22
The massive Proca field

The massive vector field is the model which describes massive vector bosons, such as the W and Z particles of the electro-weak theory.

22.1 Action and field equations

The action for the Proca field is

$$S = \int (\mathrm{d}x) \left\{ \frac{1}{4} F^{\mu\nu} F_{\mu} + \frac{1}{2} m^2 A^{\mu} A_{\mu} - J^{\mu} A_{\nu} \right\}, \tag{22.1}$$

where

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}. \tag{22.2}$$

The variation of the action gives

$$\delta A = \int (\mathrm{d}x) \ \left[-\partial^{\nu} F_{\mu\nu} + m^2 A_{\mu} - J_{\mu} \right] \delta A_{\mu} + \int \mathrm{d}\sigma^{\mu} \ F_{\mu\nu} \delta A^{\nu}. \tag{22.3}$$

This yields the field equation

$$-\partial^{\nu} F_{\mu\nu} + m^2 A_{\mu} = J_{\mu}, \tag{22.4}$$

also writable as

$$-\Box \, A_{\mu} - \partial_{\mu}(\partial^{\nu} A_{\nu}) + m^2 A_{\mu} = J_{\mu}, \tag{22.5}$$

and associated continuity conditions identical to those of the Maxwell field. The conjugate momentum ($\mathrm{d}\sigma^{\mu} = \mathrm{d}\sigma^{0}$) is

$$\Pi_i = F_{0i}. \tag{22.6}$$

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If the surface σ is taken to separate two regions of space rather than time, one has the continuity conditions for the Proca field in a vacuum:

$$\begin{aligned} \Delta F_{i0} &= 0 \\ \Delta F_{ij} &= 0, \end{aligned} \tag{22.7}$$

and we have assumed that δA_μ is a continuous function. Taking the $n+1$ divergence of eqn. (22.4), we obtain

$$\partial^\mu A_\mu = 0. \tag{22.8}$$

Here we have used the anti-symmetry of $F_{\mu\nu}$ and the assumption that the source is conserved, $\partial^\mu J_\nu$. Thus the field equations, in the form of eqn. (22.5), become

$$(-\Box + m^2) A_\mu = J_\mu \tag{22.9}$$

$$\partial^\mu A_\mu = 0. \tag{22.10}$$

In contrast to the electromagnetic field, this has both transverse and longitudinal components.

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