

favour of a long-standing conjecture asking whether Turing determinacy implies the axiom of determinacy.

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ANDREAS LIETZ, *Forcing “NS $_{\omega_1}$ is ω_1 -Dense” from Large Cardinals*. Universität Münster, Münster, Germany, 2023. Supervised by Ralf Schindler. MSC: Primary 03E57, Secondary 03E35, 03E55, 03E60, 03E25. Keywords: nonstationary ideal, forcing axioms, large cardinals, axiom (*).

Abstract

We answer a question of Woodin [3] by showing that “NS $_{\omega_1}$ is ω_1 -dense” holds in a stationary set preserving extension of any universe with a cardinal κ which is a limit of $<\kappa$ -supercompact cardinals. We introduce a new forcing axiom Q-Maximum, prove it consistent from a supercompact limit of supercompact cardinals, and show that it implies the version of Woodin’s (*)-axiom for \mathbb{Q}_{\max} . It follows that Q-Maximum implies “NS $_{\omega_1}$ is ω_1 -dense.” Along the way we produce a number of other new instances of Asperó–Schindler’s $\text{MM}^{++} \Rightarrow$ (*) (see [1]).

To force Q-Maximum, we develop a method which allows for iterating ω_1 -preserving forcings which may destroy stationary sets, without collapsing ω_1 . We isolate a new regularity property for ω_1 -preserving forcings called respectfulness which lies at the heart of the resulting iteration theorem.

In the second part, we show that the κ -mantle, i.e., the intersection of all grounds which extend to V via forcing of size $<\kappa$, may fail to be a model of AC for various types of κ . Most importantly, it can be arranged that κ is a Mahlo cardinal. This answers a question of Usuba [2].

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ZHANSAYA TLEULIYEVA, *Algorithmic Properties of Rogers Semilattices* Nazarbayev University. Supervised by Manat Mustafa and Nikolay Bazhenov. MSC: 03D45. Keywords: theory of numberings, computable numbering, Rogers semilattice, limitwise monotonic, analytical hierarchy, projective determinacy, types of isomorphism.

Abstract

The thesis uses various approaches to explore the algorithmic complexity of families of subsets of natural numbers. One of these approaches involves investigating upper semilattices

of computable numberings of a given family and their complexity in different hierarchies. These semilattices, known as Rogers semilattices, can help distinguish different structural properties of families of partial computable functions and computably enumerable sets. As a result, by using Rogers semilattices of computable numberings, we can measure the algorithmic complexity of the corresponding family.

In the first part of thesis, we focus on limitwise monotonic numberings for families of limitwise monotonic sets and define their Rogers semilattices. The chapter investigates global invariants that show differences in the algebraic and elementary properties of the Rogers semilattices of families of sets from arithmetical hierarchy and Rogers semilattices of limitwise monotonic numberings. Such invariants include cardinality, laticeness, and types of isomorphism.

Within the second part of thesis, we explore the different forms of isomorphism exhibited by Rogers semilattices of families of sets in the analytical hierarchy. Additionally, we take into account various set-theoretic assumptions. Our research demonstrates that, when set-theoretic assumption known as Projective Determinacy is assumed, there exist an infinite number of non-isomorphic Rogers semilattices at each Σ_n^1 -level of the analytical hierarchy.

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