparently very close, considering that status goods must be visible and advertising is an important way to make a good visible. Frank (1985) makes clear the point about visibility (observability) and status goods. Ireland (1994) uses a signaling model to establish the link theoretically. Chao and Schor (1998), Solnick and Hemenway (2005), and Heffetz (2011) offer important empirical evidence.

The relation between advertising and growth has been studied only recently. Grossmann (2007) and Molinari and Turino (2007) are two notable works. Both are not concerned with the role of advertising in product development. Cavenaile and Roldan-Blanco (2021) explore the relation between research and advertising, affirming a key point highlighted in the present paper: advertising increases the returns to invention by affecting the demand, but may also distract the firms' resources from innovations. Nevertheless, their paper considers a very different type of model: the innovation is the quality-improving type (ours is a variety-expansion type) and their concern is the relation between firm size and advertising (our model assumes identical firm size). Besides, all papers above are not concerned with status goods.

Finally, the series of works of Woo (2011, 2013 and 2016) have introduced a framework of status-seeking under imperfect competition. The works are useful for models of heterogeneous goods. But there is no dynamic element nor advertising involved in these works.

1.2. The Structure of This Paper

Section 2 introduces the model setting. Section 3 explores the welfare implications of this model. Section 4 concludes.

2. The model

I need a model that can be used to assess how relative status affects the growth of new products or *final goods*. GH's product development model is ideal for this purpose. Most growth models (other than GH) concentrate on the effects of expanding the variety of *intermediate goods* or improving the *quality* of goods. These two aspects, important as they are, do not have direct relation with advertisements. So they are less relevant for this paper.

There are, however, certain (perhaps odd) features involved in GH, which should not be confused with the present paper's contributions (or shortcomings). To minimize distractions from the focus of this paper, I will offer the explanations for these specific features in GH in the footnotes or a separate supplement. Meanwhile, to enable an easier comparison of my results with GH's, I will retain other features of GH as much as possible.

2.1. Consumers

Each individual is assumed to be identical. The population size is normalized to unity. At any moment of time, each one consumes n goods that are available at that time. Each intends to maximize the intertemporal utility.

2.1.1. Utility and status functions

The intertemporal utility is the same as GH's, which is logarithm separable over time:

$$U = \int_0^\infty e^{-\rho t} \ln D dt, \tag{1}$$

where ρ is a constant subjective discount rate, and D is an instantaneous utility. For ease of notations, I suppress all arguments for time t unless time is an explicit argument. Unless a variable is explicitly stated as a parameter (such as ρ), a constant (such as e) or an index (such as i), all variables are supposed to vary over time. The instantaneous utility is derived from various types of products, identifiable by a continuous index i:

$$D = \left[\int_0^n u(i) di \right]^{1/\alpha}, 1 > \alpha > 0, \tag{2}$$

where n is the range of goods (or variety) available, and α is a constant taste parameter. The specification of the product-wise utility represents the greatest difference between my model and

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GH's as I will incorporate status and advertising effect in it[3]:

$$u(i) = c(i)^{\alpha} + z(i)^{\alpha} S(c(i)x(i)^{-1}),\ i \in [0, n] \tag{3}$$

where $c(i)$ is the quantity consumed by an individual for product i, $x(i)$ is the social average consumption of product i, and S represents the status utility brought about by consuming this product, which is positively dependent on one's relative consumption $c(i)x(i)^{-1}$. In (3), the effect of status utility is present only when the good is advertised, or there is a positive value of an advertisement of product i, i.e., $z(i) > 0$.

The status function S possesses the following properties:

$$S' > 0; \tag{4}$$

$$S(1) = 0. \tag{5}$$

By (4), relative consumption increases status utility. Next, when everyone runs, one can only keep in the same (relative) place. As such, by (5), no one can get any utility from status if everyone consumes the same quantity of the good, i.e., $c(i) = x(i)$. From now on, the marginal utility of status, when $c(i) = x(i)$ happens, will be denoted as s, which is a positive constant when (4) and (5) are satisfied.[4]

$$s \equiv S'(1) > 0. \tag{6}$$

The product-wise utility (3) and the above status function are used in Woo (2011, 2013, and 2016), which demonstrate that the models are appropriate in welfare analysis with various sensible results derived. In the present context, further defences may be offered. As can be seen later in this section, tractable results can be derived in equilibrium. Without tractability, steady-state properties may not be easily characterized if one is to avoid simulation methods. To appreciate the point, consider an alternative to the present model, which assumes additivity in utility of consumption c and status S. By modifying Doi and Mino (2008), which assumes multiplicity of the two utility components, it can be easily shown that highly nontractable results will be generated if advertising is added.[5]

2.1.2. Budget constraint

Each individual holds A units of assets (saving), which pay interest at the current market rate of r, and supplies a constant amount of labor time L at a wage rate of w. The (flow) budget constraint is thus

$$\dot{A} = rA + wL - \int_0^n p(i)c(i)di, \tag{7}$$

where $p(i)$ is the consumer price of good i and the dotted value is (and all other dotted values are) used to denote the time-induced value of change.

2.1.3. Inter-temporal maximization

An individual's utility maximization problem can be solved by setting the current-value Hamiltonian H,

$$H = \frac{1}{\alpha}\ln\left[\int_0^n u(i)di\right] + \lambda\left[rA + wL - \int_0^n p(i)c(i)di\right], \tag{8}$$

where λ is the multiplier. The first-order maximization condition and the equation of motion can be derived:

$$\frac{\partial H}{\partial c(i)} = 0 \Longrightarrow \frac{1}{\alpha\int_0^n u(i)di}\left[\alpha c(i)^{\alpha-1} + z(i)^{\alpha} S'\left(c(i)x(i)^{-1}\right)x(i)^{-1}\right] = \lambda p(i);^{6} \tag{9}$$

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$$\dot{\lambda} = -\frac{\partial H}{\partial A} + \rho\lambda \Longrightarrow \frac{\dot{\lambda}}{\lambda} = \rho - r. \tag{10}$$

Taking integration of (10) and defining the average interest rate up to time t as

$$\bar{r} = \frac{1}{t}\int_0^t r(\psi)d\psi, \tag{11}$$

the multiplier can then be solved as $\lambda = e^{(\rho-\bar{r})t}$. Substituting this solution of the multiplier into (9), I can get an inverse demand function for a product from a single consumer at any moment.

2.1.4. Nash equilibrium in the status game

Since consumers are identical, in equilibrium, $c(i) = x(i)$. Using (9), (5), (6), and the solution of the multiplier introduced above, the market inverse demand is solved as

$$p(i) = \frac{e^{(\bar{r}-\rho)t}}{\alpha\int_0^n x(i)^\alpha di}\left[\alpha x(i)^{\alpha-1} + sz(i)^\alpha x(i)^{-1}\right]. \tag{12}$$

It can be easily checked that the price elasticity of each good induced by (12) is reduced by each firm's advertising.

2.2. Production

There is one type of input, namely labor, and there are three types of production in the model: manufacturing, advertising, and invention of new goods. I assume: one unit of labor can produce one unit of each good $x(i)$; $b(>0)$ units of labor can produce one unit of advertisement $z(i)$; finally, for each percentage of increase in the range of new goods, $a(>0)$ units of labor is needed.

2.2.1. Manufacturing and advertising sectors

Each (atomistic) manufacturing firm makes use of an existing blueprint to produce only one good[7] and incurs expenses on advertisements to remind consumers that the product is a status good. Therefore, each firm's profit $\pi(i)$ is

$$\pi(i) = \theta^{-1}p(i)x(i) - wx(i) - \tau wbz(i), \tag{13}$$

where θ is a constant that indicates the ratio of consumer price to producer price, and τ is a constant ratio of post-tax and pre-tax advertising cost. If $\theta = 1$, there is no consumption tax. If $\theta > 1$, there is a consumption tax. Similarly, if $\tau = 1$, there is no tax on advertisements. If $\tau > 1$, there is.

Maximizing (13) subject to the demand function (12) involves two first-order conditions[8]:

$$\frac{\partial\pi(i)}{\partial x(i)} = 0 \Longrightarrow \frac{\theta^{-1}e^{(\bar{r}-\rho)t}}{\int_0^n x(i)^\alpha di}\alpha x(i)^{\alpha-1} = w; \tag{14}$$

$$\frac{\partial\pi(i)}{\partial z(i)} = 0 \Longrightarrow \frac{\theta^{-1}e^{(\bar{r}-\rho)t}}{\int_0^n x(i)^\alpha di}sz(i)^{\alpha-1} = \tau bw. \tag{15}$$

The second-order conditions are satisfied, given our assumptions for parameter values.

2.2.2. Invention sector

The invention sector here is the same as in GH and so I only briefly introduce the key equations:

$$g \equiv \frac{\dot{n}}{n} = \frac{L_n}{a}, \tag{16}$$

$$\sigma w = \frac{vn}{a}, \text{ and} \tag{17}$$

$$\dot{v} + \pi = rv, \tag{18}$$

where g denotes the growth rate of the range of goods, L_n the labor units devoted to invention, and v the equity value of owning a blueprint by which the owner can authorize anyone to produce an existing or a newly invented good. Meanwhile, σ is a constant ratio of wage paid by inventors to the wage received by workers. If $\sigma < 1$, there is a subsidy given to inventors. If $\sigma = 1$, there is no.

The three equations above are, respectively, the invention "production" function, labor market balance condition for invention sector,[9] and no-arbitrary condition for the asset market.[10] In particular, (16) can be re-arranged as $\dot{n} = n\frac{L_n}{a}$, which shows that the existing range of goods, n, positively affects the variation of the range, $\dot{n}$. This captures the knowledge spillover effect: inventors can learn from prior inventions so that it is easier to invent new goods when more products have been invented.[11]

2.3. Equilibrium and Steady State

Firms are symmetric in the model. Hence, in equilibrium, we have $x(i) = x(i') = x$ and $z(i) = z(i') = z$ for all i, i'. To begin with, I solve each consumer's expenditure on goods at any moment as

$$E \equiv npx = e^{(\bar{r}-\rho)t}\left[1 + (\tau b)^{\frac{\alpha}{\alpha-1}}\left(\frac{s}{\alpha}\right)^{\frac{1}{1-\alpha}}\right]. \tag{19}$$

Consumption expenditure E increases with the marginal utility of status s. By choosing a measurement unit, as what GH did, this expenditure level is a constant (GH have normalized it at unity), bearing in mind that the tax factor τ is a constant. This amounts to the following normalization.

Normalization $\bar{r} = \rho$.[12]

Applying this normalization and symmetry to equations (12) to (15), in equilibrium, the levels of manufacturing output per product, advertising per firm, price for each product, and each firm's profit are, respectively, as follows (the product index i is dropped from now on):

$$x = \frac{\alpha}{\theta wn}, \tag{20}$$

$$z = \frac{\alpha}{\theta wn}(\tau b)^{\frac{1}{\alpha-1}}\left(\frac{s}{\alpha}\right)^{\frac{1}{1-\alpha}}, \tag{21}$$

$$p = \frac{\theta w}{\alpha}\left[1 + (\tau b)^{\frac{\alpha}{\alpha-1}}\left(\frac{s}{\alpha}\right)^{\frac{1}{1-\alpha}}\right], \text{ and} \tag{22}$$

$$\pi = \frac{1-\alpha}{n\theta}\left[1 + (\tau b)^{\frac{\alpha}{\alpha-1}}\left(\frac{s}{\alpha}\right)^{\frac{1}{1-\alpha}}\right]. \tag{23}$$

Price in (22) and profit per firm in (23) increase with the marginal utility of status s, which takes effect via the elasticity-reducing advertisements. Output per firm is not directly affected by s in (20), but it will be affected through wage rate w, and the equilibrium value of which will be solved later.

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The resource constraint of this model is

$$L = L_n + nx + nbz. \tag{24}$$

The left side is the total labor units available. On the right side, the first term refers to the (labor) resources used for invention, the second refers to resources for manufacturing, and the third refers to resources for advertising.

Using the normalization again, substituting (23) into (18), deriving a dynamic equation from (17), and finally (16) and (24), the steady-state growth rate[13] of new-good blueprints and the equilibrium wage rate are, respectively, found as below:

$$g^m = (1-\beta)\frac{L}{a} - \beta\rho, \text{ and} \tag{25}$$

$$w^m = \frac{\alpha}{\beta\theta(L+a\rho)}\left[1+\tau^{\frac{1}{\alpha-1}}b^{\frac{\alpha}{\alpha-1}}\left(\frac{s}{\alpha}\right)^{\frac{1}{1-\alpha}}\right], \tag{26}$$

where

$$\beta \equiv \frac{\alpha\left[1+\tau^{\frac{1}{\alpha-1}}b^{\frac{\alpha}{\alpha-1}}\left(\frac{s}{\alpha}\right)^{\frac{1}{1-\alpha}}\right]}{\left[(1-\alpha)\sigma^{-1}+\alpha\right]+\left[(1-\alpha)\sigma^{-1}\tau+\alpha\right]\tau^{\frac{1}{\alpha-1}}b^{\frac{\alpha}{\alpha-1}}\left(\frac{s}{\alpha}\right)^{\frac{1}{1-\alpha}}}. \tag{27}$$

Superscript m is used to denote the market equilibrium values. Growth and wage will be reduced by the value of β, which will be affected by policies such as tax or subsidy. Without interventions ($\sigma = \tau = 1$), we have $\beta = \alpha$. In this case, intuitive interpretation of (25) can be more readily given: more (labor) resources (higher L) and a smaller input requirement for research (lower a) support a higher growth. Furthermore, for more distinctive products (lower α), it is more attractive to conduct researches in such products and this supports a higher growth. The impacts of tax/subisdy will be discussed in more details in Section 3.2. Here, I mention only a few of them that can be more easily observed: a higher consumption tax (higher θ) will reduce the wage [in (26)] as it depresses the demand for manufacturing goods; a higher research subsidy (lower σ) will support a higher growth [via (27) on (25)].

The equilibrium output and profit values, etc. can be obtained by substituting (26) back into (20) to (23). Here, I present only the equilibrium aggregate output,

$$X^m \equiv n^m x^m = \frac{\beta(L+a\rho)}{1+b(\tau b)^{\frac{1}{\alpha-1}}\left(\frac{s}{\alpha}\right)^{\frac{1}{1-\alpha}}}. \tag{28}$$

3. Welfare implications

3.1. Social Optimum

The (first-best) social optimum is characterized by the solutions to a dynamic maximization problem as if a social planner can allocate resources freely, subject only to the resource and technological constraints. This analysis ignores the feasibility of policy implementation, which will be considered in Section 3.2.

Because of symmetry, status utility vanishes in equilibrium ($S(1) = 0$). Hence, a social planner should disregard the component S in (3). Although status-good advertisements can reduce elasticity and so induce more inventions, in a first-best setting, a social planner does not need to use such an indirect method (but the method is important in a second-best setting in Section 3.2).

Therefore, social optimum is found by setting $c = x$, $S = 0$, and $z = 0$. The utility function is thus reduced to $U = \int_0^\infty e^{-\rho t}\left(\frac{1}{\alpha}\ln n + \ln x\right)dt$, which can be rewritten, using (aggregate output)

$X = nx$, as $\int_0^\infty e^{-\rho t}\left[\left(\frac{1}{\alpha}-1\right)\ln n + \ln X\right]dt$. The equation of motion is the combination of (24) and (16), thus $\dot{n} = \frac{n}{a}(L - X)$. Then, a current-value Hamiltonian G, with multiplier μ, can be set up

$$G = \left(\frac{1}{\alpha}-1\right)\ln n + \ln X + \mu\left[\frac{n}{a}(L - X)\right]. \tag{29}$$

Now, we arrive at the same problem as in GH even if we start from a different set of assumptions (with and without status and advertising, respectively). As the solution method can be found in GH (see also the supplement), I just give the solutions here. Note that I use superscript o to denote the values in social optimum:

$$g^o = \frac{L}{a} - \frac{\alpha}{1-\alpha}\rho, \text{ and} \tag{30}$$

$$X^o = \frac{\alpha}{1-\alpha}a\rho. \tag{31}$$

To see if social optimum is attained without any interventions, set $\theta = \tau = \sigma = 1$. Then, it is easy to demonstrate that $g^m < g^o$. The sign of $X^m - X^o$ depends on the value of the marginal utility of status s. When s is high, $X^m < X^o$.[14] In short, without intervention, growth rate is smaller than optimal while aggregate output may also be too low.

3.2. The Intervention Schemes

Unlike the above, this section considers that a policy maker cannot directly arrange n, x and the growth rate in n but is bound to use feasible policy options like taxes and subsidies. However, if resources (L) are insufficient in an economy so that the optimal growth rate in (30) is not positive, the dynamic efficiency aspect in the model is irrelevant. Hence, for all policy options, the following assumption is essential. Note that a new notation γ is introduced.

Assumption 1 (Positive optimal growth). $g^o = \frac{L}{a} - \frac{\alpha}{1-\alpha}\rho \equiv \gamma > 0$.

3.2.1. Consumption tax

In the literature of status goods, consumption tax plays an important role. Normally, relative consumption induces people to consume excessively and so reducing consumption by tax is justified.

In the present model, however, consumption tax is useless. First, the aggregate output (28), and therefore output bias, will not be corrected by such a tax θ. While a tax will reduce quantity consumed in (20), it will also decrease wage (the cost of producing goods) in (26). The two effects offset each other.

Second, consumption tax also does not affect the steady-state growth rate. Both (25) and (27) will not be affected by θ. Intuitively, a higher tax on goods reduces firms' profit in equilibrium and impedes the incentive to invent new goods [see (23)]. On the other hand, the tax also reduces manufacturing activities, releasing more resources for innovations and reducing the equilibrium wage [see (26)]. This motivates innovations and so offsets the negative effect above.

In fact, GH have discovered that a subsidy to manufacturing production generates a neutral effect. Since a tax (or subsidy) on goods is equivalent to a tax (or subsidy) on producers, GH's result already implies that consumption tax (or subsidy) is useless.

3.2.2. Research subsidy

To single out the effect of research subsidy, set $\tau = \theta = 1$. It can be found that $g^m = g^o$ when $\sigma = \frac{\rho}{\gamma+\rho} < 1$. Thus, a subsidy ($\sigma < 1$) can help attain the optimal growth, a finding same as GH's.

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According to GH, a research subsidy increases the incentives to innovate, but not the incentives to manufacture goods, at the initial wage level. So, it diverts resources from manufacturing to research and boosts growth of new goods.

Nonetheless, GH have not touched upon the optimality of aggregate output in (static) equilibrium. In the present model, substituting $\sigma = \frac{\rho}{\gamma+\rho}$ into (28) and comparing it with (31), we have $X^o \gtreqless X^m \Leftrightarrow s \gtreqless 0$. Therefore, when status is concerned ($s > 0$), equilibrium aggregate output is smaller than optimal[15]; when it is not (as in GH's case), the output is optimal.

The suboptimal output level in the present model ($s > 0$) requires some policy corrections. But even in GH's model ($s = 0$), there is still a problem: how to finance the research subsidy? A tax on manufacturing products, which has a neutral effect, will not impact on the optimality in output, but we still need to ask if the tax revenue is sufficient to cover the subsidy. GH have not dealt with this problem. Thus, further explorations in this regard are warranted.

In the present context ($s > 0$), the finance problem can never be ignored. As shown above, both equilibrium growth and aggregate output are lower than optimal. If the research subsidy, which can boost growth, is financed by taxing goods, the problem of output shortfall is not corrected (as the tax has neutral effect on output). While lump-sum manufacturing subsidy may fill up the output gap, this generates another need for funding. If the required fund is collected via taxing advertisements, not only will the output be increased (as shown below), a research subsidy will also be rendered unnecessary.

3.2.3. *Tax on advertisements*

Advertising, or the value of z, does not matter to utility—$S(1) = 0$ in equilibrium while z contributes to utility only via S. Hence, if a policy maker can implement (in the first-best setting) the required values of n, x and the growth rate in n as specified in (30) and (31) above, the optimal policy of advertising is to ban it, i.e., $z = 0$.[16] Nonetheless, a policy maker cannot directly arrange n, x and the growth rate in n but is bound to use feasible policy options like tax and subsidy. Now, we have shown that both status-good tax and research subsidy have their own problems. It is then justified to consider a tax on advertising as a second-best policy to implement the optimum solutions.

To concentrate on advertising tax τ, set $\theta = \sigma = 1$. Compare the values of g^m and g^o, using (25) and (30), we have

$$g^m \gtreqless g^o \Leftrightarrow F(\tau) \gtreqless \gamma, \tag{32}$$

where

$$F(\tau) \equiv [(\tau - 1)\rho - \gamma]\tau^{\frac{1}{\alpha-1}} b^{\frac{\alpha}{\alpha-1}} \left(\frac{s}{\alpha}\right)^{\frac{1}{1-\alpha}}. \tag{33}$$

As $F(\tau)$ is not monotonic in τ, it is possible that $F(\tau) < \gamma$ for all $\tau > 1$ (if the maximum value of $F(\tau)$ is still below γ). We need to investigate the condition under which an advertising tax can stimulate growth to reach its optimal rate. The answer is given as the assumption below.

Assumption 2 (Preference for status). $s \geq \frac{\gamma}{\rho}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} b^{\alpha}\left(\frac{\gamma+\rho}{\gamma}\right)^{\alpha}$.

Assumption 2 requires that the marginal utility of status s is sufficiently high. Intuitively, if s is not high, correcting the status bias will not generate significant effects. Assumption 2 simply serves to identify the threshold where the effect is significant.

Now, we have

Proposition 1 (Existence). *Given Assumption 1 and Assumption 2, there exists a tax rate ($\tau^* > 1$) on advertisements such that the optimal growth can be attained.*

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Proposition 2 (Uniqueness). *Under Assumption 1 and Assumption 2, when there is more than one tax rate that can attain the optimal growth rate, the higher-value tax rate with* $\tau^{**} > \frac{\rho+\gamma}{\alpha\rho}$, *where* $F(\tau^{**}) = \gamma$, *is superior.*

All proofs are offered in the Appendix. Essentially, Proposition 1 is a mathematical result, which establishes the existence of the value within the targeted region, given the properties of the functions.

More importantly, what is the intuition behind this proposition? Both consumption tax and advertising tax generate a positive effect on manufacturing firms' profits and a negative effect on wage. The two effects have opposite impacts on growth. Taken together, why is it that consumption tax cannot affect growth but advertising tax can?

From (20), a tax on status goods reduces the quantity of the good proportionally but, from (21), a tax on advertisements generates nonproportional impacts. Due to this difference, in (22), advertising tax and status good tax have nonproportional and proportional effects on price, respectively. As such, the two types of tax exert different impacts on profit in (23).

Intuitively, advertising affects output and profit by reducing demand elasticity [see (12)]. The elasticity effect will not be cancelled out by competition between firms. In contrast, output decisions from firms may offset each other. To see this, inspect (14) and (15). At this stage, output and advertising decisions affect profit in a similar manner when other firms' decisions, represented by $\int_0^n x(i)^\alpha di$, are taken as given. But when symmetry between firms is taken into account, so that other firms' decisions $\int_0^n x(i)^\alpha di = nx^\alpha$ take effect, in (14), both the numerator and denominator contain x^α and are thus cancelled out (leaving only x^{-1}); but in (15), $z^{\alpha-1}$ cannot be cancelled out by x^α. Given the above, output and advertising under symmetry become (20) and (21), respectively, and the difference between output and advertisements becomes apparent at this stage.

On the other hand, the tax effect on wage depends on the amount of labor resources released by the change in the manufacturing or advertising activity. From (24), reducing status goods or advertisements releases resources for innovations in the same (linear) manner due to the assumption of constant-cost production pattern for both activities.

As taxes will reduce advertising and manufacturing activities differently but release the resources for innovation in the same manner, this explains why advertising tax is effective but consumption tax is not.

Notice that the constant-cost assumption is indeed a weaker assumption for this result. In reality, advertising may be more service-oriented, and it is more likely to involve a pattern of increasing cost. Then, the effect of diverting resources for innovation should be even more pronounced by taxing advertisements than manufacturing, strengthening the result above.

Proposition 2 has an economic implication. Mathematically, there could be more that one tax rate that can attain optimal growth [equality in (32)]. To rank these tax rates, I need another criterion—aggregate output bias. In the proof (in Appendix), it is shown that when growth is optimal, aggregate output is smaller than optimal. A higher tax rate can narrow the output gap as it will promote output (by diverting resources toward manufacturing) and is thus superior.

This brings up a problem similar to the case of research subsidy but in reverse direction: when tax is imposed optimally for growth, is the revenue so collected sufficient to cover the output shortfall identified above?

It is useful to reframe the question so as to capture the crux of the problem: The tax collected by a government is equivalent to certain labor units released from promotion activity. When these labor units are re-allocated to manufacture more goods, would it be sufficient to fill up the output shortfall? The scheme is self-sufficient if our answer is positive, or if $(\tau - 1)\, bn^m z^m \geq X^o - X^m$, which means the tax revenue is not less than the expenditure for filling the output gap. Given other conditions, the following assumption is sufficient (but not necessary) for a positive answer:

Assumption 3 (Sufficient resources). $\gamma \geq (2\alpha - 1)\rho$.

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Proposition 3 (Self-sufficiency). *Under Assumptions 1, 2, and 3, when advertisements are taxed optimally for growth, it is self-sufficient in that resources used to fill up the aggregate output shortfall can be financed solely by the tax on advertisements.*

Recall the definition of $\gamma\left(\equiv \frac{L}{a} - \frac{\alpha}{1-\alpha}\rho\right)$. When $\alpha > \frac{1}{2}$, the right side of Assumption 3 is positive and the condition is satisfied only if the resources (L) in this economy is sufficient. While Assumption 1 ascertains $\gamma > 0$ (positive optimal growth rate), Assumption 3 requires γ to be above zero by a margin ($(2\alpha - 1)\,\rho$), i.e., more resources than those required in Assumption 1. Intuitively, financial self-sufficiency requires more resources than a merely positive growth. Assumption 3 reflects this intuition.

Nevertheless, when $\alpha \leq \frac{1}{2}$, Assumption 3 is always satisfied whenever Assumption 1 ($\gamma > 0$) is satisfied. To understand why, notice that the parameter α is positively related to the "intrinsic" elasticity of substitution between goods, which is $\frac{1}{1-\alpha}$, when the superficial effect from advertising is purged (i.e. $z = 0$). As mentioned above, social status does not contribute to utility at the end and so its effect must be purged for assessing what is good for the society. Inspecting (30) and (25), both the optimal growth and equilibrium growth in the absence of taxes and subsidies (so that $\beta = \alpha$) are decreasing with α. Furthermore, as $\alpha \to 0$, optimal and equilibrium growth converge to the same (maximum) value. There is no need for any intervention schemes, including advertising tax, and also no problem about whether resources are enough to achieve self-sufficiency. If α increases from this polar value, intervention is needed but, by continuity, the problem is not severe initially until it reaches a threshold. Assumption 3 serves to identify this threshold. Intuitively, when α is very low, the incentive to invent (very) distinctive products is (very) high. Understandably, maximal distinctiveness provides sufficient incentive to invent new goods, thus overcoming the under-invention market bias caused by knowledge spillover, although this incentive fades as α increases.

4. Concluding remarks

As far as I know, the present paper is the first to address relative comparison in a model featuring a variety of consumer goods,[17] advertisements, and product inventions in an endogenous-growth setting. It contributes to the literature of status and growth by incorporating consumption variety and advertising in it. It contributes to the emerging study of the relation between growth and advertising by adding the element of social status to it. It also brings about novel policy implications—taxing advertisements, instead of taxing consumption, or subsidizing research, is possibly better for dynamic efficiency.

In particular, the paper deals with an important but ignored aspect in the existing literature on relative status: goods must be visible or they will not be taken for assessing social status. Firms' promotions of their products make their goods visible. These two elements, visibility and promotions, have not been given sufficient consideration in the existing literature of status.

Meanwhile, though existing works have studied the relation between status-seeking and growth, they often concentrate on the influence of status on saving and therefore capital accumulation. In fact, there is a more direct channel where status may affect growth: more new goods will be invented if status affects the profits of existing goods and prospective (new) goods. Promoting goods as status-bearing commodities reinforces this profit-breed-invention cycle, which fuels growth. A brand new model is therefore needed to address this issue.

The policy implications of this paper are unconventional, but it is inevitable in the present model: Taxing status goods is ineffective while subsidizing research suffers from a financial problem about self-sufficiency. Only advertising tax may be free from these problems under certain assumptions. And these assumptions do not appear to be strong.

Notes

1 Advertisement accounts for a very high share in the GDPs of advanced countries, which is often not lower than the share of R&D in GDP. Thus, though economists rarely discuss the implications of taxing it, its revenue implication should never be underestimated, not to mention its efficiency implication as introduced in this paper. According to Molinari and Turino (2007), the advertising expenditure to GDP ratios for United States, Great Britain, and Japan are, respectively, 2.26%, 1.51%, and 1.15% between 1983 and 2000. For R&D, see the World Bank data (http://wdi.worldbank.org/table/5.13), from 2005 to 2012: the world average R&D to GDP ratio is 2.13%. The corresponding ratios for high-income countries, middle-income countries, Euro area, and United States are, respectively, 2.32%, 1.24%, 2.14%, and 2.79%.

2 In fact, if we simply consider how output growth may be affected by the relativity in aggregate wealth or consumption, one may find that when relative (aggreagte) wealth matters to growth, relative (aggregate) consumption does not. See the insightful discussion of Tournemaine and Tsoukis (2008). The growth rate in consumption variety is, however, another matter. Meanwhile, though the visibility of wealth is not low in some forms, it is also not high in all forms, relative to the visibility of consumer goods. Based on Heffetz' (2011) findings, visibility of various goods, from high to low (top five) is cigarettes, cars, clothing, furniture, jewelery. "Wealth goods", such as rent in home and hotel, rank in the middle (15th and 18th, respectively). Solnick and Hemenway (2005) also find that the goods of the highest degree of positionality are—to name just a few (positional responses in parenthesis): outfits in job interviews (62%), restaurant meals (39%), and rooms in home (30%). Now, if both wealth goods and consumer goods are visible (at different degrees nonetheless) and may be taken as status goods, a question naturally arises: How will the result be affected by incorporating relativity of aggregate wealth in a model with a variety of status consumer goods? In such a model, two tiers of problems are involved: (1) the choice between aggreagte wealth and the overall consumption and (2) at any given overall consumption, the choice between variety of goods and quantity of each good. Tournemaine and Tsoukis (2008) handles only (1) but not (2). This paper handles (2) but not (1). Naturally, we expect that, for (1), overall consumption may not necessarily be excessive when relative wealth is introduced. However, for whatever result we may obtain for (1), the problem in (2) is not eliminated and is important in itself. The variety may be too little (and grow too slowly) at any overall consumption level at any moment. A discussion of this issue is interesting in itself and can be conducted as a substantial extension of this paper. I am in debt to a referee for pointing out the issue involving relative wealth.

3 The parameters of this product-wise utility function can be altered without affecting the major implications of the model. For example, different taste parameters can be assigned to c and z. Suppose this is done. We simply need to change the parameter of the advertising production function if we still want to get the result that the steady-state growth rate is a constant, as in the GH model. As such changes are not material to the major results but add complications, I prefer the present simpler specification.

4 Two examples are given below to illustrate what sorts of functions will satisfy these properties. 1. $S(cx^{-1}) = s\left(1 - e^{1-\frac{c}{x}}\right)$. $S(1) = 0$. $S'(cx^{-1}) = se^{1-\frac{c}{x}} > 0$. $S'(1) = s > 0$. 2. $S(cx^{-1}) = s\ln\left(cx^{-1}\right)$. $S(1) = 0$. $S'(cx^{-1}) = sc^{-1}x > 0$. $S'(1) = s > 0$.

5 Using the same terms but modifying product-wise utility as $u(i) = z\left[c(i)^{1-\gamma}S(i)^{\gamma}\right]^{\alpha}$, where $\gamma = z^{-1}$, the solution of output is $x = e^{(\bar{r}-\rho)t}\alpha\,(1-\gamma)\,(nw)^{-1}(\ln x)\left[1 + e^{-(\bar{r}-\rho)t}\tau bw\theta n\right]^{-1}$. There is no explict solution for it.

6 The existence of an interior solution for each product i is guaranteed by $\frac{\partial u(i)}{\partial c(i)} \to \infty$ as $c(i) \to 0$ [see (3)].

7 GH (p. 48) have offered two assumptions to justify this: each blueprint of a good is protected by a (hypothetically) permanent patent, or imitation is costly while competition in product prices is intense so that any imitators cannot recoup the imitation cost. While both assumptions are valid from a theoretical viewpoint, perhaps the second one is practically more relevant.

8 Notice that each (atomistic) firm considers it has no noticeable impact on the aggregate value of $\int_0^n x(i)^{\alpha}di$ when choosing $x(i)$ or $z(i)$.

9 The gain of invention is $v\dot{n} = vna^{-1}L_n$ while the cost is σwL_n. Adjustments in labour market ensure the gain and cost are in balance in equilibrium with $\dot{n} > 0$. So, this condition is derived.

10 The owner of a blueprint earns profit π and enjoys (or suffers) a capital gain (or loss) of $\dot{v}$ at any moment. If the funds used to purchase a blueprint are used for buying financial assets (equity), an interest income rv can be secured. When there is no arbitrage in the asset market, the two sides must be in balance.

11 While this specification is the same as GH's, GH have introduced a nonlinear and a more general specification in the appendix and identified conditions for endogenous invention to sustain in the steady state. In this paper, I focus on the simpler specification.

12 This is exactly the same normalization made by GH, who also notice that this is not a conventional treatement. However, this simplifies the calculations significantly. Furthermore, we have to bear in mind that this treatment simply reflects a choice of measurement unit, not assuming what really happens. Since I adopt the GH model as workhorse and want to compare my results with theirs, I find it appropriate to keep this normalization to faciliate an easier comparison. For more about GH's explanation, see the supplement of this paper.

13 The transitional dynamics in GH model is peculiar in that the economy jumps immediately to the steady state; see their explanation in Grossman and Helpman (1991, p. 61). The present model shares this feature. Therefore, our discussion concentrates on the performance in the steady state. For detailed derivations of these results in the steady state, see the supplement of this paper.

14 In particular, $X^s \gtreqless X^o$ if and only if $\left[(1-\alpha)\,\frac{L}{a\rho}-\alpha\right]^{1-\alpha} b^\alpha \alpha \gtreqless s$.
15 For someone who is familiar with the conventional result that status good consumption is excessive, this may be a surprising result. However, this result is not new. Woo (2016) has already demonstrated that status goods are under-consumed in the presence of relative consumption effect, product differentiation, and the preference for variety. The intuition is that relative consumption effect distorts people's perception on a good, and (artificially) reduces price elasticity, as in the present case, boosting price and suppressing the quantity consumed. Note also that this result is about static inefficiency: at every moment, aggregation output is too small. This is unrelated to the dynamic inefficiency. In fact, as status-seeking distorts consumption decisions at every moment, it will not shift the demand from one point of time to another. Nonetheless, firms would rather expend on advertising, with the purpose to mislead people to pay a higher price for their good so that firms can enjoy a higher profit. For the whole economy, promotion reduces resources for manufacturing (generating static inefficiency) and resources for innovations (generating dynamic inefficiency).
16 Notice that the model assumes only one role for advertisements—persuasive advertisements. It simply changes preferences superficially but will not (eventually) contribute to utility. If the model is modified so that the informative role of advertisements is also incorporated, utility should be enhanced by advertisements, which should not be banned even in the first-best setting. A reasonable conjecture is that advertisements, despite informative, should still be reduced (but not to zero) whenever it induces status-seeking. However, the proof of this will go beyond the framework in this paper.
17 Hof and Prettner (2019) have discussed the variety of intermediate goods in a Romer-type endogenous growth model. They assume that people are concerned with relative wealth, and wealth is composed of capital and equity ownership. Hence, their model differs from mine: my model assumes that individuals are concerned with the relative consumption in each variety of consumer goods, not intermediate goods and not capital or equity.
18 Some popular mathematics textbooks used by economists present only the version of theorem where the intermediate value lies strictly between (but does not equal to) the upper and lower bounds. The version of theorem used here, allowing equalities, can be found in, for instance, pp. 238–239 of Tao T., *Analysis I* (Third Edition), Springer.
19 To avoid over-interpretation, notice that we have adopted a normalization (as in GH) where $r=\rho$ and the condition $\tau^{**} \geq \frac{\rho+\gamma}{\alpha\rho}$ must be interpreted in light of this. Changing the normalization may affect our interpretation of the absolute level of the required tax rate. The message, however, is clear: the tax rate must be high enough so that it collects sufficient fund to finance the output shortfall.

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Appendix A

The proofs of the three propositions in Section 3.2 are given below.

Proof of Proposition 1. Note firstly that the maximum value of $F(\tau)$ occurs when $\tau = \frac{\rho+\gamma}{\alpha\rho} > 1$. Consider another value $\tau = \frac{\rho+\gamma}{\rho} > 1$, and the closed and bounded set $\left[\frac{\rho+\gamma}{\rho}, \frac{\rho+\gamma}{\alpha\rho}\right]$. We have $F\left(\frac{\rho+\gamma}{\rho}\right) = 0$. So, by Assumption 1, $F\left(\frac{\rho+\gamma}{\rho}\right) \leq \gamma$. Meanwhile, $F\left(\frac{\rho+\gamma}{\alpha\rho}\right) = \frac{1-\alpha}{\alpha}(\rho+\gamma)\left(\frac{\rho+\gamma}{\alpha\rho}\right)^{\frac{1}{\alpha-1}} b^{\frac{\alpha}{\alpha-1}} \left(\frac{s}{\alpha}\right)^{\frac{1}{1-\alpha}} \geq \gamma$ if and only if $s \geq \frac{\gamma}{\rho}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} b^{\alpha}\left(\frac{\gamma+\rho}{\gamma}\right)^{\alpha}$, which is true under Assumption 2. As $F\left(\frac{\rho+\gamma}{\rho}\right) \leq \gamma \leq F\left(\frac{\rho+\gamma}{\alpha\rho}\right)$ and F is continuous in this domain, by intermediate-value theorem,[18] there exists an $\tau^* \in \left[\frac{\rho+\gamma}{\rho}, \frac{\rho+\gamma}{\alpha\rho}\right]$ such that $F(\tau^*) = \gamma$, which by (32) implies that equilibrium growth equals optimal growth. □

Proof of Proposition 2. Unless $s = \frac{\gamma}{\rho}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} b^{\alpha}\left(\frac{\gamma+\rho}{\gamma}\right)^{\alpha}$, there is another tax rate $\tau^{**} > 1$ other than $\tau^* \in \left[\frac{\rho+\gamma}{\rho}, \frac{\rho+\gamma}{\alpha\rho}\right]$ such that $F(\tau^{**}) = \gamma$. To see this, let $\tau = \delta\left(\frac{\rho+\gamma}{\alpha\rho}\right)$, where $\delta > 1$. Then, $F\left(\delta\left(\frac{\rho+\gamma}{\alpha\rho}\right)\right) < \gamma \Leftrightarrow s < (\alpha\gamma)^{1-\alpha} b^{\alpha}(\rho+\gamma)^{\alpha} \frac{\delta}{\rho}(\delta-\alpha)^{\alpha-1}$. Since $\delta(\delta-\alpha)^{\alpha-1}$ is increasing with δ, there exists δ that is sufficiently large such that $F\left(\delta\left(\frac{\rho+\gamma}{\alpha\rho}\right)\right) < \gamma$. Furthermore, with $s > \frac{\gamma}{\rho}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} b^{\alpha}\left(\frac{\gamma+\rho}{\gamma}\right)^{\alpha}$ (the equality scenario being ruled out), $F\left(\frac{\rho+\gamma}{\alpha\rho}\right) > \gamma$. Then, as $F\left(\frac{\rho+\gamma}{\alpha\rho}\right) > \gamma > F\left(\delta\frac{\rho+\gamma}{\alpha\rho}\right)$ and F is continuous in this domain, by intermediate-value theorem, there exists an $\tau^{**} \in \left(\frac{\rho+\gamma}{\alpha\rho}, \delta\left(\frac{\rho+\gamma}{\alpha\rho}\right)\right)$ such that $F(\tau^{**}) = \gamma$.

Although there is more than one value of τ such that $F(\tau) = \gamma$, we can demonstrate that only the higher-value tax rate is optimal. Using the condition $F(\tau) = \gamma$, we have $X^m < X^o$ by (28) and (31). From (28), X^m is increasing with τ. Given the same g^o attained, a higher X increases any instantaneous utility (see Subsection 3.1). Thus, τ^{**} is superior to τ^* when both satisfy $F(\tau) = \gamma$. □

Proof of Proposition 3. The labor units collected from the tax are $(\tau-1)\, bn^m z^m$. Substituting (26) into (21), using also (28) and (31), we have $(\tau-1)\, bn^m z^m \geq X^o - X^m \Leftrightarrow \frac{(1-\alpha)\gamma}{(1-\alpha)\gamma+\rho} \geq \left\{\frac{\rho[(1-\alpha)\tau+\alpha]}{(1-\alpha)\gamma+\rho} - (\tau-1)\right\} \tau^{\frac{1}{\alpha-1}} b^{\frac{\alpha}{\alpha-1}} \left(\frac{s}{\alpha}\right)^{\frac{1}{1-\alpha}}$. Then, using $F(\tau) = \gamma$ to simplify the condition, we

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have $(\tau - 1)\, bn^m z^m \geq X^o - X^m \Leftrightarrow \tau \geq 2$. We need to confirm that the tax rate τ^{**} such that $F(\tau^{**}) = \gamma$ satisfies this (i.e. being not lower than 2). By Proposition 2 and Assumption 2, we know $\tau^{**} \geq \frac{\rho+\gamma}{\alpha\rho}$[19] (equality follows when Assumption 2 is satisfied as equality). Then, if $\frac{\rho+\gamma}{\alpha\rho} \geq 2$, $\tau^{**} \geq 2$. By Assumption 3, $\gamma \geq (2\alpha - 1)\,\rho$. So, $\frac{\rho+\gamma}{\alpha\rho} \geq 2$, and the scheme is self-sufficient. $\square$

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