



## SYMPOSIA PAPER

# The Representational Role of Sophisticated Theories

Clara Bradley

University of California Irvine, Irvine, CA, USA

Email: cbradle1@uci.edu

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## **Abstract**

Dewar (2019) argues that removing excess structure via "sophistication" can have explanatory benefits to removing excess structure via "reduction." In this paper, I argue that a more robust reason to prefer sophisticated theories is that they have representational benefits.

#### **I** Introduction

How should one remove "excess structure" from a physical theory? Dewar (2019) presents two ways to undertake such a task: First, one could move to a *reduced* version of the theory, where the models of the reduced theory are specified only in terms of structure that is invariant under the symmetries of the original theory. Second, one could move to a *sophisticated* version of the theory, where one defines additional maps between models of the original theory that preserve the structure invariant under the relevant symmetries such that symmetry-related models can be regarded as isomorphic. Dewar argues that despite these alternatives attributing the same structure to the world, the sophisticated version can have explanatory benefits over the reduced version.

Here, I provide a different argument in favor of sophistication: the models of a sophisticated theory have further resources than the models of a reduced theory for representing additional detail about physical situations such that the sophisticated theory can draw more physical distinctions than the reduced theory. While it has been argued elsewhere that isomorphic models can be used to represent distinct situations, these arguments do not directly show that the reduced version of a theory is representationally lacking. Indeed, if the reduced version of a theory posits the same structure as the sophisticated alternative, how can the sophisticated version represent a greater number of physical distinctions?

<sup>&</sup>lt;sup>1</sup> For instance, Belot (2018); Fletcher (2020); Roberts (2020).

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I will argue that this tension can be resolved by considering more carefully the ways that isomorphic models can be used to represent distinct situations. I present a division between two kinds of structure in the context of a theory that I call theoretical structure and auxiliary structure, and I demonstrate that auxiliary structure can be used to play a representational role in the sophisticated theory in a way that is absent in the reduced theory. In particular, I will argue that the sense in which isomorphic models of a sophisticated theory can represent distinct situations is that one has the freedom to define auxiliary structure within these models that can be used to represent a physical standard of comparison. It is the ability to fix this additional structure that may be lacking in the reduced version of the theory, since one may not have the same representational freedom associated with auxiliary structure within the models of the reduced theory. This will provide support for the claim that the sophisticated version of a theory can be representationally advantageous to its reduced version, without rejecting the claim that the sophisticated and reduced versions of a theory ascribe the same structure, when this structure refers to theoretical structure.

# 2 Removing excess structure

The background situation is that we have a theory that we believe has excess structure, in the following sense: there are models of the theory that are distinct, or more precisely, *non-isomorphic*, that nonetheless are related by a symmetry.<sup>2</sup> The symmetry between these distinct models is such that we think these models ought to be taken to represent a single physical situation. And yet, in being distinct models, the natural interpretation of the theory is that these models represent distinct physical situations. Therefore, the theory attributes structure to the world that we do not believe corresponds to anything physical.

There are several interpretive assumptions in the background of this argument for the presence of excess structure.<sup>3</sup> For one, there is an assumption that when models are non-isomorphic, they cannot be interpreted as representing the same physical situation. This relies on taking seriously the claims of the theory in the sense that differences between the models of the theory are taken to correspond to differences in the physical situations represented by such models. Inasmuch as we take the aim of physical theories to be that they capture the structure that the world and its parts have, this seems to be a natural assumption. Notice, however, that the argument does not seem to require that we take our theory to be a full description of all the structure of the world; only that the structure it does describe corresponds to structure in the world. We will return to this point later on.

Given that we find ourselves in such a situation, the aim is to find a way to adapt the theory such that it no longer has excess structure. In a recent paper, Dewar (2019) presents two ways to do this.

The first is *reduction*. Reduction alters the syntax of a theory by defining the theory in terms of quantities that are invariant under the relevant symmetries. This effectively equivocates between the non-isomorphic models related by the relevant symmetries in the original theory such that they correspond to identical models in

<sup>&</sup>lt;sup>2</sup> Precisely how to characterize this symmetry is debatable. Broadly, the symmetry is such that it preserves the dynamics of the theory. See Belot (2013) for discussion.

<sup>&</sup>lt;sup>3</sup> For further discussion, see Earman (2004); Ismael and van Fraassen (2003); Dasgupta (2016).

the reduced theory. This is arguably the standard method for removing excess structure from a theory.

The second is *sophistication*. Sophistication keeps the syntax of the theory the same but instead alters the semantics of the theory with respect to which the theory is interpreted so that the relevant symmetries act as isomorphisms under the new semantics.<sup>4</sup> Inasmuch as isomorphisms preserve the structure of the models of a theory, the natural interpretation of isomorphic models is that they correspond to a single physical situation. Therefore, one has removed excess structure not by redefining the models of the theory, but by redefining the symmetries of the theory.<sup>5</sup>

To see this distinction in action, we will consider an example from Dewar (2019) of a theory that describes physical situations in which every object is "handed." Call this theory  $T_H$  in the signature  $\Sigma = \{L, R\}$  and take it to be described by the following axioms, which say that every object is (excusively) either "left" or "right" handed:

$$\forall x (Lx \vee Rx),$$
  
 $\forall x \neg (Lx \wedge Rx).$ 

This theory has excess structure in the following sense. Consider, for example, two models of the theory with the same domain where in one model all objects have the property L while in the second model all objects have the property R. These models are non-isomorphic within  $T_{\rm H}$ ; the map taking L to R and vice versa doesn't preserve the extension of these properties. And yet, these models are symmetry-related: intuitively, if we imagine the physical situation represented by one of these models, it is indistinguishable from the physical situation represented by the other model; both models correspond to a situation where everything is handed in the same way. More generally, any two models that are related by "flipping" the handedness for each object are symmetric in this way. Therefore, there are distinct models in the theory that are symmetry related, and this indicates that  $T_{\rm H}$  has excess structure.

To describe a reduced version of the theory,  $T_{\rm R}$ , we need to define the theory in terms of quantities that are invariant under the relevant symmetry. An obvious candidate is a congruence relation, Cxy, that specifies whether two objects have the same handedness or not. It can be defined via

$$\forall x \forall y (Cxy \Longleftrightarrow ((Lx \land Ly) \lor (Rx \land Ry))).$$

We can then specify  $T_R$  in terms of axioms for Cxy that say that this relation is an equivalence relation with two equivalence classes.

To define a sophisticated version of the theory,  $T_S$ , we need to alter the semantics of  $T_H$  such that the symmetry-related but non-isomorphic models of  $T_H$  are treated as isomorphic. We can do this by defining an invertible homomorphism h from a model m to a model n that consists of a map  $h_1:|m|\to|n|$  and a bijection  $h_2:\{L,R\}\to\{L,R\}$  such that

<sup>&</sup>lt;sup>4</sup> One can spell out sophistication as introducing additional maps ("arrows") into the category of models of the theory and specifying their inverse and compositions with other maps. This procedure is described in Weatherall (2016), although Dewar (2019) coins the term "sophistication" for this procedure.

<sup>&</sup>lt;sup>5</sup> There is a more fine-grained distinction that Dewar gives between "internal" and "external" sophistication, where the difference is whether one uses new mathematical tools to define the models of the theory such that there is a natural isomorphism between them, or whether one stipulates what counts as an isomorphism. This distinction will not be crucial for the purposes of this paper.

$$h_1[L^m] = (h_2(L))^n,$$
  
 $h_1[R^m] = (h_2(R))^n.$ 

This homomorphism is such that it can map "left" hands to "right" hands across models (and vice versa). In this way, the homomorphism need not preserve the extension of the properties given by L and R and so there is no longer a well-defined notion of trans-model identity for the terms "left" and "right." Thus, while models related by a change in handedness for all objects were non-isomorphic in  $T_{\rm H}$ , they are isomorphic in  $T_{\rm S}$ .

## 2.1 Dewar's two claims

Dewar (2019) makes two central claims regarding the comparison between the reduced and sophisticated versions of a physical theory.

- 1. The reduced and sophisticated versions of a theory are theoretically equivalent.
- 2. The sophisticated version of a theory has explanatory benefits compared to the reduced version.

To unpack the first claim, we need a better grasp of what it is for two versions of a theory to be "theoretically equivalent." Dewar (2019) argues that the reduced and sophisticated versions of a theory are equivalent in the sense of *categorical equivalence*:

Take the category of models of the reduced theory to be  $Mod(T_R)$  and the category of sophisticated models to be mod(T). Then, there is a (reasonable) functor  $F: mod(T) \to Mod(T_R)$  that is full, faithful, and essentially surjective.

Categorical equivalence as a notion of theoretical equivalence has been defended elsewhere, but the important feature of categorical equivalence here is that it captures the relationship between models; in particular, the equivalence between models that are stipulated to be isomorphic in a sophisticated theory.

Turning to the second claim, Dewar (2019) says that:

The reduced theory treats the invariant quantities Q as primitives; this means that if some  $q \in Q$  obeys some non-trivial condition as a result of its definition (in the unreduced theory), it must be asserted to obey that condition (in the reduced theory) as a simple posit. (p. 496)

In other words, the reduced theory comes at an explanatory loss in the sense that the reduced theory must stipulate certain conditions that fall out naturally from the unreduced theory. Since the sophisticated version of the theory does not change the syntax of the theory, it does not come with the same loss.

In the handedness theory, for example, there is a fact in the sophisticated theory that must be assumed in reduced theory: the claim that the congruence relation  $C_{xy}$  is symmetric. In the reduced theory, this claim is one of the axioms, while in the

<sup>&</sup>lt;sup>6</sup> Following footnote 5, this is the external way of defining the sophisticated theory.

<sup>&</sup>lt;sup>7</sup> See Weatherall (2019) for an overview, and references therein.

sophisticated theory, it follows automatically from the axioms. Therefore, on the above account, the sophisticated theory explains this claim better.

There are some immediate worries regarding the explanatory account. First, one might question the significance of some fact being a posit in a theory rather than a consequence of a theory's axioms for the purpose of assessing the overall quality of some theory. Second, one might argue that the reduced theory also has explanatory benefits. For example, while the equivalence between certain models is stipulated in the sophisticated theory, it follows automatically if one takes the physical structure to be that given by the reduced theory. If both the reduced and sophisticated versions of a theory have explanatory benefits, how do we weigh them up to determine which theory is "better?"<sup>8</sup>

In what follows, I will not aim to resolve these worries. Instead, I will provide an alternative reason to prefer sophisticated theories and suggest that it is a more robust argument in favor of sophistication by being less susceptible these worries. One might see this alternative reason as supporting a version of the explanatory account; I will discuss this possibility briefly at the end of section 3.

# 3 The representational benefit of sophistication

The alternative reason to prefer sophisticated theories is that they can have *representational* benefits: Isomorphic models in a sophisticated theory that correspond to identical models in a reduced theory can be used to represent distinct physical situations in a way that the corresponding reduced models cannot.

Varieties of this point have been made previously. For one, there is a strand of literature on the empirical significance of symmetries that aligns closely with the idea that for representing *subsystems*, symmetry-related models can characterize empirically distinct situations. Meanwhile, Belot (2018) and Fletcher (2020) argue that there is a sense in which isomorphic models can be said to generate distinct possibilities through the maps that relate them. Finally, the partial observables approach to gauge variables pioneered by Rovelli (2004) presents a picture under which symmetry-variant features of a theory can be representationally useful. However, I take the novelty of my approach to be the focus on solving the following puzzle:

**Puzzle:** How can a sophisticated theory have representational advantages compared to a reduced theory if they are theoretically equivalent, and so have the same content?

My resolution to this puzzle draws on a distinction between two kinds of structure that one can define in the context of a theory, which I call "theoretical structure" and "auxiliary structure" respectively.

**Theoretical structure** is the structure that a theory attributes to the world in virtue of its "invariant" content: the content that is invariant under isomorphisms of the models of the theory. In other words, when models are isomorphic, they are

<sup>&</sup>lt;sup>8</sup> Dewar (2019, fn. 27) notes that his point is only that reduction has *some* explanatory deficiency, but the question of whether there is a stronger way to argue for sophistication still stands. See also Martens and Read (2021).

<sup>&</sup>lt;sup>9</sup> See in particular Wallace (2022).

equivalent in terms of theoretical structure. Moreover, one can exemplify the theoretical structure through the equivalence classes of the theory. This is indeed the standard way of explicating mathematical structure, and it is the structure that notions such as "categorical equivalence" aim to capture.

Auxiliary structure is the structure that one can define in the models of a theory in virtue of the way that one characterizes the theoretical structure. The auxiliary structure goes past the theoretical structure in that the mathematical tools one uses to characterize the theoretical structure may have further resources, such that one can differentiate more structure than simply the theoretical structure. In particular, it is only through auxiliary structure that one can talk about differences between isomorphic models. Of interest here is precisely the auxiliary structure that goes past the theoretical structure in this way.

To see this distinction in the handedness theory example, consider a model M of  $T_H$  consisting of two objects labelled a and b where  $R^M = a$  and  $L^M = b$ . Now consider a second model N of the theory that consists of permuting the domain of the first model and pushing forward the properties such that  $R^N = b$  and  $L^N = a$ . These two models agree on theoretical structure since they agree on how many objects have each handedness property. However, there is a difference between the models that one can describe using auxiliary structure: in model M object a is right-handed, while in model N, a is left-handed (and vice versa for object a).

While reference to the domain of the models is one place where auxiliary and theoretical structure can diverge, the importance of these two kinds of structure in the handedness theory example arises when thinking about how they differ between the original, reduced, and sophisticated theories. In  $T_{\rm H}$ , theoretical structure and auxiliary structure both include handedness structure, i.e. structure through which one can define "left" and "right" hands as distinct. In  $T_{\rm R}$ , neither theoretical structure nor auxiliary structure include handedness structure; while one can say whether hands are congruent or not, one cannot say which are "left" or "right." In  $T_{\rm S}$ , theoretical structure and auxiliary structure diverge in a special way: the theoretical structure does not include handedness structure but the auxiliary structure does, since, unlike in the reduced theory, one has access to the properties L and R in describing the models of the theory.

Usually, theoretical structure is understood to give the content of a theory and auxiliary structure is regarded as mere descriptional redundancy; it has to do with one's pragmatic choice of representation. On this understanding, isomorphic models have equivalent content and any differences between them described by auxiliary structure do not have any bearing on the representational capabilities of these models. However, I will argue against this view: auxiliary structure is not just descriptional redundancy but can have a representational role, and this is true even if one understands the theoretical structure to give the content of the theory. This will provide support for the claim that the sophisticated version of a theory can have representational benefits over the reduced version. I argue for this position through three claims.

 $<sup>^{10}</sup>$  In addition, one cannot reintroduce L and R in the reduced theory by fixing an element and saying any other element is L if it is congruent with the fixed element and R otherwise, since the theory doesn't allow one to identify a specific element.

**Claim 1:** Isomorphic models can equally well represent a single physical situation in virtue of being equivalent in terms of theoretical structure.

This claim captures one made elsewhere that isomorphic models have the same "representational capacities" (Weatherall 2018; Fletcher 2020). However, the emphasis here is on the role of theoretical structure—it is the theoretical structure that determines the extent to which models are able to represent some physical situation. The reason is that it is the theoretical structure that captures the physical content of the models of the theory, in the sense of being the structure attributed by the theory to the world.

The second claim is that there is an importantly different sense in which isomorphic models in a sophisticated theory can be physically distinguished.

**Claim 2:** Auxiliary structure can be used to provide physically relevant distinctions between isomorphic models in a sophisticated theory.

In order to unpack this claim, let us return to the models of the sophisticated theory that consist of only "left-handed" objects and only "right-handed" objects respectively. One might argue that there is a natural sense in which they can represent distinct situations: they can represent two physical situations where the objects are in *different* congruence classes. But how is this compatible with the fact that these models have the same invariant structure that captures the physical content of the theory?

One response is to say that one could simply stipulate the interpretation of the models to be different, or, following Wallace (2022),

symmetry-related configurations can be understood as representing different possible configurations if we hold fixed the choice of representational convention. (p. 337)

So in the above example, one might say that one could stipulate a standard of "left" across the models such that they represent different physical situations relative to this standard. But where does this representational convention come from, physically? In particular, since the theoretical structure is not sensitive to such choice, in what sense can one impose it on the models in order to distinguish the interpretation of these models?

Here is where auxiliary structure comes into play. The sense in which one can impose a physical representational convention across isomorphic models of a sophisticated theory, I argue, is that one can use the auxiliary structure of the theory, through which one can distinguish these models descriptionally, to represent further details about the physical situations represented by these models. When one can use auxiliary structure to represent an additional system that acts as a reference frame, one can distinguish the situations represented by isomorphic models that differ relative to the fixed auxiliary structure. For example, in the sophisticated models discussed above, one can use the auxiliary structure corresponding to the ability to define left- and right-handed objects to define a new object that is stipulated to be "left handed" in both models, such that, relative to this hand, the models are distinct;

one corresponds to the world where everything is congruent to this new hand, and the other corresponds to the world where everything is not congruent to the new hand. This use of auxiliary structure as defining a reference frame allows one to impose a representational convention that has a physical interpretation.

However, this would be of no interest if auxiliary structure could play the same role in the reduced theory. This leads to the final claim.

**Claim 3:** Compared to the sophisticated theory, auxiliary structure in a reduced theory is unable to provide the same physically relevant distinctions between isomorphic models.

The reasoning is as follows. The reduction process equivocates between isomorphic models in the sophisticated theory that are non-isomorphic in the original theory; they correspond to identical models in the reduced theory. Therefore, while one could say that certain isomorphic models in the sophisticated theory correspond to distinct physical situations by using auxiliary structure to define a reference system, one cannot say the same of the corresponding identical models in the reduced theory.

To see this clearly, consider the same models as before and consider adding a new object that is handed in some way. We do not have the resources to stipulate that it represents a "left" or "right" hand in the reduced theory; we can only stipulate that it represents, for example, some hand congruent to all the hands in a model. But now, inasmuch as the models are identified in the reduced theory, the additional structure will not be able to distinguish these models by fixing its interpretation across them; if it is congruent to the hands in the first model, it is congruent in an identical model. Therefore, while physically relevant distinctions could be drawn between isomorphic models in the sophisticated theory using auxiliary structure, one cannot draw such distinctions in the corresponding reduced models.

To say that the additional hand is congruent to the objects in one model but not in an identical model, one would need to stipulate some additional fact about the difference between the two models, namely that all the hands in one model are in a different equivalence class to those of the other. But this fact can only be specified when talking about the models as subsets of a "larger" model that includes the objects of both models, where the difference between the congruence of the objects can be specified in terms of *theoretical* structure. But this move is something that can also be made in the sophisticated theory; what is lacking in the reduced theory is the ability to distinguish these models without stipulating additional inter-model relations that rely on theoretical structure.

The combination of these three claims highlights that isomorphic models in a sophisticated theory play a multi-faceted role: they can be used to represent a single physical situation (Claim 1), and they can be used to represent distinct situations through a physical interpretation of the auxiliary structure as, for example, a fiducial system (Claim 2).<sup>11</sup> Inasmuch as this multi-faceted role is beneficial, Claim 3

<sup>&</sup>lt;sup>11</sup> The fact that a physical interpretation is given to auxiliary structure does not imply that it should be promoted to theoretical structure. If one did so, one would no longer be able to use the models to represent a single situation, and one would return to the issue of a theory with excess structure.

demonstrates that the sophisticated version of a theory can have a representational advantage over the reduced version. Moreover, these three claims are compatible with the sophisticated and reduced theories being theoretically equivalent in the sense of attributing the same theoretical structure. We therefore have solved the puzzle stated earlier: there is no tension between a sophisticated theory having representational benefits while also being theoretically equivalent to a reduced theory.

This representational argument for sophistication is arguably more robust than the explanatory argument given by Dewar (2019), for the following reasons. First, one can point to precisely why a theory is superior if its models have the resources to represent a greater number of physical distinctions than the models of another theory: inasmuch as these physical distinctions are ones that one thinks a theory ought to capture, a theory whose tools prevent one from representing them is lacking. Second, unlike the fact that one can point to explanatory benefits of the reduced theory, there is not a corresponding representational benefit that can attributed to the reduced theory, inasmuch as reduction always constrains auxiliary structure. Therefore, the two worries raised previously for the explanatory account are bypassed.

However, the representational argument is aligned with the explanatory argument in the sense that part of the explanatory benefit of sophistication might be seen to come from its representational benefits. For example, consider the fact that two subsystems that each consist only of congruent hands can be such that their combined system does not consist only of congruent hands. This fact can be explained in the sophisticated theory through the difference in auxiliary structure between the representation of the subsystems: using auxiliary structure as a physical representational convention distinguishes the subsystems even though the models are equivalent in terms of theoretical structure. In the reduced theory, there are no relevant differences in auxiliary structure between the models representing these subsystems, and so the difference can only be given by asserting further statements about the relation between the subsystems. Therefore, one might argue, this fact falls out naturally from the sophisticated theory but must be stipulated in the reduced theory. And so, the representational power of sophisticated theories lends itself to explanatory benefits.

#### 4 Conclusion

We began with discussing the question of how to get rid of excess structure. The idea was that we should care only about what is invariant under certain symmetry transformations, and so we should remove any features of the theory that are not invariant. But in fact, we have seen that what varies under a symmetry transformation can be representationally useful when it is part of the *auxiliary structure* of a theory. And so, while it might be true that what we care about when talking about *theoretical structure* are the features invariant under symmetry transformations, we have shown that what might be regarded as "surplus" from this perspective need not be surplus from a wider perspective that includes the representational role of auxiliary structure.

In discussions on theory building and excess structure, it is often assumed that the only consideration is that of determining theoretical structure, where the way that one characterizes this structure is something that has only pragmatic value. This

obscures the role of auxiliary structure and leads to confusion regarding cases where some structure appears "surplus" and yet also seems to play an essential role. If we consider theory building to not just be about correctly characterizing theoretical structure, but also auxiliary structure, then this suggests that closer attention ought to be paid to the choices of auxiliary structure and the physical implications these choices have.

In light of this, arguments about excess structure *do* require consideration of whether one takes a theory to be a full description of all of the structure in the world: if one wants to allow for a theory to represent incomplete physical situations such as subsystems or situations relative to some reference frame, then certain kinds of auxiliary structure may be indispensable in a way that they are not if the theory is only used to represent the whole universe. We have discussed one example in this paper; however, further work is necessary to characterize precisely the choices of auxiliary structure for some theory and the way that one associates representational aims with these choices.

### References

Belot, Gordon. 2013. "Symmetry and Equivalence". In *The Oxford Handbook of Philosophy of Physics*, edited by Robert Batterman, 318–39. Oxford: Oxford University Press.

Belot, Gordon. 2018. "Fifty Million Elvis Fans Can't Be Wrong". Noûs 52 (4):946-81. https://doi.org/10. 1111/nous.12200

Dasgupta, Shamik. 2016. "Symmetry as an Epistemic Notion (Twice Over)". The British Journal for the Philosophy of Science 67 (3):837-78. https://doi.org/10.1093/bjps/axu049

Dewar, Neil. 2019. "Sophistication about Symmetries". The British Journal for the Philosophy of Science 70 (2):485-521. https://doi.org/10.1093/bjps/axx021

Earman, John. 2004. "Laws, Symmetry, and Symmetry Breaking: Invariance, Conservation Principles, and Objectivity". *Philosophy of Science* 71 (5):1227–41. https://doi.org/10.1086/428016

Fletcher, Samuel C. 2020. "On Representational Capacities, with an Application to General Relativity". Foundations of Physics 50 (4):228-49. https://doi.org/10.1007/s10701-018-0208-6

Ismael, Jenann and Bas C. van Fraassen. 2003. "Symmetry as a Guide to Superfluous Theoretical Structure". In *Symmetries in Physics: Philosophical Reflections*, edited by Katherine Brading and Elena Castellani, 371–92. Cambridge: Cambridge University Press. https://doi.org/10.1017/CB09780511535369.024

Martens, Niels C. M. and James Read. 2021. "Sophistry about Symmetries?" Synthese 199 (1-2):315-44. https://doi.org/10.1007/s11229-020-02658-4

Roberts, Bryan W. 2020. "Regarding 'Leibniz Equivalence". Foundations of Physics 50:250–69. https://doi.org/10.1007/s10701-020-00325-9

Rovelli, Carlo. 2004. Quantum Gravity. Cambridge: Cambridge University Press.

Wallace, David. 2022. "Observability, Redundancy, and Modality for Dynamical Symmetry Transformations". In *The Philosophy and Physics of Noether's Theorems: A Centenary Volume*, edited by James Read and Nicholas J. Teh, 322–53. Cambridge: Cambridge University Press. https://doi.org/10.1017/9781108665445.014

Weatherall, James Owen. 2016. "Are Newtonian Gravitation and Geometrized Newtonian Gravitation Theoretically Equivalent?" *Erkenntnis* 81:1073–91. https://doi.org/10.1007/s10670-015-9783-5

Weatherall, James Owen. 2018. "Regarding the 'Hole Argument". The British Journal for the Philosophy of Science 69 (2):329-50. https://doi.org/10.1093/bjps/axw012

Weatherall, James Owen. 2019. "Part 2: Theoretical Equivalence in Physics". *Philosophy Compass* 14 (5): e12592. https://doi.org/10.1111/phc3.12591

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