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Characterizations of Real Hypersurfaces in a Complex Space Form

In-Bae Kim, Ki Hyun Kim and Woon Ha Sohn

Abstract. We study a real hypersurface M in a complex space form $M_n(c)$, $c \neq 0$, whose shape operator and structure tensor commute each other on the holomorphic distribution of M.

1 Introduction

A complex n-dimensional Kaehler manifold of constant holomorphic sectional curvature c is called a *complex space form* and is denoted by $M_n(c)$. A complete and simply connected complex space form consists of a complex projective space $P_n(\mathbb{C})$, a complex Euclidean space $\mathbb{C}^n$ or a complex hyperbolic space $H_n(\mathbb{C})$, according to $c > 0$, $c = 0$ or $c < 0$. The shape operator or second fundamental tensor of a real hypersurface in $M_n(c)$ will be denoted by A, and the induced almost contact metric structure of the real hypersurface by $(\phi, \langle\cdot\,,\cdot\rangle, \xi, \eta)$.

R. Takagi [9] classified all homogeneous real hypersurfaces in $P_n(\mathbb{C})$ into six model spaces A_1, A_2, B, C, D and E (see also [10]). J. Berndt [3] has completed the classification of homogeneous real hypersurfaces with principal structure vector fields ξ in $H_n(\mathbb{C})$, which are divided into the model spaces A_0, A_1, A_2 and B. A real hypersurface of type A_1 or A_2 in $P_n(\mathbb{C})$ or that of A_0, A_1 or A_2 in $H_n(\mathbb{C})$ is said to be *of type A* for simplicity.

A typical characterization for a real hypersurface M of type A in a complex space form $M_n(c)$ was given under the condition

$$\langle (A\phi - \phi A)X, Y\rangle = 0 \quad \text{for any tangent vector fields } X \text{ and } Y \text{ on } M \tag{1.1}$$

by M. Okumura [8] for $c > 0$ and S. Montiel and A. Romero [6] for $c < 0$. Namely,

Theorem A *Let M be a real hypersurface in a complex space form $M_n(c)$, $c \neq 0$, $n \geq 3$. If it satisfies* (1.1)*, then M is locally congruent to a real hypersurface of type A.*

The *holomorphic distribution* T_0 of a real hypersurface M in $M_n(c)$ is defined by

$$T_0(p) = \{X \in T_p(M) \mid \langle X, \xi\rangle_p = 0\},$$

where $T_p(M)$ is the tangent space of M at p. A (1,1) type tensor field T of M is said to be *η-parallel* if $\langle(\nabla_X T)Y, Z\rangle = 0$, and *cyclic η-parallel* if

$$\mathfrak{S}\langle(\nabla_X T)Y, Z\rangle = \langle(\nabla_X T)Y, Z\rangle + \langle(\nabla_Y T)Z, X\rangle + \langle(\nabla_Z T)X, Y\rangle = 0$$

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for any X, Y and Z in the holomorphic distribution T_0. Real hypersurfaces with η-parallel or cyclic η-parallel shape operator or Ricci operator have been studied by many authors (see [7]). Of course, the condition

$$\langle (A\phi - \phi A)X, Y\rangle = 0 \tag{1.2}$$

for any X and Y in T_0 is weaker than (1.1). It is known that a ruled real hypersurface in $M_n(c)$ satisfies (1.2) and has the η-parallel shape operator [1, 5, 7]. Recently, C. Baikoussis [2] studied real hypersurfaces in $M_n(c)$ with certain conditions related to the Ricci operator and the structure tensor ϕ. U-H. Ki and Y.-J. Suh [4] investigated a real hypersurface satisfying (1.2) and having a special η-parallel shape operator.

The purpose of this paper is to give some characterizations of a real hypersurface satisfying (1.2) and having the cyclic η-parallel shape operator or Ricci operator. We shall prove the following.

Theorem 1.1 *Let M be a real hypersurface in a complex space form $M_n(c)$, $c \neq 0$, $n \geq 3$. If M has the cyclic η-parallel shape operator and satifies* (1.2)*, then M is locally congruent to either a real hypersurface of type A or a ruled real hypersurface.*

Theorem 1.2 *Let M be a real hypersurface in a complex space form $M_n(c)$, $c \neq 0$, $n \geq 3$. Assume that M satisfies* (1.2)*. Then M is locally congruent to a ruled real hypersurface if and only if it satisfies one of the following:*

$$\langle AX, Y\rangle \phi U + \langle Y, \phi U\rangle AX + \langle X, \phi U\rangle AY = \beta\{\langle X, U\rangle\langle Y, \phi U\rangle + \langle Y, U\rangle\langle X, \phi U\rangle\}\xi, \tag{1.3}$$

$$(\nabla_X A)Y = \{\beta^2(\langle X, U\rangle\langle Y, \phi U\rangle + \langle Y, U\rangle\langle X, \phi U\rangle) - \frac{c}{4}\langle \phi X, Y\rangle\}\xi \tag{1.4}$$

for any X, Y in the holomorphic distribution T_0, where $\beta(\neq 0)$ is the length of $\phi\nabla_\xi\xi$ and $U = -\frac{1}{\beta}\phi\nabla_\xi\xi$, ∇ being the Riemannian connection of M.

Theorem 1.3 *Let M be a real hypersurface in a complex space form $M_n(c)$, $c \neq 0$, $n \geq 3$. If M has the cyclic η-parallel Ricci operator and satifies* (1.2)*, then M is locally congruent to a real hypersurface of type A.*

2 Preliminaries

Let M be a real hypersurface immersed in a complex space form $(M_n(c), \langle\cdot\,,\cdot\rangle, J)$ of constant holomorphic sectional curvature c, and let N be a unit normal vector field on an open neighborhood in M. For a local tangent vector field X on the neighborhood, the images of X and N under the almost complex structure J of $M_n(c)$ can be expressed by

$$JX = \phi X + \eta(X)N, \quad JN = \xi,$$

where ϕ defines a linear transformation on the tangent space $T_p(M)$ of M at any point $p \in M$, and η and ξ denote a 1-form and a unit tangent vector field on the neighborhood, respectively. Denoting the Riemannian metric on M induced from the metric on $M_n(c)$ by the same symbol $\langle \cdot\,, \cdot \rangle$, it is then easy to see that

$$\langle \phi X, Y \rangle + \langle \phi Y, X \rangle = 0, \quad \langle \xi, X \rangle = \eta(X)$$

for any tangent vector field X and Y on M. The collection $(\phi,\ \langle \cdot\,, \cdot \rangle, \xi, \eta)$ is called an *almost contact metric structure* on M, and satisfies

$$\begin{aligned} \phi^2 X = -X + \eta(X)\xi, \quad \phi\xi = 0, \quad \eta(\phi X) = 0, \\ \eta(\xi) = 1, \quad \langle \phi X, \phi Y \rangle = \langle X, Y \rangle - \eta(X)\eta(Y). \end{aligned} \tag{2.1}$$

Let ∇ be the Riemannian connection with respect to the metric $\langle \cdot\,, \cdot \rangle$ on M, and A be the shape operator in the direction of N on M. Then we have

$$\nabla_X \xi = \phi A X, \quad (\nabla_X \phi) Y = \eta(Y) A X - \langle A X, Y \rangle \xi. \tag{2.2}$$

Since the ambient space is of constant holomorphic sectional curvature c, the equations of Gauss and Codazzi are given by

$$\begin{aligned} R(X,Y)Z = \frac{c}{4}\Big\{ \langle Y, Z \rangle X - \langle X, Z \rangle Y + \langle \phi Y, Z \rangle \phi X - \langle \phi X, Z \rangle \phi Y \\ - 2\langle \phi X, Y \rangle \phi Z \Big\} + \langle AY, Z \rangle AX - \langle AX, Z \rangle AY, \end{aligned} \tag{2.3}$$

$$(\nabla_X A)Y - (\nabla_Y A)X = \frac{c}{4}\{\eta(X)\phi Y - \eta(Y)\phi X - 2\langle \phi X, Y \rangle \xi\} \tag{2.4}$$

for any tangent vector fields X, Y and Z on M, where R is the Riemannian curvature tensor of M. We shall denote the Ricci operator of M by S. Then it follows from (2.3) that $SX = \frac{c}{4}\{(2n+1)X - 3\eta(X)\xi\} + mAX - A^2 X$, where $m = \operatorname{trace} A$ is the mean curvature of M. The covariant derivative of S is given by

$$\begin{aligned} (\nabla_X S)Y = -\frac{3c}{4}\{\langle \phi A X, Y \rangle \xi + \eta(Y) \phi A X\} + (Xm)AY \\ + m(\nabla_X A)Y - (\nabla_X A)AY - A(\nabla_X A)Y. \end{aligned} \tag{2.5}$$

If the vector field $\phi \nabla_\xi \xi$ does not vanish, that is, the length β of $\phi \nabla_\xi \xi$ is not equal to zero, then it is easily seen from (2.1) and (2.2) that

$$A\xi = \alpha \xi + \beta U, \tag{2.6}$$

where $\alpha = \langle A\xi, \xi \rangle$ and $U = -\frac{1}{\beta}\phi \nabla_\xi \xi$. Therefore, U is a unit tangent vector field on M and $U \in T_0$. If the vector field U cannot be defined, then we may consider $\beta = 0$ identically. Therefore $A\xi$ is always given in (2.6)

From now on, we assume that the condition (1.2) holds on M, that is,

$$\langle (A\phi - \phi A)Y, Z\rangle = 0$$

for any Y and $Z \in T_0$. With respect to any tangent vector field X on M, if we differentiate this relation covariantly in the direction of X and make use of (1.2), (2.1), (2.2) and (2.6), we obtain

(2.7)
$$\langle (\nabla_X A)Y, \phi Z\rangle + \langle (\nabla_X A)Z, \phi Y\rangle = \beta\big\{ \langle Y, U\rangle\langle AX, Z\rangle + \langle Z, U\rangle\langle AX, Y\rangle - \langle Y, \phi U\rangle\langle \phi AX, Z\rangle - \langle Z, \phi U\rangle\langle \phi AX, Y\rangle \big\},$$

where we have used the equation

$$\nabla_X Y = \langle \nabla_X Y, \xi\rangle\xi + (\nabla_X Y)_0 = -\langle \phi AX, Y\rangle\xi + (\nabla_X Y)_0, \quad (\nabla_X Y)_0 \in T_0.$$

Putting $Y = Z$ into (2.7) and using the Codazzi equation (2.4), we have

$$\langle (\nabla_Y A)\phi Y, X\rangle = \langle \beta\{\langle Y, U\rangle AY + \langle Y, \phi U\rangle A\phi Y\} - \frac{c}{4}\langle Y, Y\rangle\xi, X\rangle$$

for any tangent vector field X on M and $Y \in T_0$, which implies

$$(\nabla_X A)\phi X = \beta\{\langle X, U\rangle AX + \langle X, \phi U\rangle A\phi X\} - \frac{c}{4}\langle X, X\rangle\xi, \tag{2.8}$$

$$(\nabla_{\phi X} A)X = \beta\{\langle X, U\rangle AX + \langle X, \phi U\rangle A\phi X\} + \frac{c}{4}\langle X, X\rangle\xi \tag{2.9}$$

for any $X \in T_0$.

Next we consider that the vector fields X, Y and Z in (2.7) belong to the holomorphic distribution T_0. In the equation (2.7), we shall replace X, Y and Z cyclically and then add the equation to (2.7), from which we subtract the third one. By use of the Codazzi equation and the relation (1.2), we have (see also [4])

$$\langle (\nabla_X A)Y, \phi Z\rangle = \beta\{\langle AX, Y\rangle\langle Z, U\rangle - \langle A\phi X, Z\rangle\langle Y, \phi U\rangle - \langle A\phi Y, Z\rangle\langle X, \phi U\rangle\}.$$

Putting $Z = \phi Z$ into the above equation and using (1.2), we obtain

$$\langle (\nabla_X A)Y, Z\rangle = \beta\mathfrak{S}\langle AX, Y\rangle\langle Z, \phi U\rangle \tag{2.10}$$

for any X, Y and Z in T_0, or equivalently

$$(\nabla_X A)Y = \beta\{\langle AX, Y\rangle\phi U + \langle Y, \phi U\rangle AX + \langle X, \phi U\rangle AY\} + \{\langle (\nabla_X A)Y, \xi\rangle - \beta^2(\langle X, U\rangle\langle Y, \phi U\rangle + \langle X, \phi U\rangle\langle Y, U\rangle)\}\xi. \tag{2.11}$$

3 Proofs of Theorems

In this section, we shall prove Theorems 1.1, 1.2 and 1.3. Let M be a real hypersurface in a complex space form $M_n(c)$, $c \neq 0$, $n \geq 3$. We first prove some lemmas.

Lemma 3.1 *Assume that M has the cyclic η-parallel shape operator and satisfies* (1.2). *If* (1.3) *does not hold, then ξ is principal, that is $A\xi = \alpha\xi$.*

Proof Assume that there is a point p of M such that the scalar field β given in (2.6) is not equal to zero at p. Then there exists an open neighborhood $\mathcal{U}$ of p such that $\beta \neq 0$ on $\mathcal{U}$, that is, ξ is not principal on $\mathcal{U}$. Since we have $\langle AU, \phi U\rangle = 0$ and $A\phi U = \phi AU$ from (1.2) and $n \geq 3$, we see from (2.6) that there are scalar fields γ and δ on $\mathcal{U}$ and a unit tangent vector field V orthogonal to ξ, U and ϕU such that

$$AU = \beta\xi + \gamma U + \delta V, \quad A\phi U = \gamma\phi U + \delta\phi V. \tag{3.1}$$

Under the assumption (1.2), it is easy to see from (2.10) that the shape operator A is cyclic η-parallel if and only if A is η-parallel. Multiplying (2.11) by ϕU and using (3.1) and the η-parallel shape operator, we have

$$\langle AX, Y\rangle + 2\gamma\langle X, \phi U\rangle\langle Y, \phi U\rangle + \delta\{\langle X, \phi U\rangle\langle Y, \phi V\rangle + \langle X, \phi V\rangle\langle Y, \phi U\rangle\} = 0. \tag{3.2}$$

If we put $X = Y = U$, or $X = U$ and $Y = V$ into (3.2) and make use of (3.1), then we see that

$$\gamma = \delta = 0 \tag{3.3}$$

on $\mathcal{U}$, and hence (3.2) is reduced to

$$\langle AX, Y\rangle = 0. \tag{3.4}$$

It follows from (2.6), (3.1), (3.3) and (3.4) that

$$A\xi = \alpha\xi + \beta U, \quad AX = \beta\langle X, U\rangle\xi \tag{3.5}$$

for any $X \in T_0$, which shows that M is locally congruent to a ruled real hypersurface (see [1, 5]).

Since M does not satisfy (1.3), there are some vector fields X and Y in T_0 such that

$$\langle AX, Y\rangle\phi U + \langle Y, \phi U\rangle AX + \langle X, \phi U\rangle AY \neq \beta\{\langle X, U\rangle\langle Y, \phi U\rangle + \langle Y, U\rangle\langle X, \phi U\rangle\}\xi. \tag{3.6}$$

It is easily seen from (3.5) that (3.6) gives a contradiction. Therefore ξ must be principal. ∎

Lemma 3.2 *M is locally congruent to a ruled real hypersurface if and only if it satisfies* (1.3).

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Proof If M is locally congruent to a ruled real hypersurface, then the scalar field β does not vanish and M satisfies (3.5). It is clear that (1.3) is given by (3.5).

Conversely, we assume that (1.3) holds on M. Putting $Y = \phi U$ into (1.3), we have

$$\langle A\phi U, X\rangle \phi U + AX + \langle X, \phi U\rangle A\phi U = \beta\langle X, U\rangle\xi \tag{3.7}$$

for any $X \in T_0$. If we put $X = \phi U$ into (3.7) and multiply it by ϕU, then we see that $A\phi U = 0$. Therefore (3.7) is reduced to $AX = \beta\langle X, U\rangle\xi$ for any $X \in T_0$, which together with (2.6) means that M is locally congruent to a ruled real hypersurface. ■

Lemma 3.3 *Assume that M satisfies* (1.2). *Then the equations* (1.3) *and* (1.4) *are equivalent.*

Proof If (1.3) is given on M, then we see from Lemma 3.2 that (3.5) holds on M. Differentiating the first equation of (3.5) covariantly along $X(\in T_0)$ and using (2.2) and (3.5), we have

$$(\nabla_X A)\xi = (X\alpha)\xi + (X\beta)U + \beta\nabla_X U, \tag{3.8}$$

and, from the second of (3.5), we get

$$(\nabla_\xi A)X = -A\nabla_\xi X + \beta^2\langle X, U\rangle\phi U + \xi(\beta\langle X, U\rangle)\xi.$$

From the above two equations and the Codazzi equation (2.4), we obtain

$$(X\beta)U + \beta\nabla_X U = \{\xi(\beta\langle X, U\rangle) - X\alpha\}\xi + \beta^2\langle X, U\rangle\phi U - \frac{c}{4}\phi X - A\nabla_\xi X. \tag{3.9}$$

Since we have $\langle(\nabla_X A)Y, \xi\rangle = \langle(X\beta)U + \beta\nabla_X U, Y\rangle$ from (3.8), it is easily seen from (2.1), (2.2), (3.5) and (3.9) that

$$\langle(\nabla_X A)Y, \xi\rangle = \beta^2(\langle X, U\rangle\langle Y, \phi U\rangle + \langle X, \phi U\rangle\langle Y, U\rangle) - \frac{c}{4}\langle\phi X, Y\rangle \tag{3.10}$$

for any X and Y in T_0. Substituting (3.5) and (3.10) into (2.11), we then have (1.4).

Conversely, we assume that (1.4) is given on M. Then it follows from (1.4) and (2.11) that

$$\begin{aligned}\beta\{\langle AX, Y\rangle\phi U &+ \langle Y, \phi U\rangle AX + \langle X, \phi U\rangle AY\} \\ &= \{2\beta^2(\langle X, U\rangle\langle Y, \phi U\rangle + \langle X, \phi U\rangle\langle Y, U\rangle) - \frac{c}{4}\langle\phi X, Y\rangle - \langle(\nabla_X A)Y, \xi\rangle\}\xi,\end{aligned}$$

which implies

$$\langle AX, Y\rangle\langle Z, \phi U\rangle + \langle Y, \phi U\rangle\langle AX, Z\rangle + \langle X, \phi U\rangle\langle AY, Z\rangle = 0 \tag{3.11}$$

for any X, Y and Z in T_0. If we put $Y = Z = \phi U$ into (3.11), then we see that

$$2\langle AX, \phi U\rangle + \langle X, \phi U\rangle\langle A\phi U, \phi U\rangle = 0.$$

Putting $X = \phi U$ into the above, we get $\langle A\phi U, \phi U\rangle = 0$ and hence the above is given by

$$\langle AX, \phi U\rangle = 0 \quad \text{for any } X \in T_0. \tag{3.12}$$

By putting $Z = \phi U$ into (3.11) and using (3.12), we obtain $\langle AX, Y\rangle = 0$, which implies that (3.5) holds on M. The equation (1.3) is induced from (3.5) by Lemma 3.2. ■

Proof of Theorem 1.1 If M satisfies (1.3), then M is locally congruent to a ruled real hypersurface by Lemma 3.2.

If (1.3) does not hold on M, then the scalar field β vanishes identically on M, that is, ξ is principal by Lemma 3.1. We see from (2.1) and (2.6) that $(A\phi - \phi A)\xi = 0$, which together with our assumption (1.2) implies (1.1), that is, $A\phi = \phi A$ on M. Thus, Theorem A shows that M is locally congruent to a real hypersurface of type A. ■

Proof of Theorem 1.2 If M is a ruled real hypersurface, then we have (1.3) by Lemma 3.2. Under the assumption (1.2), we also have (1.4) by Lemma 3.3.

If M satisfies (1.3) or (1.4), then we have $AX = \beta\langle X, U\rangle\xi$ for any X in T_0 as seen in the proof of Lemmas 3.2 and 3.3, which means that M is locally congruent to a ruled real hypersurface. ■

The following is due to Ki and Suh [4] and immediate from Theorems 1.1 and 1.2.

Corollary 3.4 ([4]) *If a real hypersurface M in a complex space form $M_n(c)$, $c \neq 0$, $n \geq 3$ satisfies* (1.2) *and $(\nabla_X A)Y = -\frac{c}{4}\langle \phi X, Y\rangle\xi$ for $X, Y \in T_0$, then M is locally congruent to a real hypersurface of type A.*

Proof of Theorem 1.3 We assume that there is a point p of M such that $\beta(p) \neq 0$. Then there exists an open neighborhood $\mathcal{U}$ of p such that $\beta \neq 0$ on $\mathcal{U}$. We see from (1.2) and (2.6) that there are scalar fields γ and δ on $\mathcal{U}$ and a unit tangent vector field V orthogonal to ξ, U and ϕU satisfying (3.1).

Putting $X = Y = U$, $Z = \phi U$ and $X = Y = U$, $Z = \phi V$ into (2.10) and taking account of (3.1), we have

$$\langle (\nabla_U A)U, \phi U\rangle = \beta\gamma, \quad \langle (\nabla_U A)U, \phi V\rangle = 0 \tag{3.13}$$

on $\mathcal{U}$, respectively. As a similar argument as the above, by putting $X = Y = \phi U$, $Z = \phi U$ or $Z = \phi V$ into (2.10), we also obtain

$$\langle (\nabla_{\phi U} A)\phi U, \phi U\rangle = 3\beta\gamma, \qquad \langle (\nabla_{\phi U} A)\phi U, \phi V\rangle = 2\beta\delta. \tag{3.14}$$

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If we substitute $X = U$ into (2.8) and (2.9) and make use of (3.1), we get

$$(3.15)\quad (\nabla_U A)\phi U = (\beta^2 - \frac{c}{4})\xi + \beta\gamma U + \beta\delta V, \quad (\nabla_{\phi U} A)U = (\beta^2 + \frac{c}{4})\xi + \beta\gamma U + \beta\delta V,$$

respectively.

Since the Ricci operator S is cyclic η-parallel, it follows from (2.5) and (2.10) that

$$(3.16)\quad \mathfrak{S}\{(Xm)\langle AY, Z\rangle + 3m\beta\langle AX, Y\rangle\langle Z, \phi U\rangle - \langle(\nabla_X A)Z + (\nabla_Z A)X, AY\rangle\} = 0$$

for any vector fields X, Y and Z in T_0. Putting $X = Y = U$ and $Z = \phi U$ into (3.16) and taking account of (3.1), (3.13) and (3.15), we get

$$(3.17)\qquad \gamma((\phi U)m) + 3m\beta\gamma - 4\beta^3 - 6\beta\gamma^2 - 4\beta\delta^2 = 0.$$

Moreover, if we put $X = Y = Z = \phi U$ into (3.16) and make use of (3.1) and (3.14), we have $\gamma((\phi U)m) + 3m\beta\gamma - 6\beta\gamma^2 - 4\beta\delta^2 = 0$, which together with (3.17) gives a contradiction.

Therefore ξ must be principal and our conclusion follows from Theorem A. ■

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Department of Mathematics
Hankuk University of Foreign Studies
Seoul 130-791
Korea
e-mail: ibkim@hufs.ac.kr
rlgusm@hanmail.net
mathsohn@hufs.ac.kr